

**F.R.I. PROJECT RECORD**

**No. 3278**

**A COMPARISON of METHODS to PREDICT  
INDIVIDUAL TREE DIAMETER GROWTH**

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# FRI/INDUSTRY RESEARCH COOPERATIVES

## **EXECUTIVE SUMMARY**

In order to evaluate current methods for projecting individual radiata pine tree diameters through time, a series of sample plots were used to compare the predicted diameters against actual measurements. Three methods of projection were compared:

'Simple' method : All trees grow in basal area at the same rate.

'W&H' method : An equation is used to predict the relative basal area of each tree, from which the projected diameter can be calculated. (Woollons and Hayward 1985)

'Manley' method : Diameter increment is estimated directly from relative diameter and stand parameters, with site index as a multiplicative variable.

The variance of the distributions were predicted more accurately by the Manley method whereas the Simple and W&H methods both under-estimated the variance. The under-estimate increases with increasing projection period and stocking.

The method with the most potential of those compared is Manley's method, based on predicting individual diameter increment using stand variables, site index and relative tree size.

In order to develop a robust method for projecting inventory data, a more detailed review of the literature is necessary to obtain an overview of current methods and their efficacy. With this overview and the results of the comparisons made here, we should have a good basis for constructing a reliable method for projecting individual tree diameters obtained from inventory data.

## INTRODUCTION

Most measures of stand parameters are derived by aggregating measurements of trees. In a bounded plot for example, tree counts and diameters are used to form an estimate of stand stocking and basal area. This process of aggregation results in some loss of information, in particular, the relative size of each individual in a list of trees.

Stand-based growth models may project growth accurately, but they are normally driven by aggregated information and so can only disaggregate a projected stand into a stand table in a generalised fashion. Where tree lists are available (as in inventory measurements) it is desirable to use the additional detail to produce a stand table that reflects varying growth rates due to the initial relative sizes of the trees. This would mean MARVL inventory data (Deadman and Goulding 1978, MicroMARVL 1989), for example, could be "grown" forward in a manner more sensitive to the shape of the initial diameter distribution than by simply scaling individual tree basal areas by the stand basal area increment.

Information on tree spacing and inter-tree distances is not usually available when tree lists are formed from inventory data. Furthermore, the distribution of tree sizes within the list can be affected by the plot size used in the inventory from which the list was assembled (Garcia 1988, 1991). Many authors consider that knowledge of tree coordinates does not appear to be critical for predicting individual tree growth (Hann 1981, Munro 1974). Relative size does seem to be important however, which means having a precise and stable estimate of the parameter to which tree size will be related.

In order to evaluate current methods for projecting individual radiata pine tree diameters through time, a series of sample plots were used to compare the projections against actual measurements. Three methods of projection were compared.

## NOTATION

$G$	stand basal area ( $\text{m}^2/\text{ha}$ )
$T$	stand age (years)
$N$	stand stocking (stems/ha)
$\bar{h}_{100}$	mean top height (m)
$S$	site index (m)
$d_i$	breast height (1.4m) diameter over bark of $i$ th tree in the list (cm)
$\bar{d}_{100}$	mean top diameter (cm)
$g_i$	tree basal area ( $\text{m}^2$ )
$n$	number of trees per plot

## PROJECTION METHODS

The projection methods tested were considered as adjuncts to the stand growth models. It was assumed for the purpose of comparison that stand growth can be projected without error, and all projected tree basal areas were adjusted so that  $G_2$  as measured equalled  $G_2$  as projected.

### Simple Projection

If all trees grow in basal area at a same rate, the projected diameter is given by:

$$\hat{d}_{2i} = d_{1i} \sqrt{\frac{G_2}{G_1}} \quad (1)$$

### Woollons and Hayward

This method (Woollons and Hayward 1985) was derived to project the diameter class mid-points of a stand table. An equation is used to predict the relative basal area,  $R = \frac{g}{\left(\frac{G}{N}\right)}$ ,

of each tree, from which the projected diameter can be obtained:

$$\hat{d} = \sqrt{\left(\frac{40000}{\pi}\right) R \left(\frac{G}{N}\right)}$$

Relative basal area is predicted by

$$\hat{R}_{2i} = \left( \frac{\left(\frac{G_1}{N_1}\right)}{\left(\frac{G_2}{N_2}\right)} \right)^{\beta_1} \left( \frac{g_{1i}}{\left(\frac{G_1}{N_1}\right)} + \frac{\beta_2}{\left(\frac{G_1}{N_1}\right)} (T_2 - T_1) \right) \quad (2)$$

where  $\beta_1 = 0.1473$

$\beta_2 = 0.00032$

(Woollons and Hayward *op.cit.*, equation 21b for thinned stands).

## Manley

In Manley's distance-independent tree growth model (Manley 1981), tree diameter increment is estimated directly from relative diameter and stand parameters, using site index as a multiplicative variable. The equation is

$$\Delta \hat{d}_i = \left( \beta_0 + \beta_1 G + \beta_2 G^2 + \frac{\beta_3}{H\sqrt{N}} + \beta_4 \frac{d_i}{\bar{d}_{top}} + \beta_5 H \right) S^{\beta_6} \quad (3)$$

where  $\beta_0 = 0.3103$

$\beta_1 = -0.01766$

$\beta_2 = 0.000153$

$\beta_3 = 21.5041$

$\beta_4 = 0.5509$

$\beta_5 = -0.002662$

$\beta_6 = 1.0853$

## DATA

Plots were selected from the PSP data base for Kaingaroa Forest and the Nelson region to form a data set for this comparison. At least 15 trees measured for diameter were required in each plot to ensure consistent estimates of variance. This ruled out very small plots particularly where the stocking was low. Plots were required to have had two or more consecutive measurements, with the first measurement somewhere between ages 15 and 25 inclusive. Only plots with "normal" levels of mortality (i.e. excluding windthrow, poison thinnings etc.) were chosen, and all thinning operations must have been completed prior to the initial measurement. An estimate of  $\bar{h}_{100}$  was required.

A total of 46 plots were selected from Kaingaroa and 65 from Nelson which met these criteria. They are listed, together with the data, in Appendix 1.

## COMPARATIVE STATISTICS

To compare the projected diameters with the actual measured values several statistics were formed for each plot and projection-period combination. Because of the assumptions made,  $G_2$  and hence  $\bar{g}_2$  were correct, so attention could be focused on comparing other measures of the actual and predicted diameter distributions.

The root mean square (RMS) error is defined as:

$$\sqrt{\frac{\sum_{i=1}^n (\hat{g}_i - g_i)^2}{n}}$$

and measures the average error in the projected diameters, disregarding sign. Large deviations receive more weight than small deviations.

To highlight changes in the variance caused by the projection method, the ratio of the variance of the projected tree basal areas to the variance of the actual tree basal areas was used. For comparison purposes, the log of this ratio was plotted on the assumption that a distribution with a variance only half the correct size is as undesirable as one twice the size it should be.

From the ratio an approximate  $\chi^2$  statistic was formed as :

$$\chi^2 = \frac{(n-1)s_{projected}^2}{s_{actual}^2}$$

Finally the ratio of the skewness of the projected tree basal areas to the actual tree basal areas was examined. Skewness is the tendency of one tail of the population to be longer than the other i.e, a positive value indicates a long tail on the right and vice versa. It is calculated as:

$$\frac{n}{(n-1)(n-2)} \frac{\sum_{i=1}^n (g_i - \bar{g})^3}{s^3}$$

By examining the ratio, any changes in skewness as a result of projection can be identified and their relative sizes compared.

## RESULTS

An initial examination of three "representative" plots from Nelson illustrates the types of changes that can occur to the diameter distribution after projection (Figures 1,2 and 3). Underestimates of the variance are clearly related to the initial plot stocking. The highly stocked plot shown in Figure 3 has the distributions for the second projection period (age 16.5 to 29.0) translated 10 cm to the right for clarity.

The RMS error, variance ratio and  $\chi^2$  probability were plotted, by method, over the projection period.

The RMS error increased with projection period so direct comparisons of methods were not easily made (Figure 4). Attempts to find significant differences by method were inconclusive. A linear model with projection period as a covariate and plot and method as factors produced some evidence of differences between the least-square means, but significant interactions between plot and projection period cast doubts on the validity of this analysis. The ordering of the least-square means differed between Kaingaroa and Nelson.

In contrast, the variance of the distribution was clearly predicted more accurately by the Manley method, whereas the Simple and W&H methods increasingly under-estimated the variance with increasing projection period (Figures 5 and 6).

Plotting the approximate probability of the  $\chi^2$  value over the projection period further supported this result for the Kaingaroa plots, although the picture was not as clear in the Nelson data (Figure 7).

In order to detect any effects related to the state of the stand at  $T_1$ , the comparative statistics were plotted over  $G_i$ ,  $N_i$ ,  $\bar{h}_{100,1}$  and  $S$ . Plots of the variance ratio over  $N_i$  show the Manley method predicting with little bias over the range of  $N_i$  (Figures 8 and 9). In contrast, the other methods consistently under-predict the variance, with increasing error as  $N_i$  increases. There were no clear relationships between the variance ratio by method and any other initial stand variable.

No effects by method on skewness were detected.

## DISCUSSION

Projecting the plots using Manley's method gave the best estimate of the variance of the diameter distribution. This method appears to be reasonably reliable over long projection periods and different initial stand conditions. Plots from both Nelson and Kaingaroa showed the same trends although the Manley method tended to overestimate variance in the Nelson plots.

### Relative Competitive Status

Both the Woollons and Hayward and the Manley methods use a base measure of the stand diameter to form a relative diameter, which is then used to determine the growth rate of individual trees/diameter classes. The Woollons and Hayward method uses *mean diameter* as the base, whereas the Manley method uses *mean top diameter* ( $\bar{d}_{100}$ ). The rationale for using top diameter is that it is more stable and less sensitive to mortality and thinning from below. Unfortunately, the stability of both these measures is influenced by the sample size used to estimate them. This is particularly marked in the comparisons performed here, where each plot was taken as a unique starting point.

