A new mortality function for New Zealand Douglas-fir

M.O. Kimberley, R.L. Knowles

A NEW MORTALITY FUNCTION FOR NEW ZEALAND DOUGLAS-FIR

Report No. 55   February 2007

M.O. Kimberley and R.L. Knowles

ABSTRACT

A new mortality function based on Reineke's 'line of self-thinning' concept has been developed for New Zealand Douglas-fir. The model was derived using data from more than 1,300 permanent sample plots from throughout New Zealand. All measurements made since 1970 were used in the development of the model. Reineke's 3/2 power rule is defined on a graph of log (Stocking) versus log(DBH), and the thinning line is invariant with site. However, analysis of the data indicated that $DBH^2 \times MTH$ performed better than DBH as the tree size variable in the model. The self-thinning line was found to vary with site with more productive sites able to carry higher stockings of trees of a given size compared to lower productivity sites. Mortality levels are therefore lower for stands of similar stocking and tree size on sites with high values of $SBAP$, and to a lesser extent, SI. The model can be used in Douglas-fir throughout New Zealand. However, there is some evidence that it over-predicts mortality slightly in the South Island and under-predicts in the North Island (by about plus and minus 12% respectively). In the implementation of the model in the Douglas-fir Calculator, the user can enter a percentage adjustment to account for such differences.

INTRODUCTION

Models for predicting mortality in forest stands are important components of forest growth modelling systems. For New Zealand Douglas-fir, an interim mortality function was developed several years ago for use with the Douglas-fir National Growth Model. This model was described briefly at the 2001 meeting (Meeting Proceedings, page 49), and is currently implemented in the Douglas-fir Calculator. This report describes a new and improved version of this mortality function.

Mortality in forest plantations can be categorised into three types:
- Attritional mortality – low-level mortality not due to rare catastrophic events in stands which are not at excessively high stockings.
- Catastrophic mortality – mortality occurring as a result of rare and catastrophic events such as major storms, fires, etc.
- Competition-induced mortality – this occurs in highly-stocked stands when competition becomes intense causing the smaller, less vigorous trees to die.

There is some debate as to whether mortality functions implemented in growth modelling systems should predict the effects of all three types of mortality. Arguably, it is better to handle catastrophic mortality independently of the growth model. For example, it could be assumed that there is a certain probability of a stand being destroyed or severely damage as a result of a major storm event. These risks could be considered separately when assessing the likely benefits and risks from long-term activities such as forestry. For predicting the likely stocking at the end of a rotation, and its effect of mean stem size, it is probably better to ignore such catastrophic events. Growth Model mortality functions should therefore predict the effects of attritional and competition-induced mortality only.

However, it can be difficult to decide whether mortality is 'catastrophic' rather than attritional or competition-induced. For example, to eliminate catastrophic mortality, Garcia removed observations from his model dataset when there were more than two tree deaths during a growth increment (Goulding, pers. com.). However, when we applied this criterion to the New Zealand Douglas-fir data, it was apparent that growth increments with more than two tree deaths, tended to be clustered around the 'self-thinning' boundary line and were clearly mainly caused by competition rather than being due to catastrophic events. Therefore, it was decided not to attempt to screen out plots on the basis of catastrophic mortality when developing the model described in this report. The only mortality excluded from the analysis was of trees identified in the database as being 'windthrown', but these form only a minor proportion of total mortality in the database. However, although some catastrophic mortality may be included in our data, it is likely that plots in seriously damaged or destroyed stands will have often been abandoned without re-measurement. Therefore, to some extent catastrophic mortality may have been eliminated by default from the database.

Reineke (1933) noticed that in fully-stocked plots, a graph of log(Stocking) against log(*DBH*) typically has a straight line with slope approximating -3/2 (the 3/2 power rule). This result was found to hold generally for a wide range of forest species in North America. In order for this relationship to hold, there must be a rise in mortality when the mean diameter of a stand approaches this self-thinning boundary. Reineke also developed a Stand Density Index (*SDI*) which categorises stands in terms of their distance from this self-thinning line. Reineke believed that each species could attain a certain maximum *SDI*, which is largely invariant to site. A graph of log(Stocking) versus log(*DBH*) for our New Zealand Douglas-fir data clearly shows the

existence of a Reineke-type relationship. It was therefore considered desirable to incorporate this concept into the new mortality model, as suggested by Dave Marshall, the mensurationist based at US Forest Service, Olympia, Washington, and more recently with Weyerhaeuser at Tacoma. This report describes the model and the methodology used to develop it.

