# DIAMETER DISTRIBUTIONS FOR THE REGIONAL STAND GROWTH MODELS

## M.E. LAWRENCE

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: This material is unpublished and must not be cited as

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#### **EXECUTIVE SUMMARY**

New equations have been developed to predict the coefficients for the Weibull probability density function for seven regional stand growth models: Central North Island pumice plateau (KGM3, PPM88), Hawkes Bay, Canterbury, North Island sands (SANDS), Nelson, North Island clays (CLAYS, CLAYSFERT), and Southland (SGM2, SGM3).

As part of the development, both two and three parameter versions of the Weibull function were tested, as were different methods for calculating the variance parameter for the function. The most robust version is a three-parameter Weibull, using the predicted minimum and maximum DBH to establish the variance of the diameters. This is similar to the function developed by Goulding and Shirley (1979) which is currently in use.

An alternative procedure (Garcia, 1984) which predicts the coefficent of variation of diameter squared for use in either a two or three parameter Weibull was found to be not as reliable in some situations.

#### INTRODUCTION

The Weibull probability density function is often used as a flexible, simply derived, diameter distribution model for forestry. The model normally requires three parameter estimates: parameter a is the location parameter, b the scale parameter, and c the shape parameter. The function can then be evaluated in its cumulative form:

$$F(x) = 1 - \exp\left(\frac{-(x-a)}{b}\right)^{c}$$

where a <= x

A simpler two parameter version can be obtained by fixing the location parameter to zero. The parameters are estimated in two steps. Firstly, the minimum DBH, mean DBH and variance of the tree basal areas are predicted. Secondly, these three values are used in an iterative procedure to obtain the Weibull coefficients a,b and c. Finally, the function can be evaluated at suitable diameter class intervals to obtain the required distribution.

This report is in two parts. Part I describes the derivation of new equations for the Central North Island pumice plateau, suitable for use with growth models KGM2, KGM3 and PPM88. It involves a test of different versions of the Weibull function itself, and of different methods for deriving the variance parameter for the function. Part II describes the application of the "best" method found in Part I to derive equations suitable for the other growth models.

PART I

## FORM OF WEIBULL; METHODS OF PARAMETER ESTIMATION DERIVATION OF NEW COEFFICIENT EQUATIONS

#### DATA

The data base is essentially that used for the construction of the "interim" Kaingaroa growth model KGM3, (Dunningham and Lawrence, 1987), and more latterly, the pumice plateau growth model PPM88. It consists of more than 3000 measurements from 300 Permanent Sample Plots established in Kaingaroa and Tarawera Forests after 1955 (see Table 1).

Table 1
RANGE OF DATA

Variable	Min	Max	Mean
Age (yrs)	2.8	36.0	
Site index (m)	22.5	39.4	31.6
Basal area (m2/ha)	0.5	75.6	
Stocking (sph)	89	5239	
Top height (m)	2.6	50.6	

For each measurement, basal area, stocking, top height, minimum, mean and maximum diameter and the variance of the individual diameters squared were obtained from the PSP system. The coefficient of variation for diameter squared and the variance of the individual tree basal areas were derived from the above for use as dependent variables in the regression analysis.

#### ANALYSIS

Three versions of the Weibull function were fitted and evaluated. The first method follows that of Goulding and Shirley, and is currently used in the stand volume generator, Program PROD (Goulding and Shirley, 1979). This method involves fitting a multiple regression based on the stand parameters (or their transformation) to the dependent variables DMIN, DMAX and DVAR - corresponding to the stand minimum and maximum diameters, and the variance of the individual tree basal areas respectively.

The second method is that alluded to by Garcia (1984), whereby a two parameter Weibull function is fitted after predicting the natural logarithm of the coefficient of variation of diameter squared.

The third method was a modification of the second. In this version, the function to predict DMIN developed for the first method was used to estimate the location parameter 'a' in a three parameter Weibull based on Garcia's procedure.

The dependent variables DMIN, DMAX, DVAR and lnCV were initially plotted against all the independent variables. A number of transformations were then derived from them and subsequently used in a stepwise multiple regression procedure (GENSTAT 5). At all stages the results were compared with the graphical analyses to ensure the correct selection of significant variables.