For example, plot RO 681 /0 /8 /0 is 0.0405 ha in size and contains 12 trees (296 stems/ha). The mean diameter was estimated from all 12 trees, whereas top diameter was calculated from only 5 trees, of which the first 4 supplied over 98% of the weight. As the largest two trees are considerable bigger than the rest, their growth was over-predicted. Had the mean

top diameter estimate been made from a much larger sample (e.g. all the plots in an inventory stratum) it may have been somewhat larger, as the size of the two largest trees suggests, in which case their predicted growth rate would have been closer to the actual.

Figures 10 and 11 show the actual and predicted diameter growth for this plot. The increase in precision that could be expected when these methods are applied using stand estimates based on large samples could be greater for methods based on mean top diameter rather than mean diameter.

FIGURE. 10 Actual Tree Diameter Growth

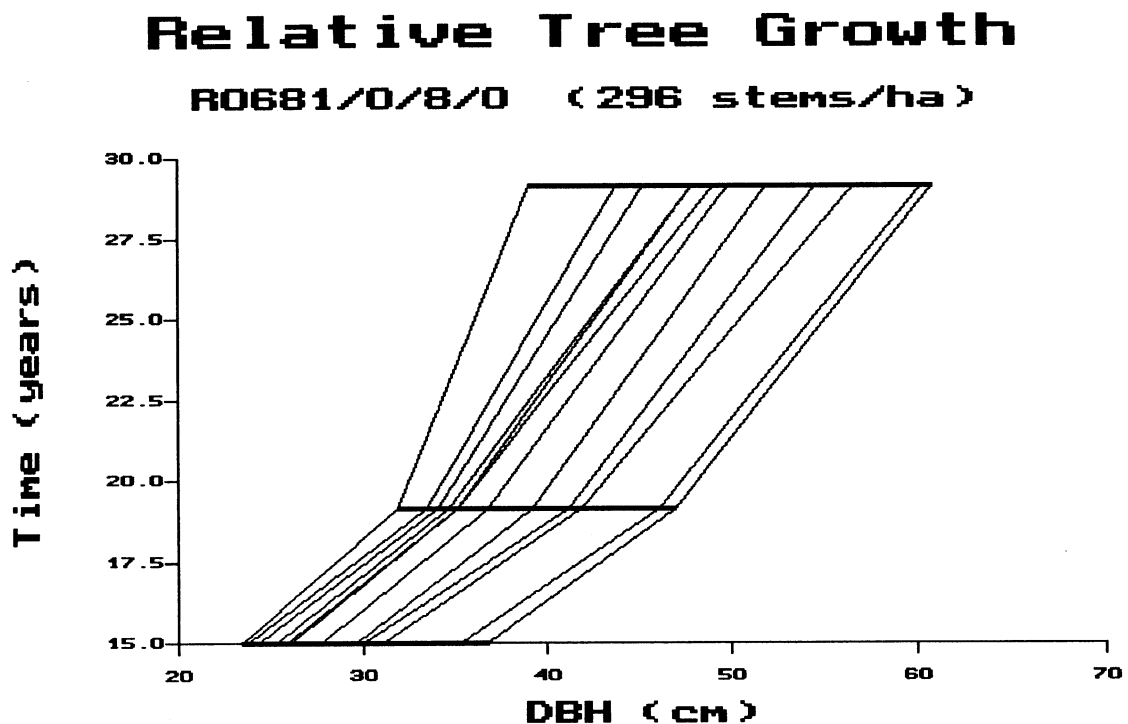
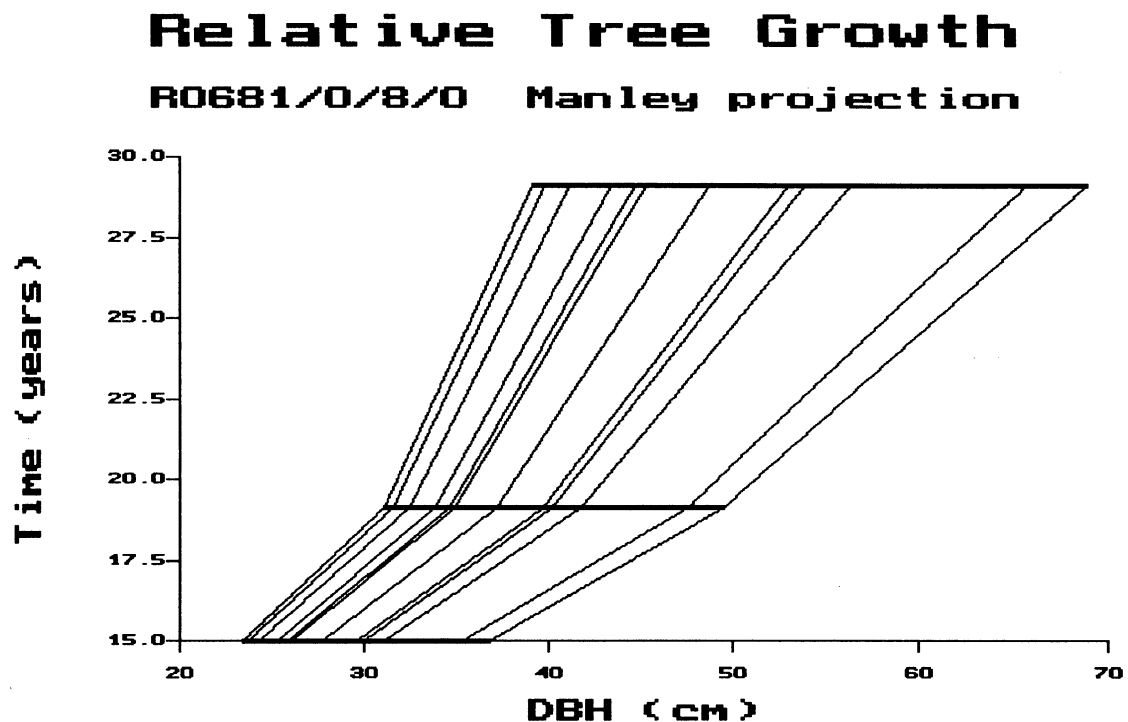


FIGURE. 11 Projected (Manley) Tree Diameter Growth



### Stand Density

Manley's method allows for different levels of density by including a specific term in the regression. Relative spacing is the ratio of average between-tree spacing to  $\bar{h}_{100}$ , and gives a measure that can be considered inversely proportional to competition. Plotting the variance ratio over relative spacing demonstrates the methods ability to produce reasonable predictions over a range of stand densities (Figures 12 and 13).

## CONCLUSIONS

Projecting individual tree basal area at a constant rate results in under-estimates of the variance of the tree diameter distribution. The under-estimate increases with increasing projection period and initial stand stocking.

The method with the most potential of those compared here is based on predicting the individual tree basal area increment using stand variables, site index and relative tree size.

To develop a robust method for projecting inventory data the literature must be reviewed in more detail in order to build an overview of methods in current use and their efficacy. With this overview and the results of the comparisons made here, we should have a good basis for constructing a reliable method for projecting individual tree diameters from inventory data.

## REFERENCES

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## APPENDIX 1

Plot ID	Area (ha)	$S$	No. Meas. <sup>1</sup>	$T_1$	$N_1$	$G_1$	$\bar{h}_{100,1}$
RO 416 0 0 0	0.2023	32.9	3	17.0	450	38.48	29.9
RO 421 0 0 0	0.2023	30.7	3	21.0	366	41.07	32.9
RO 464 0 0 0	0.2023	35.3	5	22.0	356	37.93	36.4
RO 488 0 3 0	0.2023	31.1	2	15.3	193	22.20	23.9
RO 693 0 0 0	0.2023	31.0	3	16.0	158	16.99	25.5
RO 721 0 0 0	0.1000	29.0	3	18.0	280	26.08	25.9
RO 746 0 0 0	0.1012	33.4	4	17.1	455	32.58	27.9
RO 681 0 21 0	0.0405	33.1	4	15.0	543	37.57	25.4
RO 681 0 22 0	0.0405	31.5	4	15.0	543	30.96	23.5
RO 681 0 23 0	0.0405	33.4	4	15.0	568	33.54	24.6
RO 681 0 24 0	0.0405	34.0	4	15.0	519	38.07	25.9
RO 681 0 37 0	0.0405	34.4	4	15.0	741	47.24	26.4
RO 681 0 39 0	0.0405	33.1	4	15.0	716	33.93	25.5
RO 681 0 41 0	0.0405	33.1	4	15.0	519	35.61	25.6
RO 681 0 42 0	0.0405	34.4	4	15.0	543	36.18	27.7
RO 681 0 43 0	0.0405	33.5	4	15.0	543	38.77	25.7
RO 681 0 44 0	0.0405	33.4	4	15.0	543	33.46	26.5
RO 685 2 7 0	0.0405	24.3	4	15.1	741	35.58	18.0
RO 685 2 8 0	0.0405	23.6	3	15.1	716	33.64	18.1
RO 685 4 16 0	0.0405	22.8	4	15.1	716	29.72	16.7
RO 690 0 3 0	0.0809	35.0	4	15.1	494	36.77	26.8
RO 690 0 4 0	0.0809	35.3	3	15.1	457	34.77	27.0
RO 695 1 16 0	0.1012	28.0	3	15.1	198	17.82	21.6
RO 695 2 4 0	0.1012	29.7	3	15.1	296	22.18	22.2
RO 695 2 23 0	0.1012	29.2	3	15.1	287	22.66	23.3
RO 695 3 11 0	0.1012	28.9	3	15.1	395	25.19	22.9
RO 695 3 14 0	0.1012	29.5	3	15.1	385	28.64	23.1
RO 695 4 10 0	0.1012	29.9	3	15.1	494	31.50	22.4
RO 695 5 18 0	0.1012	30.1	3	15.1	563	32.34	22.4
RO 695 5 21 0	0.1012	29.8	3	15.1	583	32.77	23.2
RO 695 6 17 0	0.1012	27.6	3	15.1	682	34.38	21.2
RO 696 1 5 0	0.1012	26.3	3	15.1	198	18.75	20.5
RO 696 1 15 0	0.1012	27.7	3	15.1	198	21.01	20.7
RO 696 2 2 0	0.1012	28.9	3	15.1	395	33.34	22.3
RO 696 3 10 0	0.1012	27.8	3	15.1	375	29.52	22.1
RO 696 4 16 0	0.1012	27.0	3	15.1	336	25.49	20.5
RO 696 4 24 0	0.1012	28.6	3	15.1	385	30.09	22.5
RO 696 5 1 0	0.1012	27.9	3	17.1	573	43.19	24.3
RO 696 5 20 0	0.1012	28.6	3	15.1	583	37.63	22.8
RO 696 6 11 0	0.1012	29.9	3	17.1	534	39.79	25.2
RO 696 6 21 0	0.1012	28.2	3	15.1	593	35.83	21.9
RO 696 8 6 0	0.1012	27.7	3	15.1	771	37.52	21.5
RO 902 0 5 0	0.0600	32.0	4	15.0	400	33.59	24.6
RO 911 1 1 0	0.2023	34.5	4	17.0	198	35.04	30.4
RO 911 1 3 0	0.2023	33.3	4	15.0	193	25.87	25.1
RO 911 1 4 0	0.2023	32.3	4	17.0	208	34.31	28.2