DATA

The data used in this analysis consisted of measurements from permanent sample plots (PSPs) from throughout New Zealand. Only measurements made since 1970 were included in the analysis, and plots with less than 150 stems/ha were excluded. Mortality, expressed as an annual percentage, was calculated for each growth increment using:

$$[1] \quad M = 100 \times \left[1 - \left(N_1 / N_0 \right)^{1/\Delta T} \right]$$

where N_0 and N_1 are stocking in stems/ha at the beginning and end of the increment period, and ΔT (= T_1 - T_0) is the length of the increment in years. A total of 8,004 growth increments from 1,316 PSPs were available for analysis (Table 1). The mean increment length was 2.2 years. The average mortality in the complete data set was just over 0.7% per annum. When trees identified in the database as windthrown were excluded from the calculation, the mean mortality reduced to slightly over 0.6%.

Table 1. Data used to develop mortality function.

Region	Number of plots	Number of increments	Mean mortality (%)	Mean mortality excluding windthrow (%)
Bay of Plenty	425	3566	0.64	0.56
Waikato	50	191	0.75	0.71
Gisborne	6	51	0.94	0.89
Hawkes Bay	148	598	1.74	1.72
Wairarapa/Manawatu	23	90	1.14	1.13
Wellington	15	27	1.47	1.07
Nelson	226	1358	0.62	0.56
Marlborough	13	62	0.60	0.47
West Coast	21	107	0.86	0.20
Canterbury	210	1086	0.40	0.33
Otago	127	641	0.52	0.44
Southland	52	227	0.88	0.76
Total	1,316	8,004	0.70	0.62

MODELLING METHODOLOGY

A simple model of attritional mortality model which assumes a constant annual mortality rate, M, which is unaffected by stand parameters or environmental factors, can be represented by the following equation:

$$[2] \quad N_1 = N_0 (1 - M/100)^{\Delta T}$$

where N_0 and N_1 are stocking in stems/ha at the beginning and end of the increment period, and ΔT is the length of the increment in years. To incorporate the Reineke relationship into a mortality function, the mortality rate M must increase for stands positioned near the self-thinning boundary. This effect can be achieved using the following function:

$$[3] \quad M = 100 \times (a + b \times SDI^c)$$
$$\text{where} \quad SDI = N \times D^d$$

In this above function, M is percentage annual mortality, and SDI is the Stand Density Index as defined by Reineke. The SDI is defined in terms of the stocking, N, and the quadratic mean DBH, D, of the stand. The model parameters are a, b, c and d and each have an interpretation: a is the minimum mortality rate and can be regarded as the attritional mortality (possibly including also a component from rare catastrophic events) in stands not subject to serious competition; b represents the intercept of the self-thinning boundary line; c controls the rate at which mortality increases as a stand approaches the boundary; and d is the slope of the self thinning boundary (without the minus sign) which according to Reineke should have a value of about 3/2. Note that to enable a more general form of the relationship, the parameter d was estimated from the data rather than assumed to have a value of 3/2. To use the model for predicting a change in stocking, the mortality predicted by [3] is applied in Model [2] to predict stocking N_1 from the previous stocking N_0 over a time step length ΔT.

When implemented, the model will not be invariant to step length. It is necessary to take account of this when fitting the model to the data. For example, if the model is to be implemented with a step length of one year, the correct procedure would be to fit the model using the above equation, with N and D as the stocking and diameter at the beginning of the increment, only if the great majority of increments were of one year. On the other hand, if the model is to be implemented with a very short step length, the terms N and D in the above equation should be replaced by ($N_0 + N_1$) / 2 and ($D_0 + D_1$) / 2, respectively when fitting the equation to the data. As the National Douglas-fir Growth Model is implemented with a short step length of one month, this approach was adopted.

To estimate the parameters of the combined Models [2] and [3], it was desirable to take account of the distributional form of the chosen dependent variable n_1, the number of stems in a plot at the end of an increment period. This can be assumed to follow a binomial distribution with expected value $N_0 (1 - M/100)^{\Delta T}$, where N_0 is the number of stems at the beginning of the increment, and M is as defined in Equation [3]. The model was fitted using the SAS (Version 9) Nonlinear Mixed Modelling procedure NLINMIX with an allowance for over-dispersion. This fitting procedure automatically adjusts for the effects of increment length, mean mortality level, and plot size on the residual variance.