The residuals for each of the equations were plotted to check for bias and predictions of DMIN and DMAX were checked for consistency i.e. DMIN < DMEAN < DMAX. The final equations are:

```
DMIN = -0.545 + 1.03064 * DMEAN - 0.13705 * BAREA - 0.4172 * AGE
R^2 = 0.91
DMAX = 1.135 + 0.8301 * DMEAN + 0.13085 * BAREA + 0.649 * AGE
R^2 = 0.97
DVAR = 0.00007924 + 0.80514 * (((DMIN^2 - DMAX^2) * 0.00007854)/4)^2
R^2 = 0.92
InCV = -0.0923 + 0.3375 * InH + 0.24665 * InB + 0.02605 * AGE
```

#### RESULTS

Several tests were made to test the 'goodness of fit' of the function derived from the new coefficients. Initially, a simple consistency check was carried out by plotting the predicted minimum diameter against the mean diameter, to ensure that it was always less than the mean. This was repeated with predicted maximum diameter against the mean, to ensure it was always larger.

Eight sample plots were then selected from a trial at Tarawera Forest (R0955) to test the ability of the three versions to model actual distributions. The eight plots were selected to duplicate a range of stockings and where possible, include more than 20 trees per plot. The plot statistics are given in Table 2 below.

Table 2
TEST PLOT STATISTICS

Plot	#Trees	Age	Barea	Sph	Top Ht
RO955/6/22/0	12	25	48.82	198	43.1
RO955/4/09/0	18	25	50.65	297	41.7
RO955/4/14/0	23	25	58.94	379	41.7
RO955/4/07/0	23	25	56.26	379	41.7
RO955/4/03/0	36	25	63.08	593	41.5
RO955/4/04/0	37	25	67.33	610	42.0
RO955/4/10/0	57	25	69.50	939	41.7
RO955/4/05/0	85	25	69.82	1400	41.2

For each of the eight plots, the predicted diameter distributions were calculated from the three different functions. The predicted distributions were then plotted over the actual diameter distributions. The resulting graphs are shown over page. There is little to choose between the 'Garcia' two and three parameter functions - which confirms earlier work carried out in developing the Golden Downs growth model GDNS81 (O.Garcia, pers.comm.)

In general, the new three parameter function based on the methodology adopted by Goulding and Shirley would seem to be more robust, particularly at higher stockings.

These indications were initially confirmed by a Chi-squared analysis of the differences between the predicted and actual distributions. In every case, the 'Goulding & Shirley' method was superior to the other two.

A more efficient test that is commonly employed as an alternative is the Kolmogorov-Smirnov statistic. This test is based on the largest difference between two cumulative frequency distributions — in this case the actual distribution and one of the alternative 'fitting' methods. The statistic that is calculated can then be compared with a tabulated value at the desired level of significance. The results of this test on the eight plots and associated distributions are given in Table 3.

Table 3

KOLMOGOROV-SMIRNOV TEST : CRITICAL VALUES

Plot	D (L)	D (G2)	D (G3)
RO955/6/22/0	0.0692	0.2222	0.1389
RO955/4/09/0	0.0714	0.2088	0.1485
RO955/4/14/0	0.0987	0.1995	0.1533
RO955/4/07/0	0.0858	0.1530	0.1272
RO955/4/03/0	0.0610	0.2095 <b>*</b>	0.1680
RO955/4/04/0	0.1185	0.1639	0.1567
RO955/4/10/0	0.0472	0.3180 <b>**</b>	0.2851 <b>**</b>
RO955/4/05/0	0.0841	0.4290 <b>**</b>	0.4276 <b>**</b>

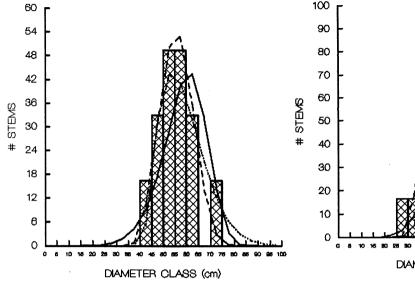
- \* exceeds critical value at p=0.10
- \*\* significantly different, p=0.01

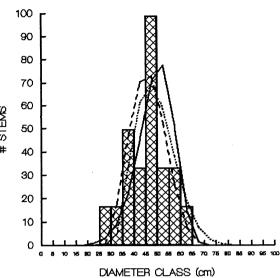
It can be seen that the new function based on the 'Goulding and Shirley' methodology has a lower critical value in every case, and that the two functions based on the 'Garcia' approach do not fit the actual distributions well at high stockings. This is illustrated in the following eight graphs of the three functions versus actual data. It is concluded that the methodology originally adopted by Goulding and Shirley (1979) is the more robust for deriving the Weibull parameters and can be used with some confidence.

The final set of eight graphs compares the new equations (Lawrence) with the original set produced by Goulding and Shirley. They are very similar in most respects; the new distribution being slightly more 'peaked' and not having quite the 'spread' of the original. This is perhaps to be expected given the more intensive management of the current crop and hence less variation in tree diameter.