<sup>1</sup> Number of measurements used in the analyses.

Plot ID	Area (ha)	$S$	No. Meas.	$T_1$	$N_1$	$G_1$	$\bar{h}_{100,1}$
NN 234 0 3 0	0.0405	23.9	4	18.0	1235	55.82	22.4
NN 278 1 9 0	0.0809	27.4	3	16.1	198	14.81	21.8
NN 278 1 13 0	0.0405	26.3	3	16.1	1457	50.73	22.1
NN 278 1 14 0	0.0405	26.6	2	16.1	1457	48.62	20.2
NN 376 0 1 0	0.0500	22.7	3	15.0	780	29.77	17.2
NN 376 0 2 0	0.0405	28.3	3	15.0	617	44.25	21.0
NN 421 0 9 0	0.0600	23.3	2	15.8	767	18.94	18.5
NN 421 0 10 0	0.0600	23.3	2	16.0	417	9.96	18.8
NN 421 0 12 0	0.0600	20.0	3	15.2	1100	21.12	15.0
NN 421 0 16 0	0.0600	28.1	3	15.3	933	35.76	22.4
NN 446 1 68 3	0.1012	24.2	2	18.0	326	25.82	21.8
NN 446 1 68 4	0.1012	24.7	3	16.2	306	18.62	19.9
NN 446 1 68 6	0.1012	23.6	3	16.0	267	19.66	19.2
NN 446 1 75 1	0.1012	26.4	3	18.0	208	19.26	22.5
NN 446 1 75 3	0.1012	27.3	3	18.2	366	30.41	24.7
NN 446 1 75 4	0.1012	27.6	3	17.0	306	22.21	22.2
NN 446 1 75 5	0.1012	26.0	2	16.0	366	16.46	20.1
NN 446 1 75 6	0.1012	26.7	2	17.2	306	21.29	23.1
NN 446 1 75 7	0.1012	27.6	2	16.0	217	16.68	21.7
NN 446 1 76 2	0.1000	24.1	3	17.0	290	26.27	19.7
NN 446 1 76 3	0.1000	28.4	3	15.0	230	15.66	20.6
NN 446 1 76 5	0.1000	25.5	3	16.0	220	14.66	19.4
NN 446 1 76 6	0.1000	25.1	3	15.0	240	14.07	18.1
NN 446 1 76 9	0.1000	25.5	3	15.0	250	15.57	18.6
NN 446 1 76 10	0.1000	27.2	3	17.0	350	21.38	22.3
NN 446 1 76 11	0.1000	24.7	3	16.0	260	16.09	20.4
NN 446 1 76 13	0.1000	28.2	3	17.0	350	22.59	23.3
NN 446 1 76 14	0.1000	28.2	3	16.0	330	21.37	22.1
NN 446 1 77 1	0.1000	28.4	2	15.0	470	22.20	21.9
NN 446 1 77 2	0.1000	26.2	3	15.0	230	13.44	18.5
NN 446 1 77 11	0.1000	27.9	2	17.1	500	24.64	23.5
NN 446 1 77 12	0.1000	23.4	2	17.2	240	13.76	20.4
NN 446 1 78 26	0.1000	29.2	3	16.0	230	15.87	24.0
NN 446 1 78 51	0.0800	28.3	3	16.0	350	20.55	23.2
NN 446 2 68 2	0.1012	28.2	3	16.2	415	25.70	20.9
NN 446 2 76 4	0.1000	27.0	3	15.0	220	12.19	20.2
NN 446 2 76 7	0.1000	28.1	4	15.0	210	14.28	20.5
NN 446 2 76 8	0.1000	25.1	4	15.0	190	9.87	18.3
NN 446 2 76 12	0.1000	27.3	4	15.0	280	12.85	19.6
NN 446 2 76 15	0.1000	29.4	3	15.0	410	27.30	22.5
NN 446 2 77 5	0.1000	27.1	3	15.0	210	14.42	20.7
NN 446 2 77 8	0.1000	28.3	3	15.0	180	10.58	21.0

Plot ID	Area (ha)	$S$	No. Meas.	$T_1$	$N_1$	$G_1$	$\bar{h}_{100,1}$
NN 379 1 2 0	0.1000	28.7	3	15.6	280	18.15	22.6
NN 379 1 6 0	0.1000	29.5	3	15.6	330	21.50	23.2
NN 379 1 7 0	0.1000	29.9	3	15.6	320	20.39	23.2
NN 379 1 8 0	0.1000	28.4	3	15.6	260	16.96	22.1
NN 379 1 9 0	0.1000	29.9	3	15.6	350	21.57	23.7
NN 379 1 10 0	0.1000	28.8	3	15.6	310	17.97	23.2
NN 379 1 11 0	0.1000	29.2	3	15.6	190	14.35	22.7
NN 379 1 12 0	0.1000	27.3	3	15.6	320	18.09	21.2
NN 462 0 69 4	0.0405	27.5	4	16.0	1235	53.51	22.3
NN 462 0 69 5	0.0405	22.8	4	16.0	1630	56.27	18.6
NN 462 0 69 6	0.0405	26.6	2	17.3	420	25.04	23.8
NN 462 0 69 7	0.0405	28.1	3	16.5	864	53.60	24.3
NN 462 0 69 9	0.0405	27.3	3	16.0	617	28.58	20.1
NN 462 0 69 10	0.0405	26.8	4	17.0	741	44.84	21.5
NN 462 0 72 1	0.1012	26.6	3	18.0	385	25.85	23.4
NN 462 0 78 2	0.1000	25.3	3	17.1	670	30.10	21.8
NN 462 0 78 4	0.1000	29.3	3	19.1	330	29.13	28.3
NN 462 0 78 5	0.1000	28.1	3	19.1	240	24.99	27.1
NN 183 0 1 0	0.0397	23.5	3	20.5	680	55.42	23.7
NN 184 0 1 0	0.0316	23.2	2	20.5	981	63.39	23.6
NN 514 1 4 0	0.1807	27.0	3	20.0	194	18.46	28.3
NN 514 3 1 0	0.1506	26.5	2	20.1	518	37.01	26.9
NN 514 3 2 0	0.1120	31.1	2	16.1	875	59.44	24.3
NN 514 3 7 0	0.1120	30.0	2	16.1	580	37.59	23.8