Several variations of Model [3] were tested. Although Reineke used the ln(N) versus ln(D) graph to establish an appropriate self-thinning boundary, other authors have suggested that tree size parameters other than *D* may perform better. For example, Yoda et al. (1963) suggest using mean plant biomass, while Zeide (1995) suggested that mean diameter at the base of the green crown (*DBGC*) may perform better than *DBH*. We tested the use of mean top height (*H*), D^2H (as a surrogate for biomass), and *DBGC*, in place of *D* in Equation [3]. Estimates of *DBCG* were established for each measurement using the D-fir crown length function and a taper equation.

Reineke believed that the self-thinning boundary was invariant to site for each species. However, other researchers had suggested that the boundary moves to the right on more productive sites, which can therefore carry a higher stocking of trees for a given mean diameter. Two measures of site productivity were tested for inclusion in the mortality model, namely, Site Index (*SI*), and Site Basal Area Potential (*SBAP*). These were derived for each plot used in the analysis using an automated routine developed by Lars Hansen which operates in conjunction with the latest version of the D-fir Calculator. These productivity indices were incorporated into the *SDI* term in Equation [3], for example:

[4] $SDI = \exp(f \times SBAP + \ln N + d \ln D)/1{,}000$

All models were compared using -2(log likelihood) as the goodness of fit criteria. Generally, the model with the smallest value of this statistic has the best fit.

Some authors have suggested that the self-thinning boundary is non-linear. To test this, an additional term in $(\ln D)^2$ was included in Model [3], but this provided no significant improvement in fit. Also, tests of different rates of mortality in different regions of New Zealand were obtained by fitting the model to regional subsets of the data with all parameters other than *b* (or, in some cases, *a*) fixed to their national estimates. The estimates of *b* obtained for each region could be converted into 'multiplier' terms suitable for each region if these were considered necessary. A similar procedure was used to test for differences in mortality rates between measurement years.