\*\*\*\*\*\*

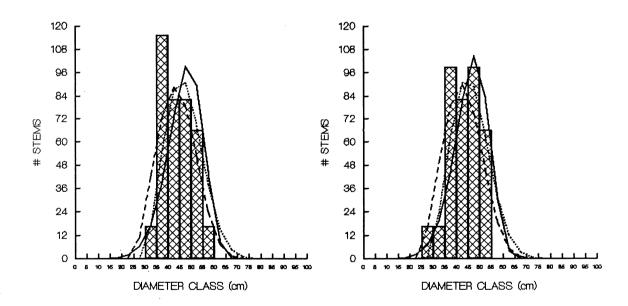
## GRAPHICAL COMPARISON OF METHODS AND PLOT DATA



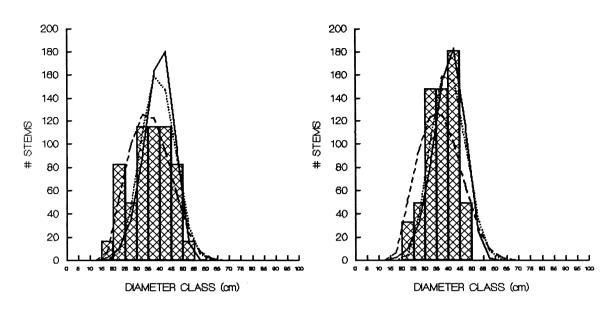


RO955/4/14/0 (379sph)

RO955/4/7/0 (379sph)

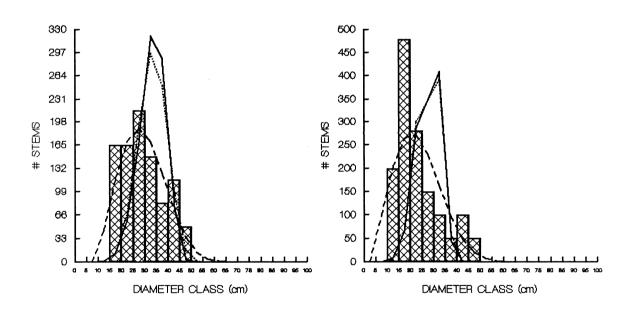


'Garcia' 2 parameter fn
'Garcia' 3 parameter fn
Lawrence 3 parameter fn



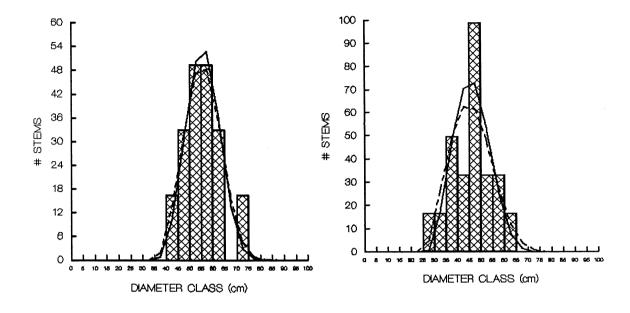
RO955/4/10/0 (939sph)

RO955/4/5/0 (1400sph)



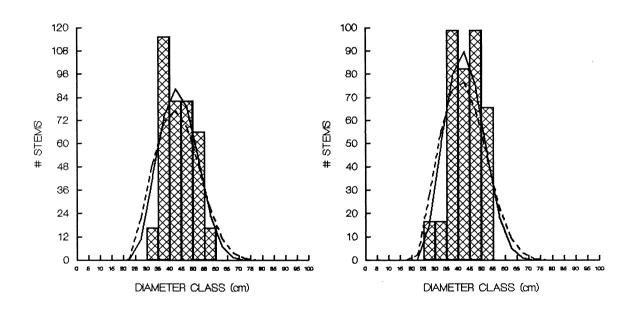
'Garcia' 2 parameter fn
'Garcia' 3 parameter fn
Lawrence 3 parameter fn

## COMPARISON OF 'OLD' vs 'NEW' FUNCTIONS and DATA



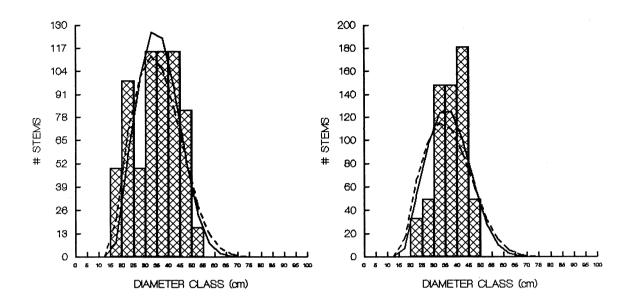
RO955/4/14/0 (379sph)

RO955/4/7/0 (379sph)