stems per hectare

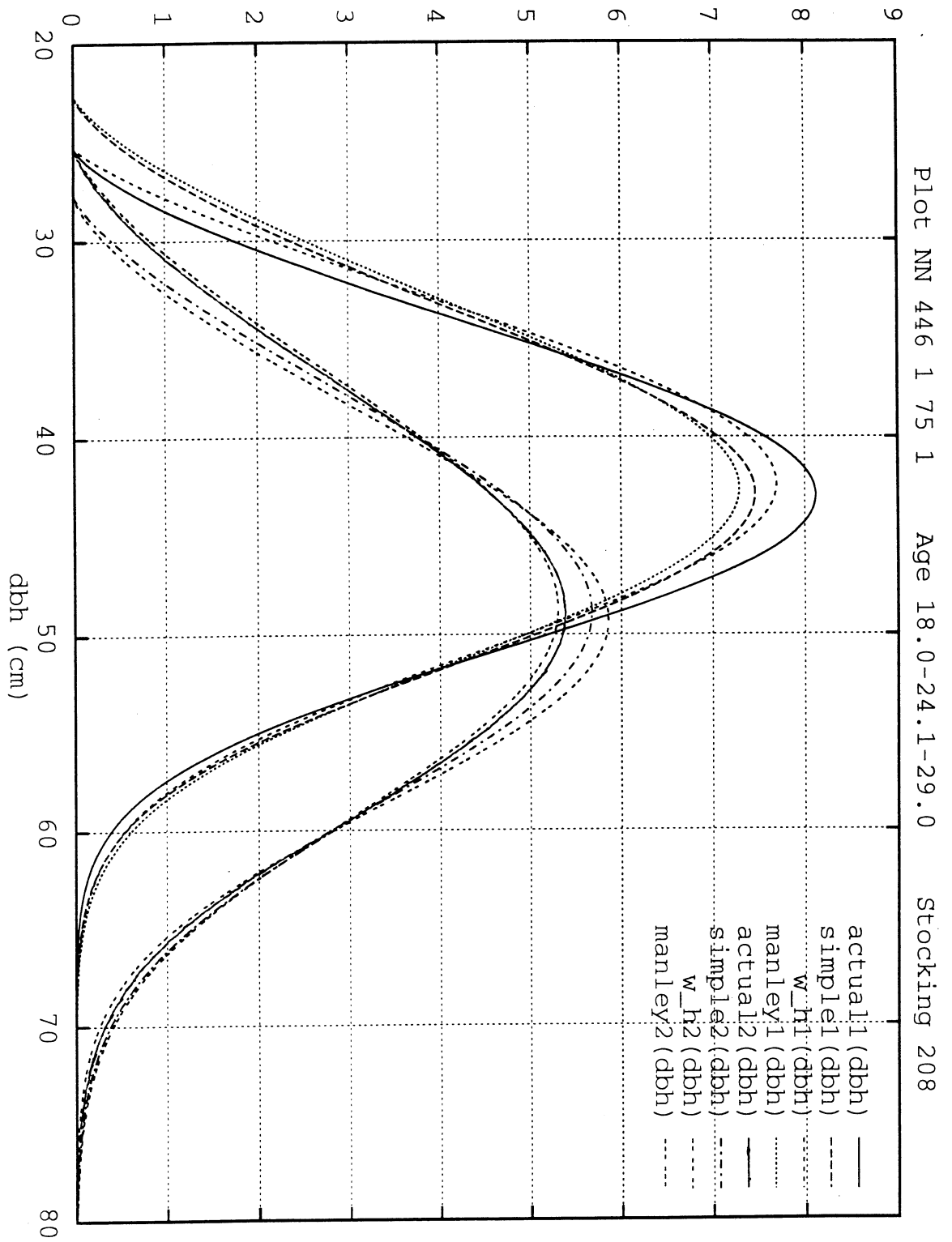


FIGURE 2.

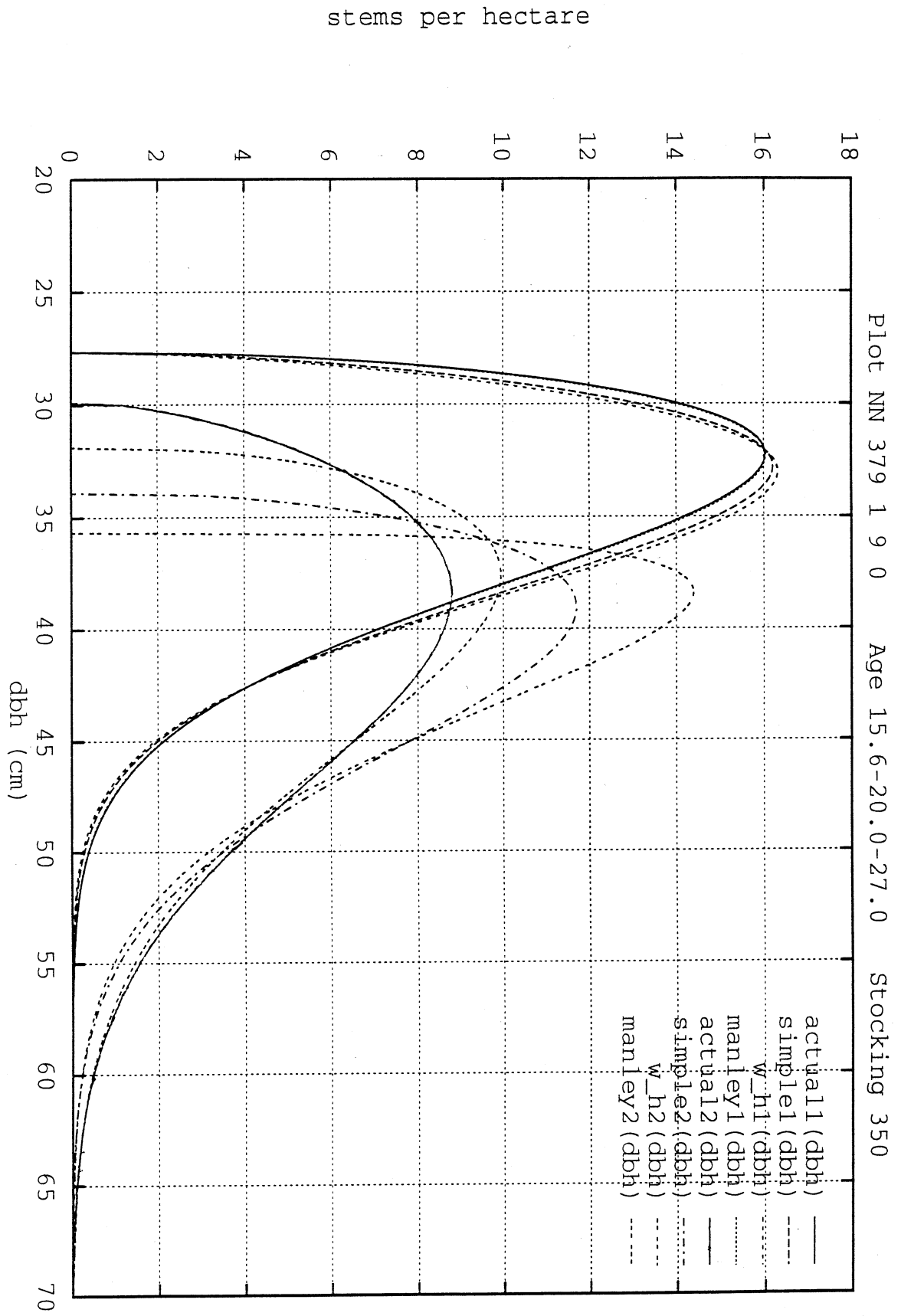


FIGURE 3.

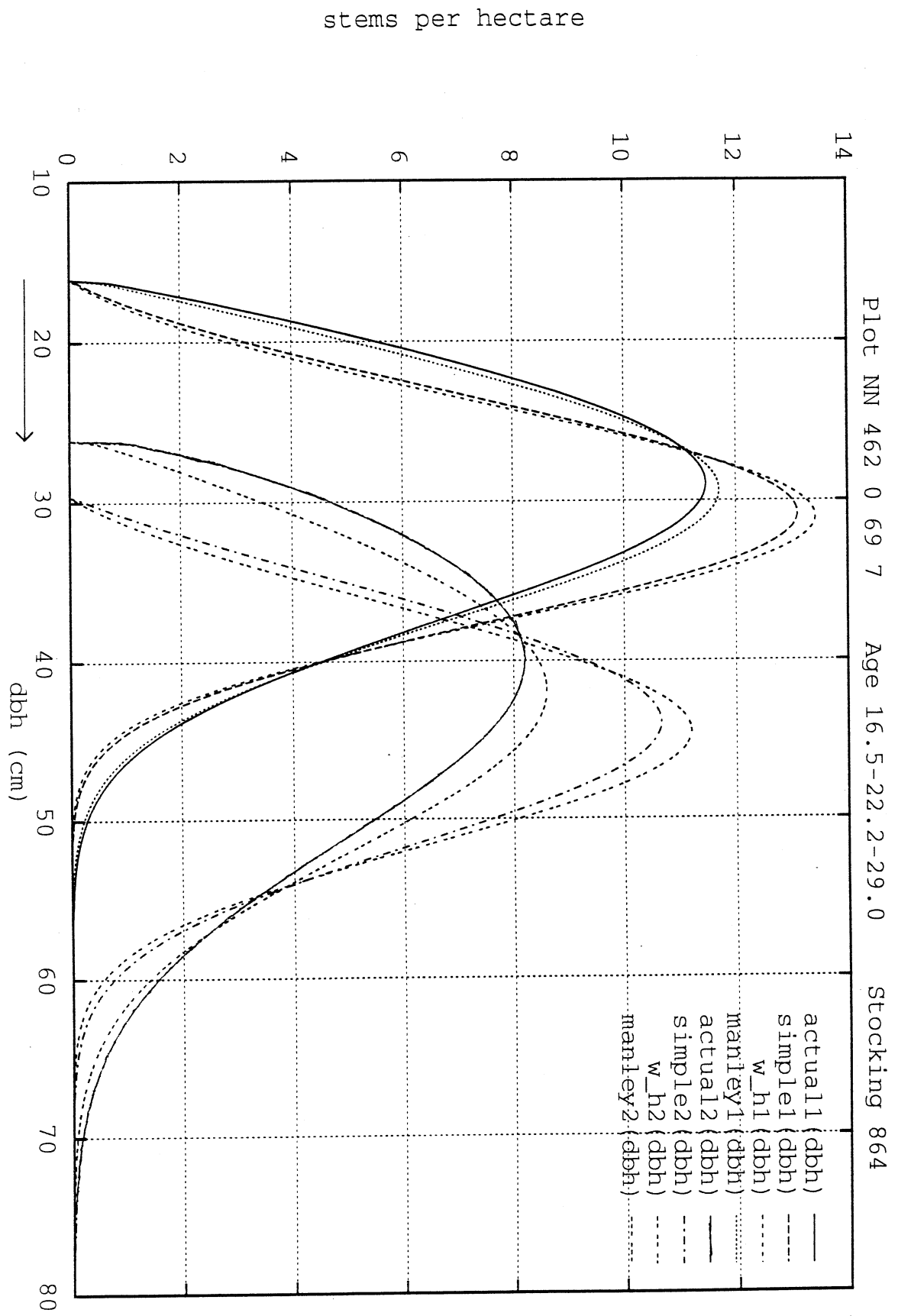


FIGURE 4.