RESULTS

In Fig. 1, a plot of ln(Stocking) vs ln(*DBH*) is shown for the PSP data, with each point representing a PSP growth increment. Points are classified into four classes based on mortality. In this graph, the self-thinning boundary is clearly evident, and the mortality increases markedly when a stand approaches this boundary. The corresponding graph using *DBGC* in place of *DBH* (Fig. 2) suggests that this variable is not as useful as *DBH*. Although there is some relationship between ln *N* and ln *DBGC*, many plots not undergoing mortality lie close to the 'self-thinning' line on this graph.
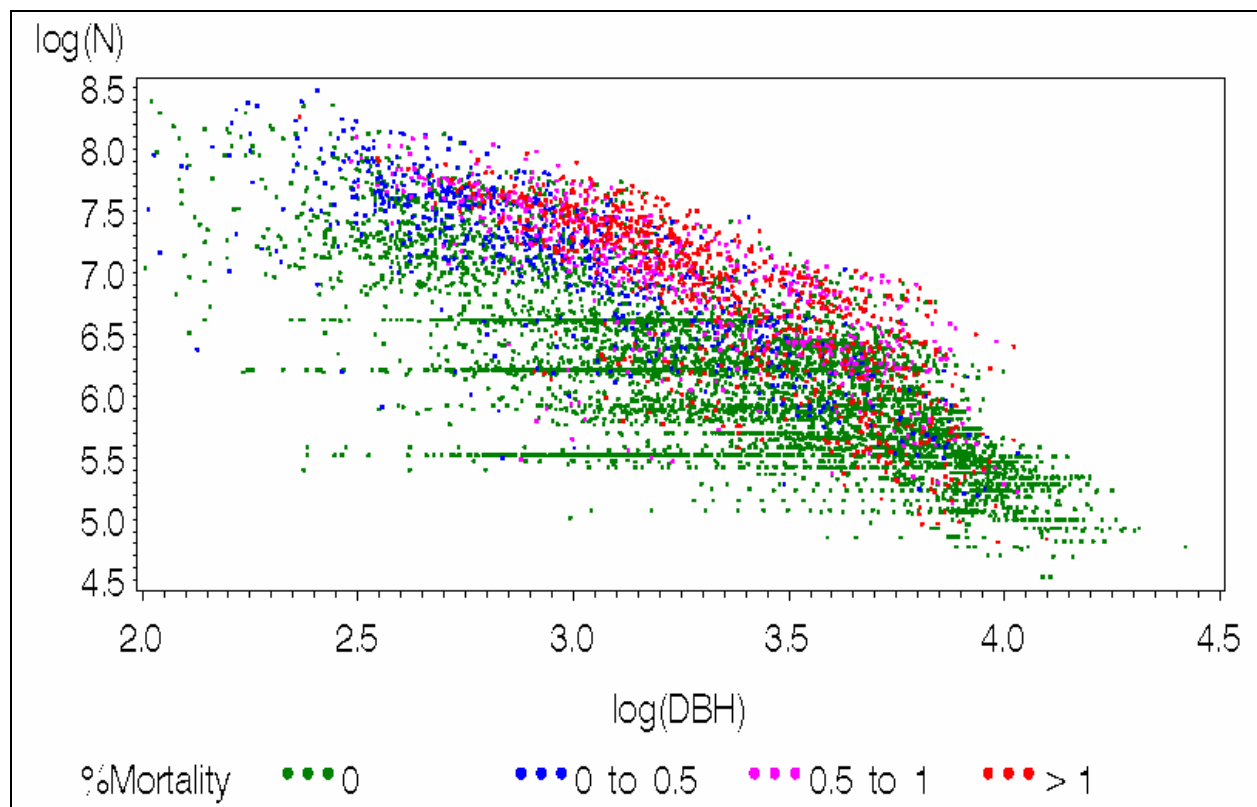
Fig. 1. Annual mortality rate plotted on the ln *N* versus ln *D* graph for New Zealand Douglas-fir.
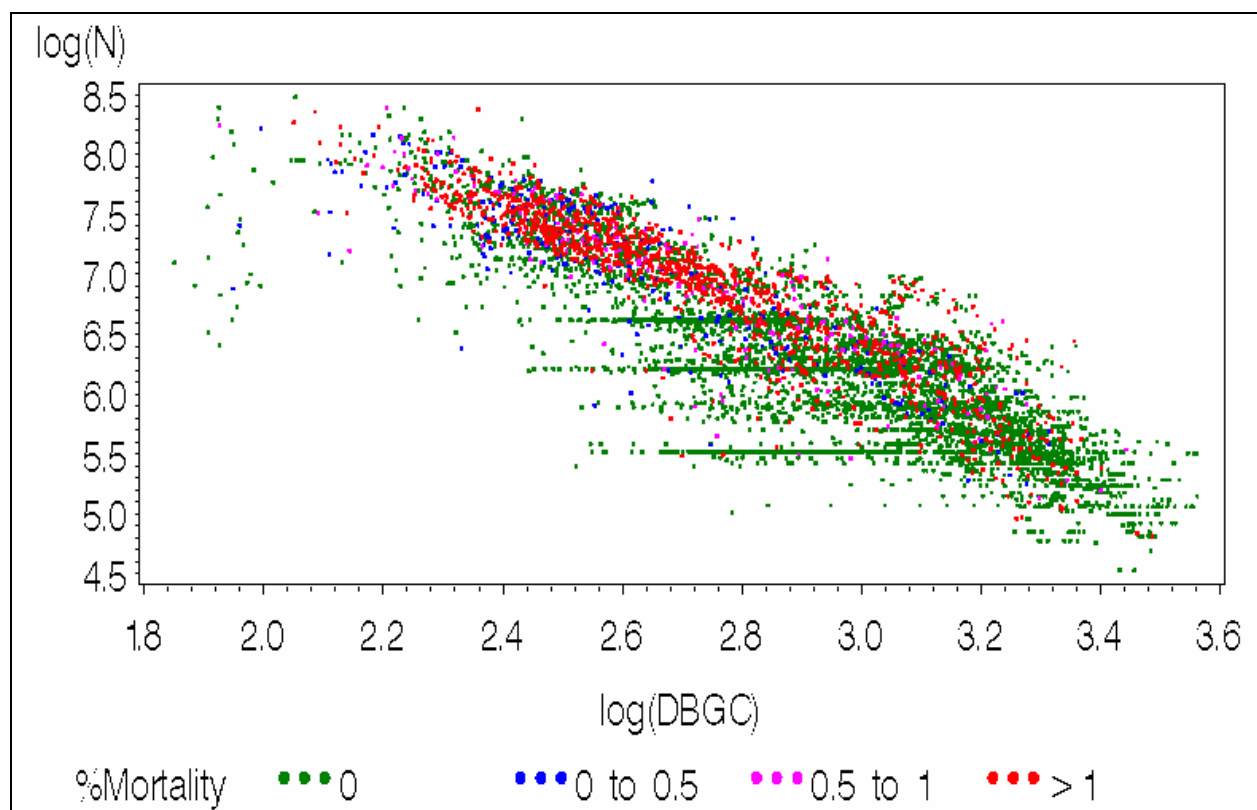

Fig. 2. Annual mortality rate plotted on the ln *N* versus ln *DBGC* graph for New Zealand Douglas-fir.