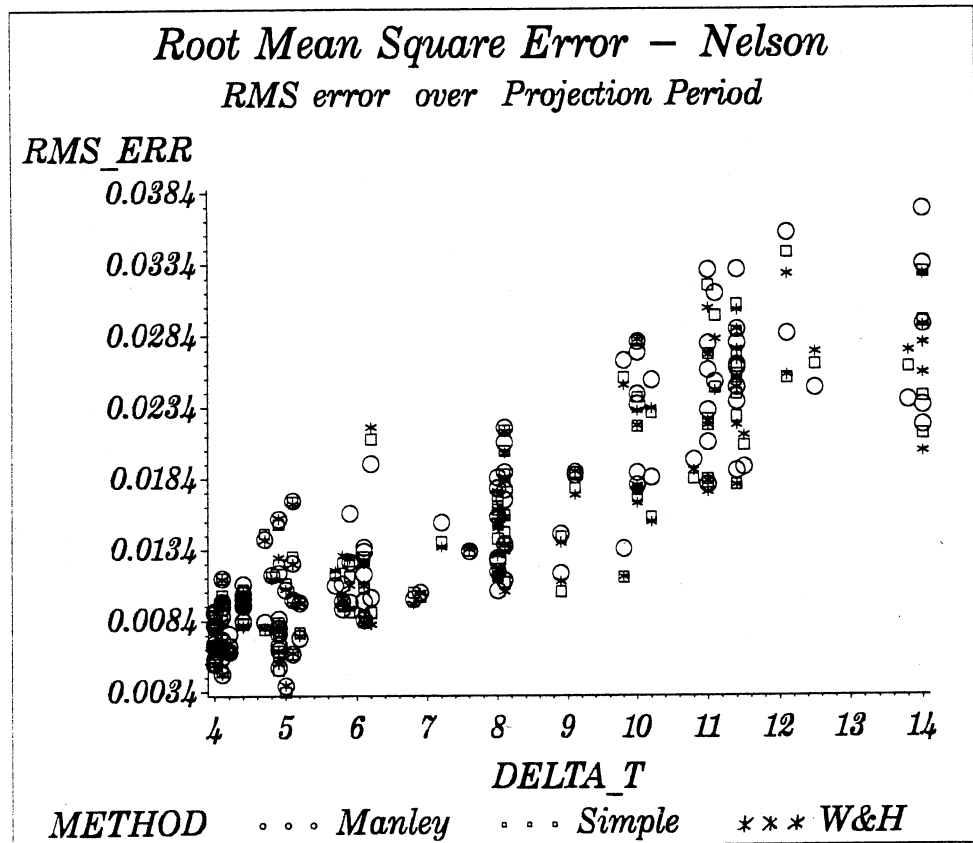
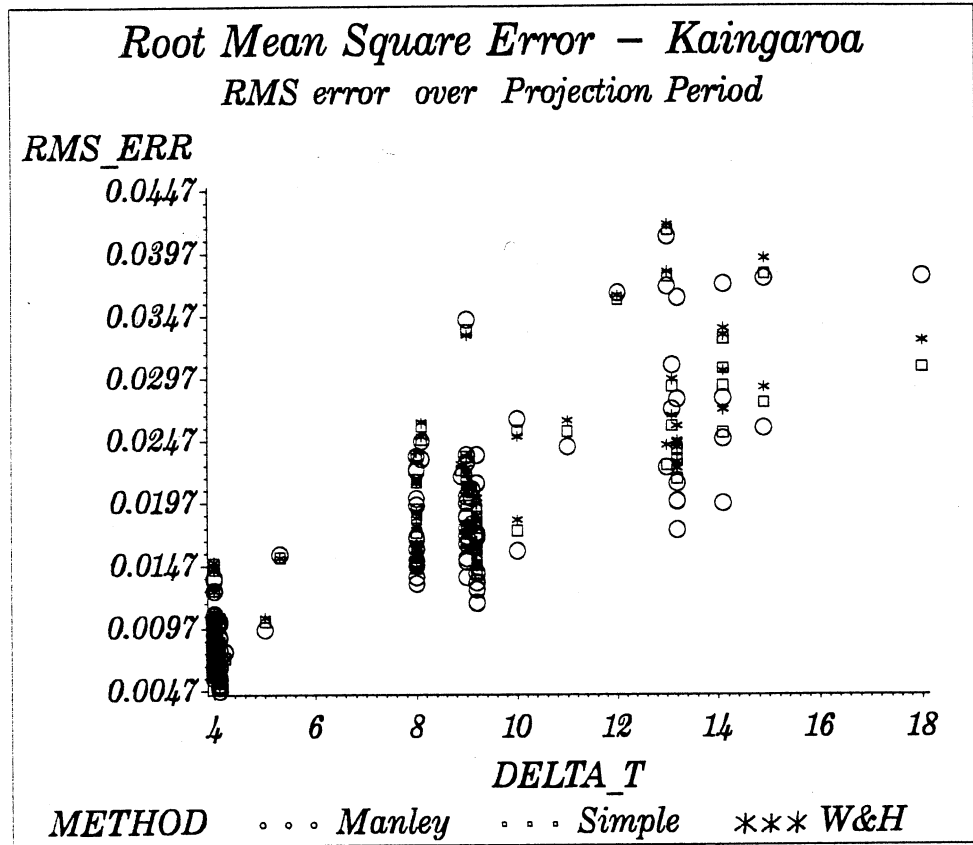


FIGURE 5.

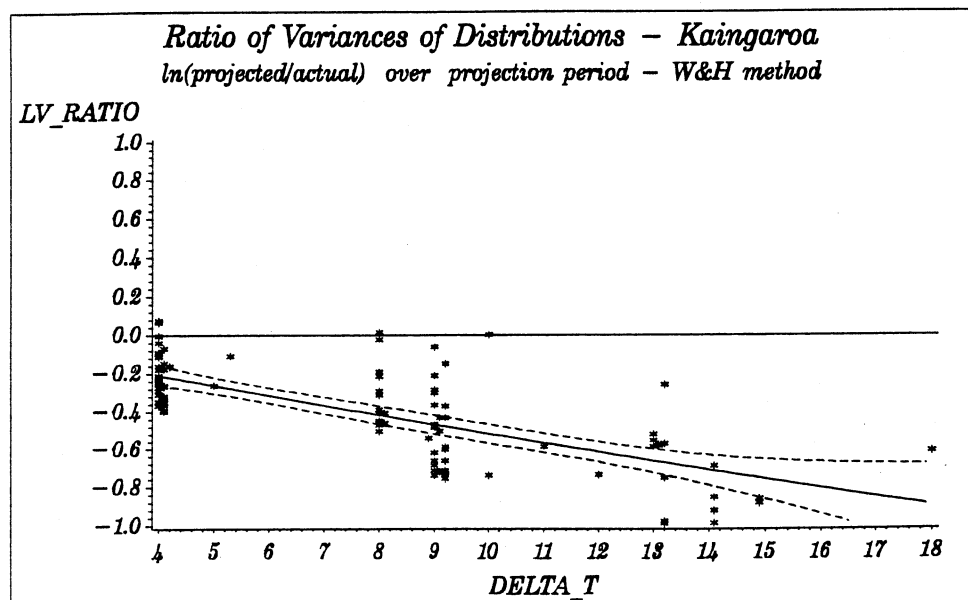
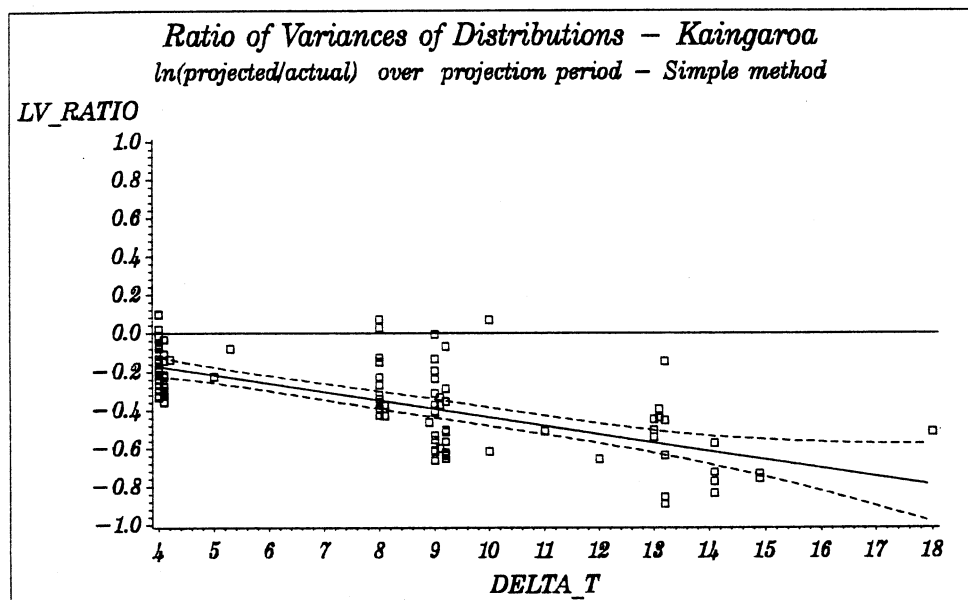
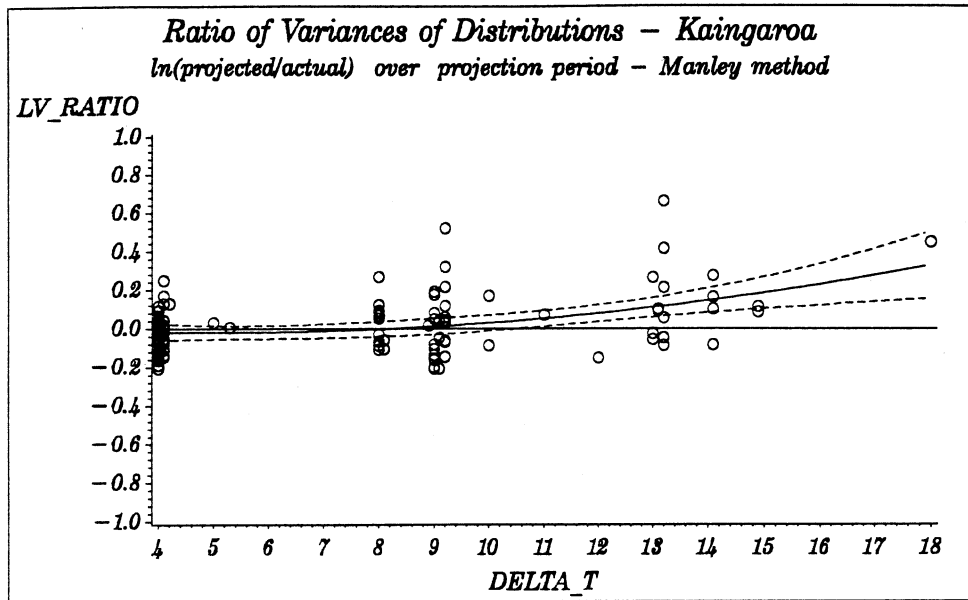


FIGURE 6.

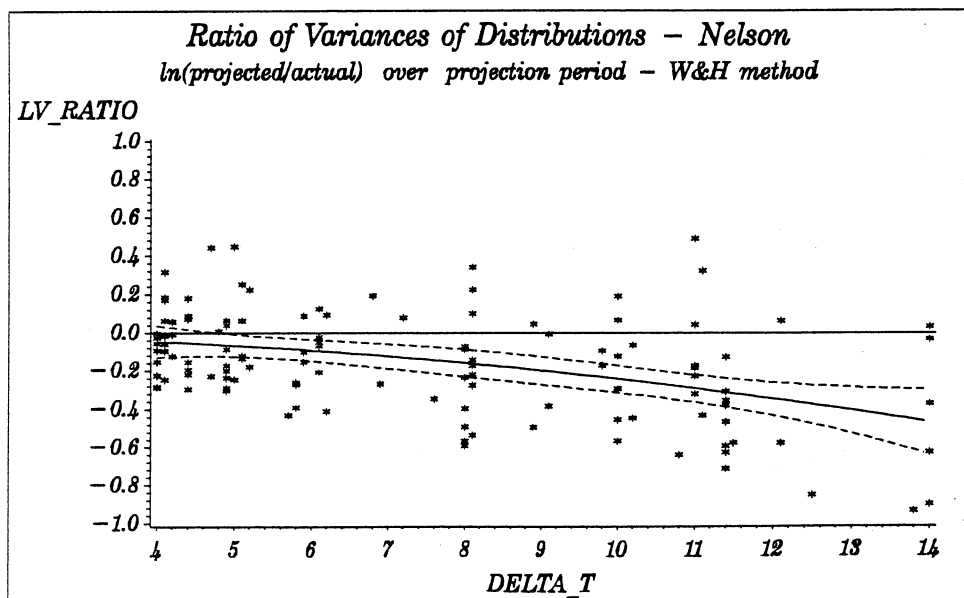
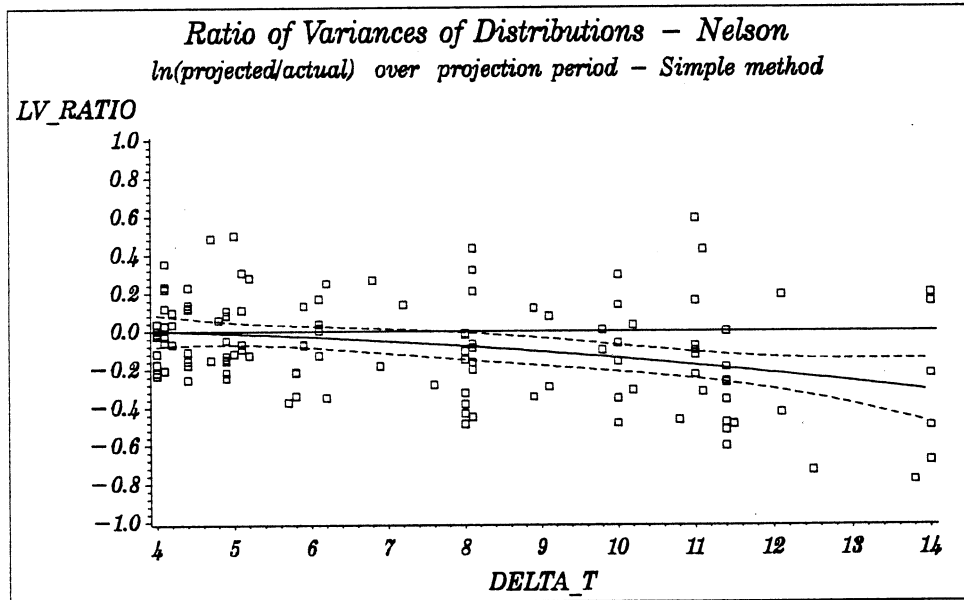
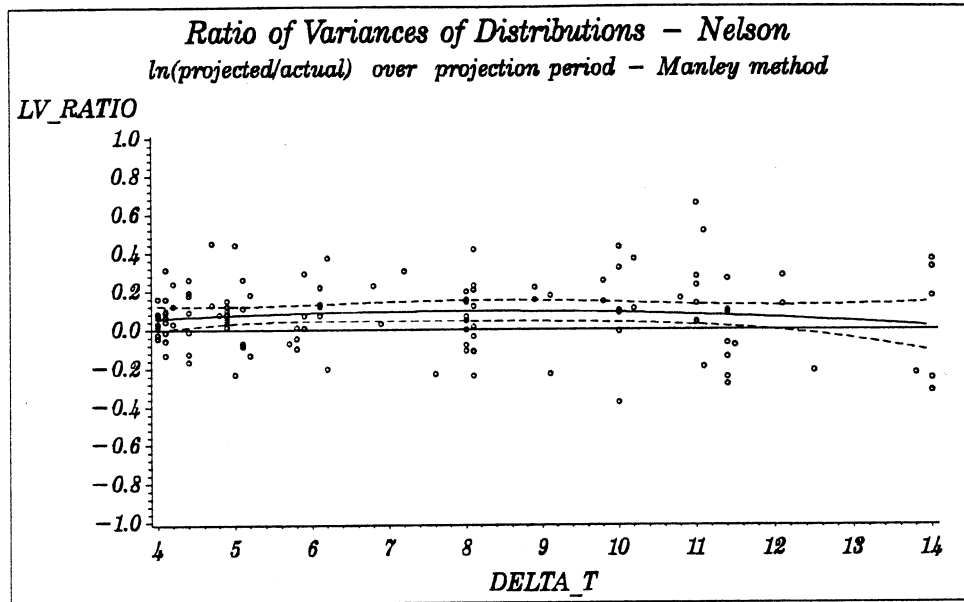


FIGURE 7.

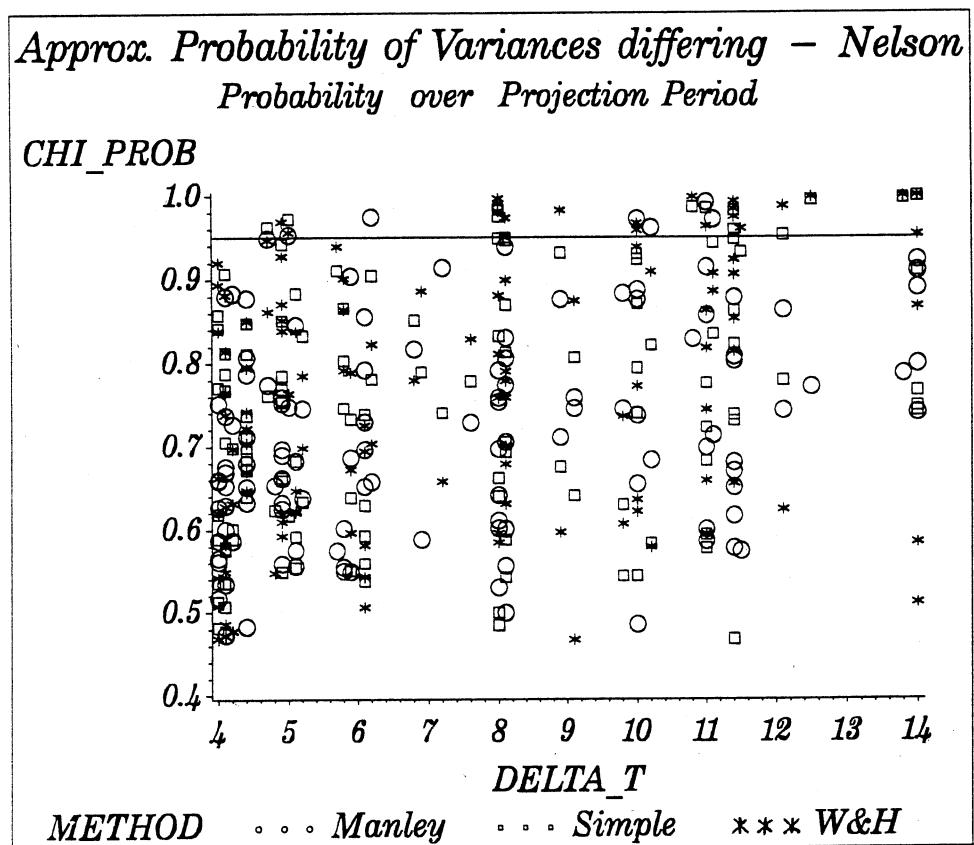
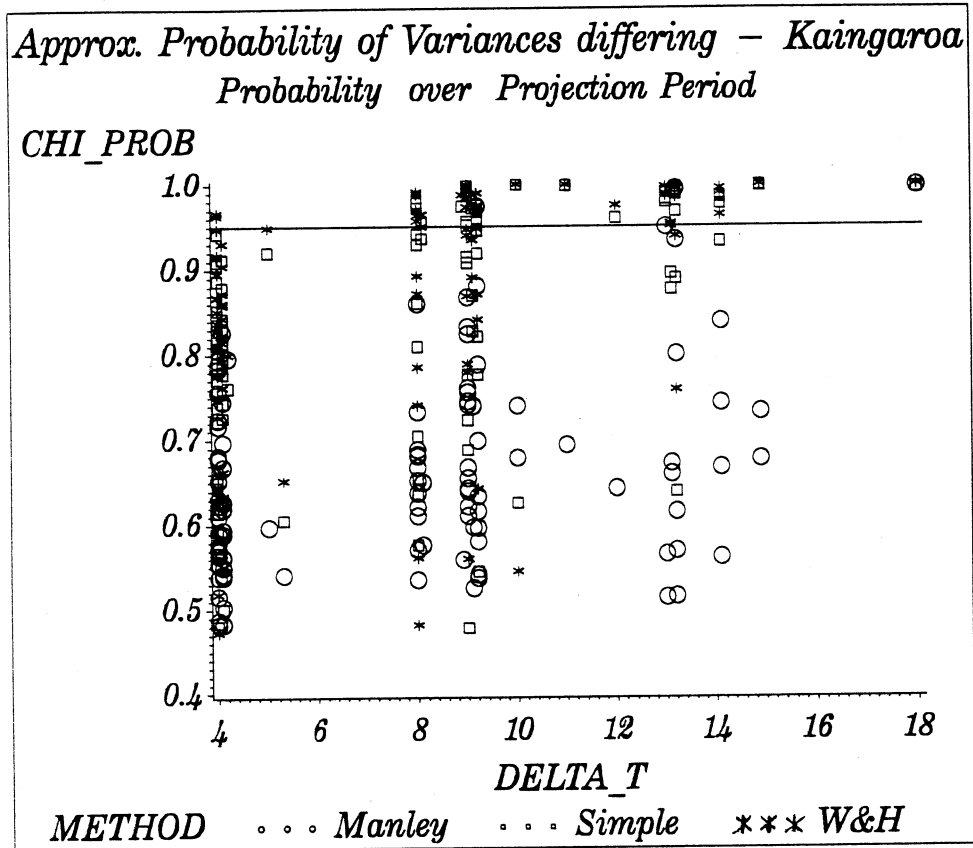