The fits of the various models tested are given in Table 2. When no site productivity variables were included in the model, mean top height (H) provided the best performing model, with substantially better fit than DBH or D^2H. As expected from Fig. 2, $DBGC$ performed poorly. When $SBAP$ was included in the model there was a large gain in performance demonstrating that the self-thinning boundary is not invariant with site. Site Index gave much less improvement in fit than $SBAP$, but there was a small benefit in including SI in addition to $SBAP$ in the model. When productivity indices were included in the model, DBH performed better than MTH as the tree size variable, but D^2H performed slightly better than DBH.

Table 2. Fit of various mortality models as indicated by -2 log likelihood statistics. Note that in general the smaller the value, the better the fit.

Tree size variable	Productivity variables included in model			
	None	**SBAP**	**SI**	**SBAP & SI**
D	19,207	17,303	19,206	17,303
H	18,643	17,861	18,575	17,816
D^2H	18,709	17,259	18,688	17,241
$DBGC$	21,279	20,150	21,271	20,073

The importance of including site productivity in the model is demonstrated clearly in Fig. 3 where mortality is plotted against SDI based on the basic Reineke Model (Equation [3]), with the data split into three $SBAP$ productivity classes. For the same SDI, mortality is lower on more productive sites and higher on less productive sites. This shows that higher productivity sites can sustain a higher stocking of trees for a given mean size than lower productivity sites, or equivalently, that they can sustain the same stocking of larger sized trees.
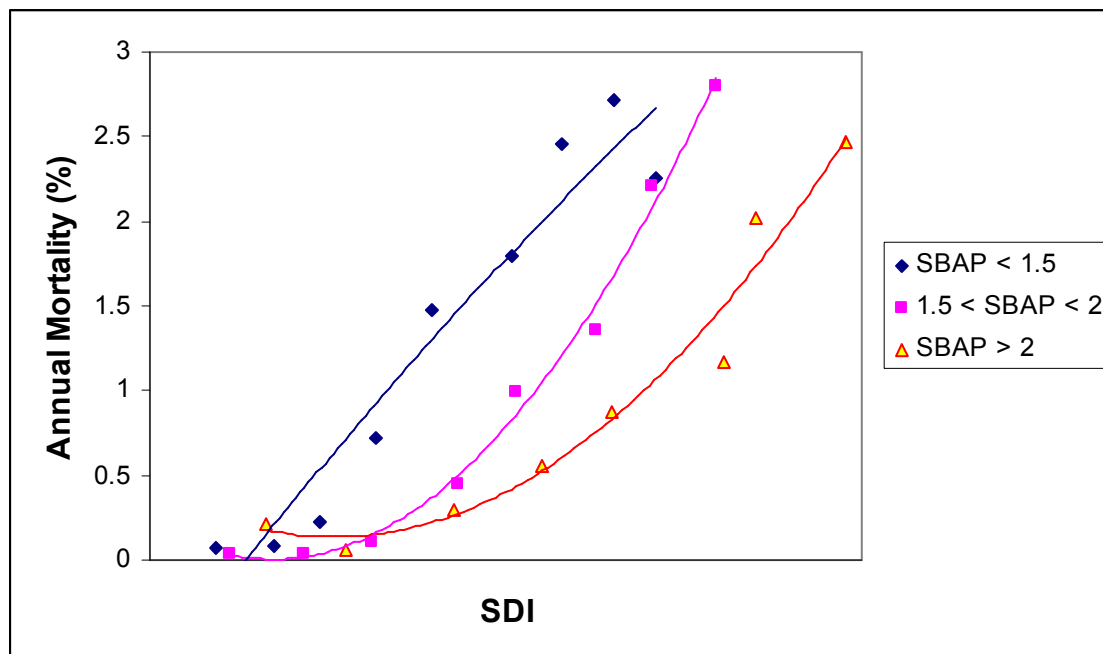

Fig. 3. Mean mortality in plots classified into SDI classes and into three site productivity classes on the basis of $SBAP$. In this graph, a simplified version of SDI was used ($SDI = N \times D^{1.17}$).