FIGURE 8.

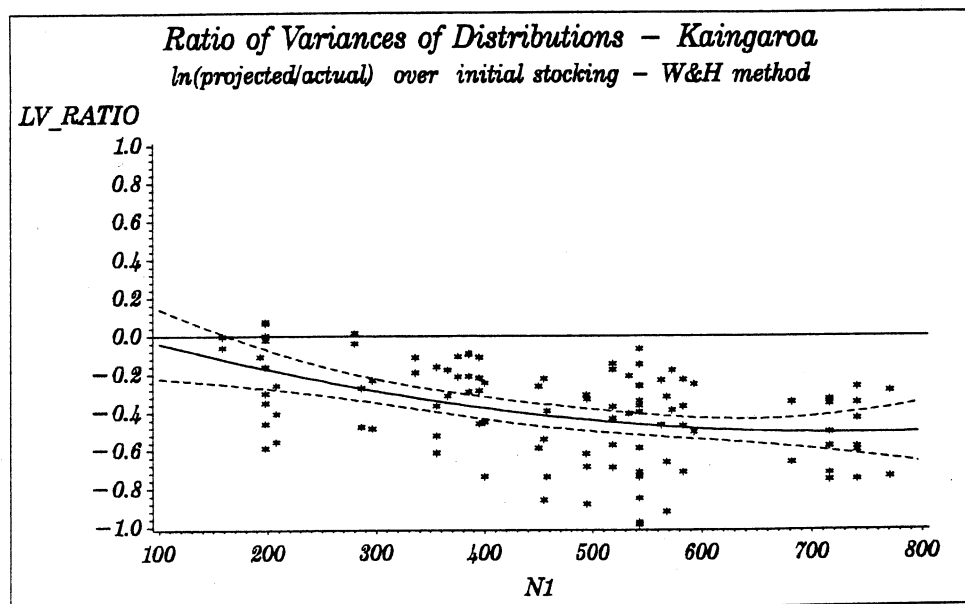
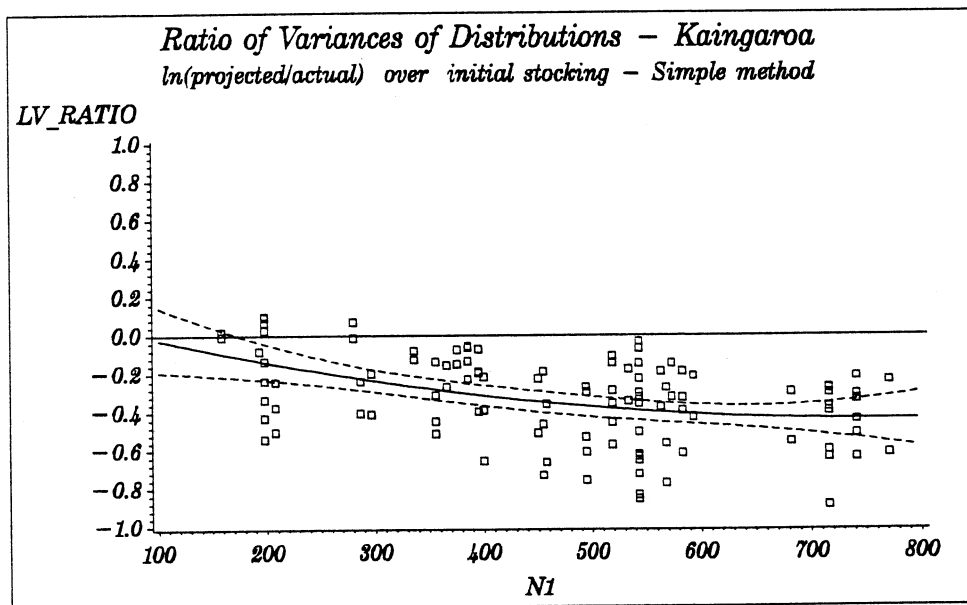
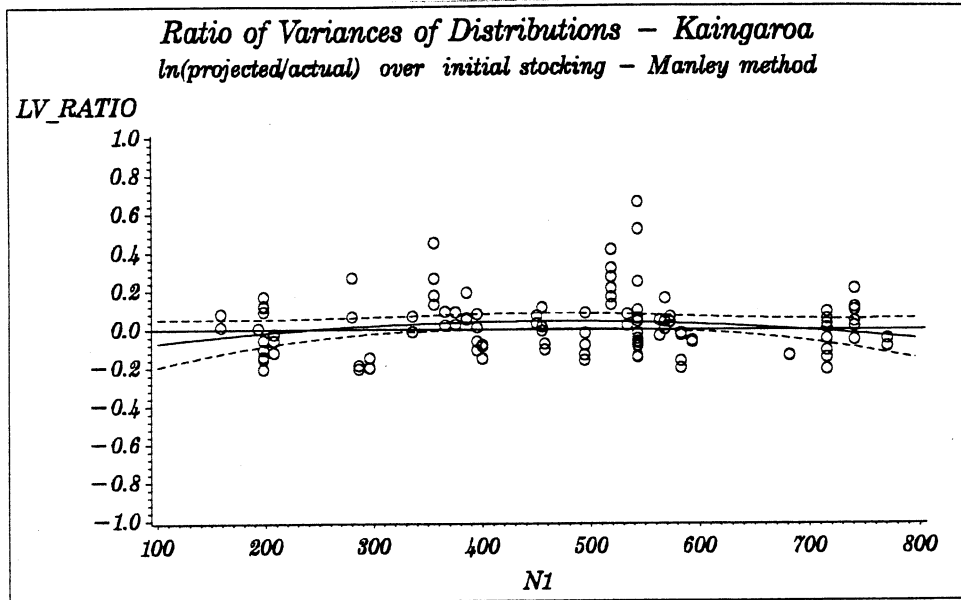


FIGURE 9.

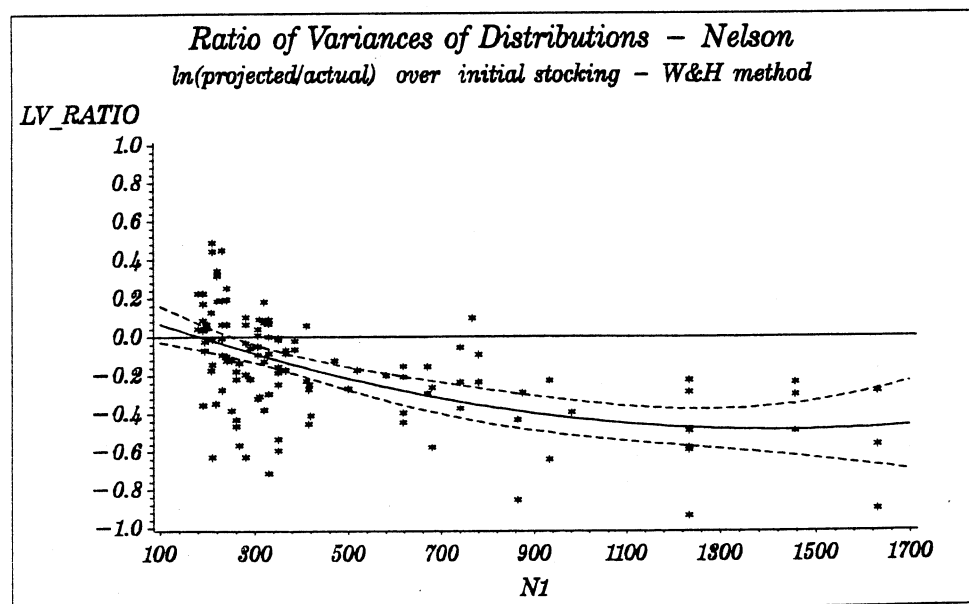
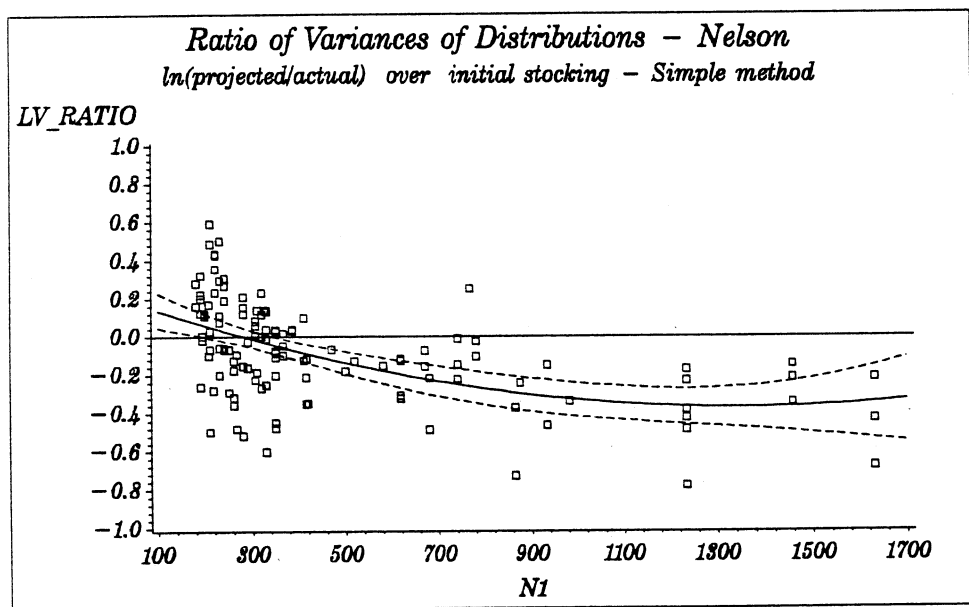
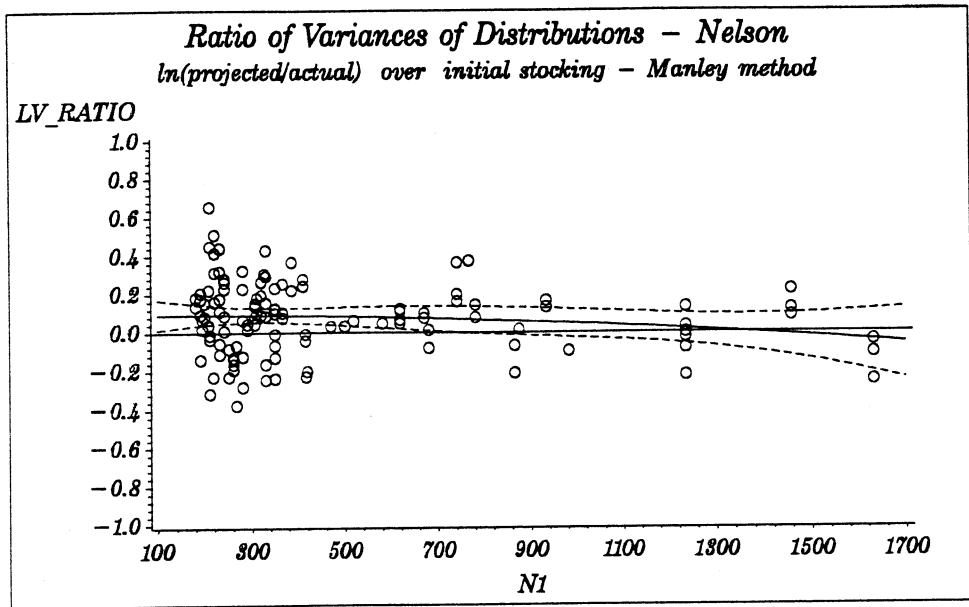


FIGURE 12.

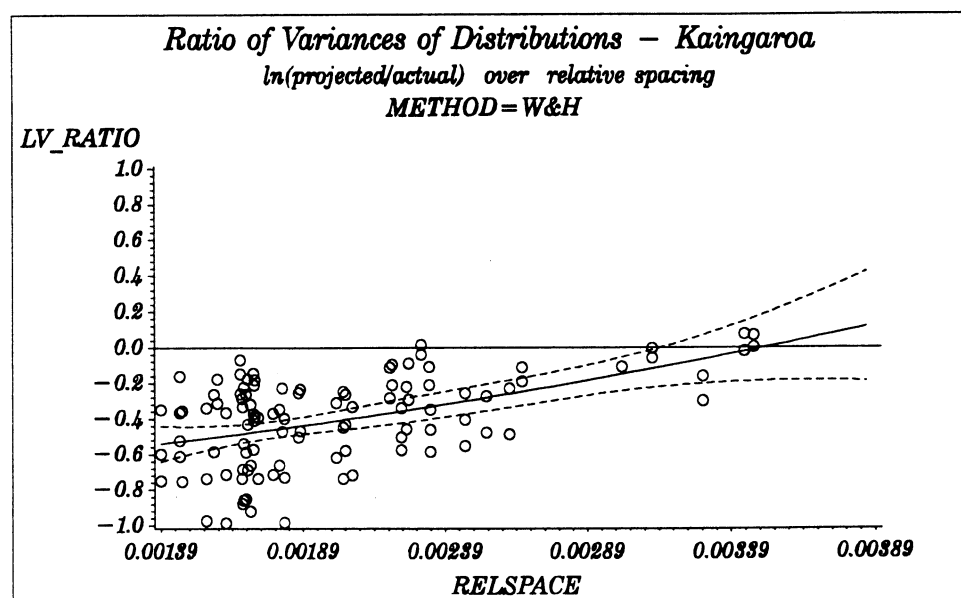
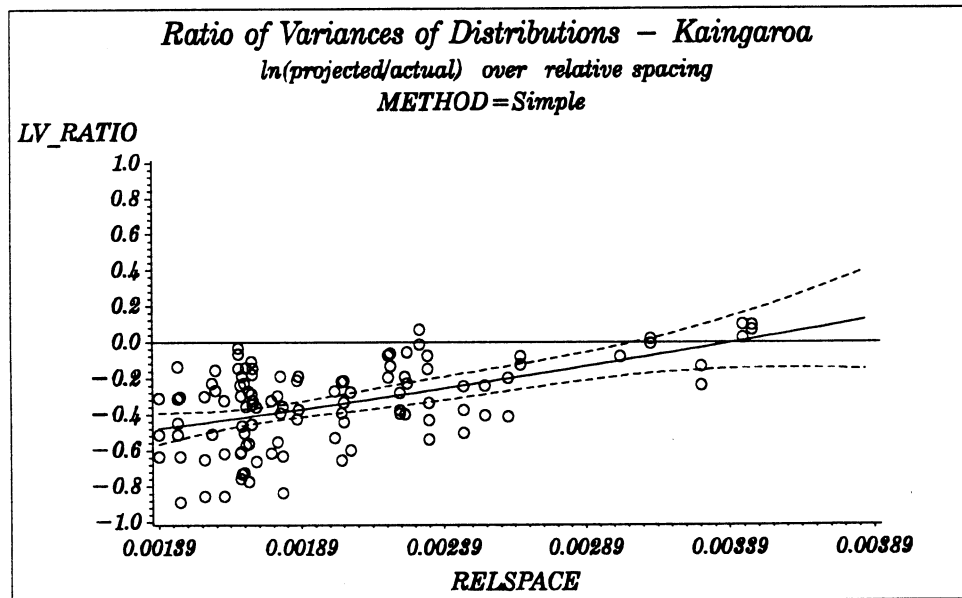
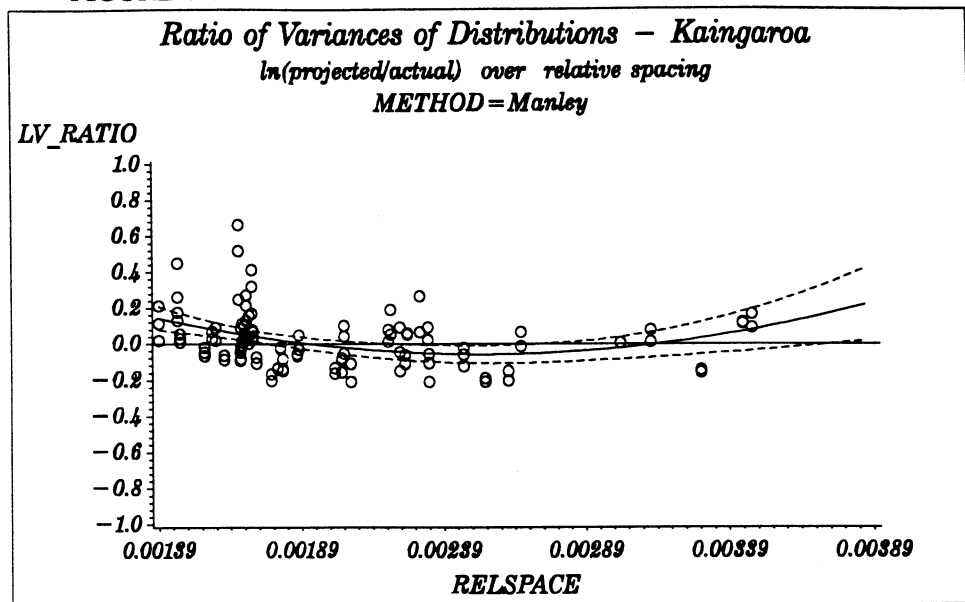


FIGURE 13.