The best model was therefore:

$$N_1 = N_0 (1 - M/100)^{\Delta T}$$

[5]   where $M = 100 \times (a + b \times SDI^c)$

and $SDI = \exp(f \times SBAP + g \times SI + \ln(N) + d\ln(D^2 H))/1{,}000$

Parameter estimates for the model are given in Table 3. The attritional mortality parameter, a, is estimated to be zero (it was constrained to be non-negative in the fitting process). The self-thinning boundary slope, d, has a value very different from 1.5 because of the use of the size variable $D^2 H$ in place of D. The equivalent model using D in place of $D^2 H$ has a slope of 1.37 (std. error 0.02), slightly below Reineke's theoretical value of 1.5. Note that both D and H are expressed in metres in this formulation.

Table 3. Parameter estimates of Model [5].

Parameter	Estimate	Standard error
a	0	0
b	0.0452	0.0068
c	1.990	0.043
d	0.4766	0.0080
f	-0.487	0.020
g	-0.00675	0.0022

An examination of the frequency distribution of *SDI* within the dataset showed that it had a maximum of about 1.0, but that very few stands had an *SDI* greater than 0.8 which may be taken to be a more realistic maximum (Fig. 4). The maximum *SDI* in well managed stands will be much lower than this. When we restricted our data to a maximum age of 50 years and a stocking of 800 stems/ha, the highest *SDI* was about 0.5.
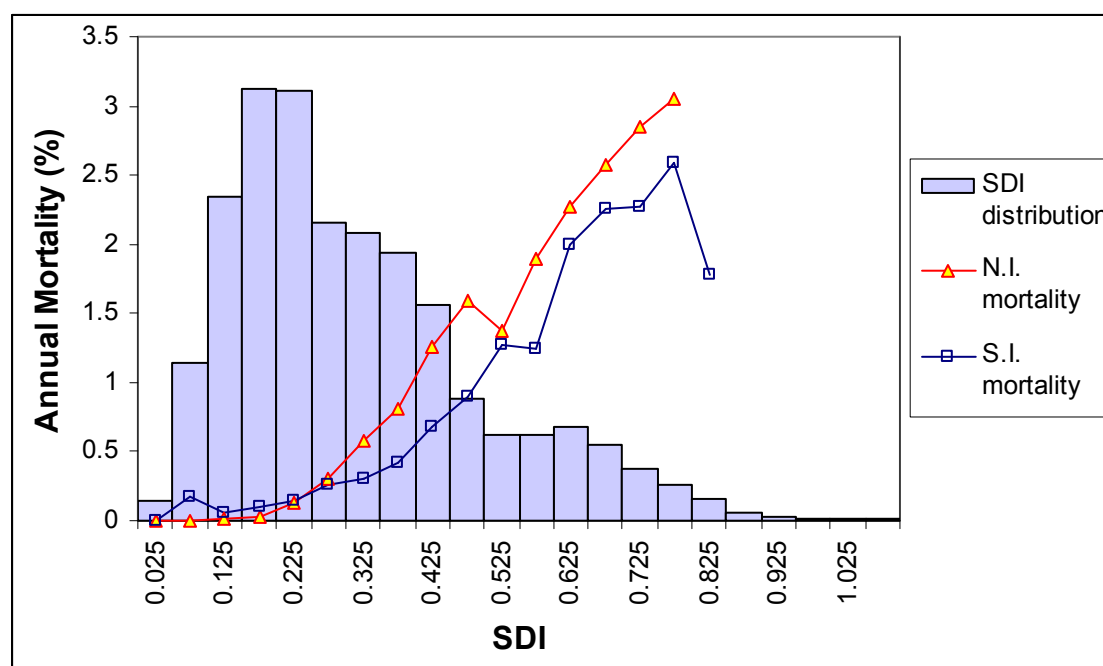

Fig. 4. National frequency distribution of *SDI*, and mean mortality in 0.05 *SDI* steps for North Island and South Island PSPs.

Figure 4 also shows the mean mortality of the data for 0.05 steps in the *SDI* for both the North and the South Islands. As expected, mortality increases with increasing *SDI*. It appears that North Island stands may generally have higher levels of mortality than South Island stands of equivalent *SDI*. The apparent slight reduction in mortality for the highest *SDI* class in Fig. 4 for the South Island data is probably due to the limited number of plots representing this data point.

Although it may be possible for Douglas-fir stands in New Zealand to achieve an *SDI* of 0.8, Fig. 4 shows that high levels of mortality will occur at much lower values. In fact, the mortality begins to rise appreciably when the *SDI* is at a quarter to a third of its maximum. According to Long & Daniel (1990), competition begins at 35% maximum *SDI*, and self-thinning begins at 60% maximum *SDI*. Fig. 4 shows that increased levels of mortality can occur at well below the 60% level, and suggests that 'self-thinning' is a much more gradual developing process than is often believed and does not have a sharp threshold.

To explore the issue of regional variation in mortality, we added an additional multiplier term to the second term in Model [5] so that it became:

[6] $\qquad M = 100 \times \left(1 + R/100\right) \times b \times SDI^{c}$

Note that the *a* parameter is not shown as it has a value of zero (Table 3). The *R* parameter allows this model to predict higher or lower levels of mortality than the standard model. A value of $R = 0$ gives no adjustment while a value of $R = 100$ will double the mortality and a value of $R = -100$ will reduce the mortality to zero. The other parameters in the model retain their estimated values as given in Table 3. This model was fitted to subsets of the data and parameter estimates of *R* obtained for each subset. Fig. 5 shows estimates of *R* for the major geographical regions represented in the database for each decade of data.

There was some tendency for the *R* values of the South Island regions to be negative, and the North Island regions positive, although the pattern was somewhat inconsistent between decades. For example, the highest mortality (50% above model prediction) was in Canterbury during the 1980's although other decades were below average for this region. It appears, however that Nelson had a consistently lower mortality than model predictions.
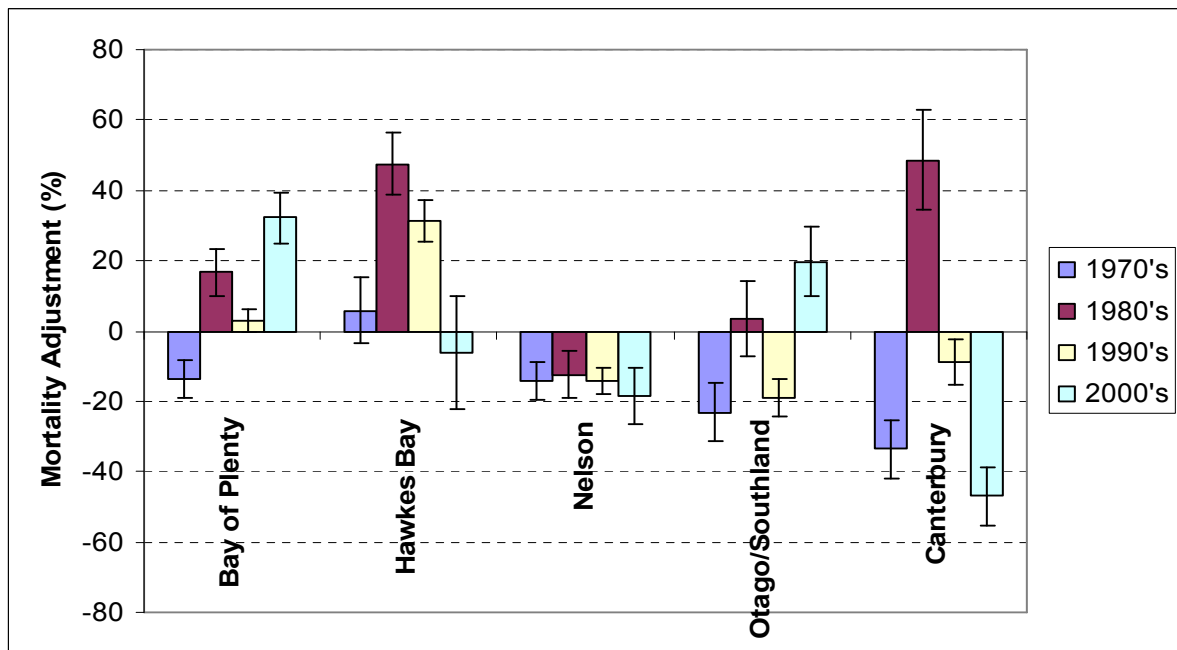
Fig. 5. Mortality adjustments *R* estimated for each of the important regions in the database by decade of measurement. Error bars show standard errors.

The model was also fitted to each year of data separately for the North and South Islands (Fig. 6). This showed that mortality varied considerably between years, but that North Island levels were generally higher than South Island levels. For the North Island, *R* averaged 12 while for the South Island it averaged -13 (i.e., mortality was 12% greater in the North Island and 13% lower in the South Island than predicted by the model). In 27 of the 37 years, the North Island *R* value was higher than the South Island value. This is a statistically significant difference (Sign Test; p = 0.008), although these yearly observations are not statistically completely independent. However, we believe that this provides strong evidence that mortality levels are somewhat higher in the North than the South Island. In the implementation of the model in the Douglas-fir Calculator, the user will be able to enter their own value of *R* as a percentage adjustment to account for such differences if required.
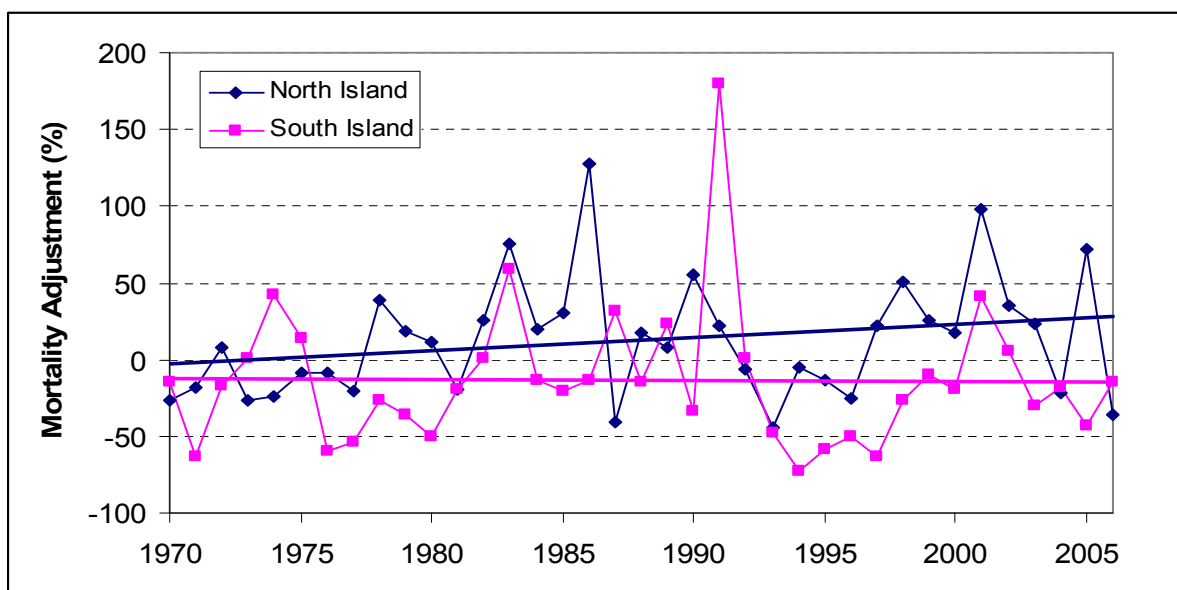

Fig. 6. Mortality adjustments *R* estimated for each measurement year of the South Island and North Island.

CONCLUSIONS

A new mortality function based on Reineke's line of self-thinning concept has been developed for New Zealand Douglas-fir. The model predicts mortality as a function of log(Stocking) and log($DBH^2 \times MTH$). The level of mortality is also influenced by site productivity with more productive sites able to carry higher stockings of trees of a given size compared to lower productivity sites. This effect is incorporated into the model using *SBAP* and *SI*. The model can be used for predicting mortality in Douglas-fir stands throughout New Zealand. However, there is some evidence that it over-predicts mortality slightly in the South Island and under-predicts in the North Island (by about plus and minus 12% respectively). In the implementation of the model in the Douglas-fir Calculator, the user will be able to enter a percentage adjustment to account for such differences.

ACKNOWLEDGEMENTS

REFERENCES

Long, J.N., Daniel, T.W. 1990. Assessment of a growing stock in uneven-aged stands. Wet. J. Appl. For., 5, 93-96.

Reineke, L.H. 1933. Perfecting a stand-density index for even-aged forests. J. Agric. Res., 46 627-638.

Yoda, K., Kira, T., Ogawa, H. and Hozumi, K. 1963. Self-thinning in overcrowded pure stands under cultivated and natural conditions. J. Biol.Osaka City Univ., 14 107-129.

Zeide, B. 1995. A relationship between size of trees and their number. Forest Ecology & Management 72 265-272.