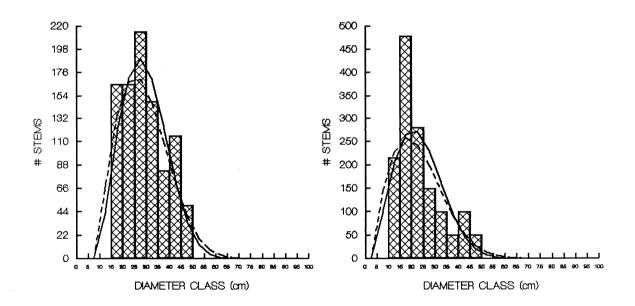
Lawrence (1989)

Goulding & Shirley (1979)



RO955/4/10/0 (939sph)

RO955/4/5/0 (1400sph)



Lawrence (1989)

----- Goulding & Shirley (1979)

#### PETTERSON HEIGHT COEFFICIENT

Given a diameter distribution function and a method for calculating its parameters, a corresponding height function is required before use in stand volume generators such as Program PROD or STANDPAK (1988). The traditional function used in New Zealand is the Petterson height function which takes the form:

$$\frac{1}{(H-1.4)^{0.4}} = \alpha + \frac{b}{D}$$

where: H = top height (m)

D = DBH (cm)

'a' and 'b' coefficents define the shape of the curve

In order to predict the average height of trees in each diameter class, an equation specific to each function must be used to calculate the intercept 'a'. Firstly, the mean top diameter (MTD) is calculated from the diameter distribution. Top height, MTD and the predicted intercept are then substituted in the height equation to solve for 'b'. Finally, the Petterson equation can be evaluated over all the diameter classes to derive the average height of trees in each class.

Using the same database involved in the development of the Weibull parameters, a similar stepwise regression technique was employed to derive an equation for the intercept term. Graphical examination of the data indicated a clear relationship between top height and 'a', and this was subsequently included via a ratio with the other significant variables. The resulting equation is:

$$a = (1.2555 * lnA + 0.039341 * BAREA + 0.05003 * DMEAN) / MTH  $R^2 = 0.83$$$

\*\*\*\*\*

## PART II

WEIBULL PARAMETER ESTIMATION EQUATIONS FOR THE REGIONAL STAND GROWTH MODELS

#### HAWKES BAY

The Hawkes Bay growth model (NAPIRAD) was developed in 1983, based on approximately 200 plots from Wharerata, Patunamu, Mohaka, Esk, Gwavas and Kaweka Forests. The model covers a reasonable range of sites and conditions (see Table 4), although there was limited data available from unthinned/highly stocked stands and from the poorer sites in the region.

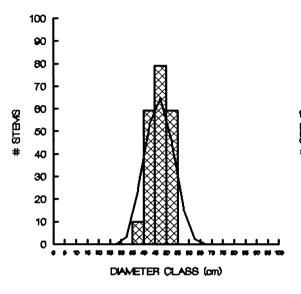
Table 4
RANGE OF DATA

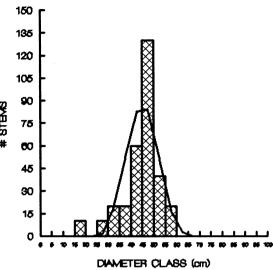
Variable	Min	Max	Mean
Age (yrs)	4.1	29.2	
Site index (m)	20.6	38.5	29.8
Basal area (m2/ha)	0.6	90.3	
Stocking (sph)	99	2772	
Top height (m)	3.6	47.0	

This data was used in conjunction with the "best" method identified in Part I to construct equations (below) for deriving the Weibull coefficients for Hawkes Bay. Examples of their application are given on the next page; plot details are contained in Appendix 1.

 $R^2 = 0.78$ 

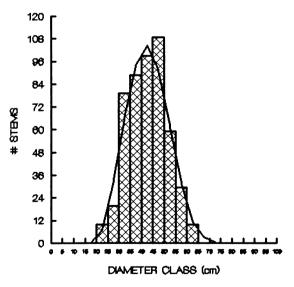
DMIN = 
$$-2.482 + 0.9429 *$$
 DMEAN -  $0.7027 *$  AGE +  $1104.6$  / STOCKING  $R^2 = 0.90$  DMAX =  $2.302 + 0.9144 *$  DMEAN +  $0.08995 *$  BAREA +  $0.3316 *$  MTH  $R^2 = 0.98$  DVAR =  $0.94946 *$  (((DMIN<sup>2</sup> - DMAX<sup>2</sup>) \*  $0.00007854$ )/4)<sup>2</sup>  $R^2 = 0.92$  a =  $0.64058 - 0.15254 *$  lnH +  $0.01692 *$  lnB +  $0.001143 *$  DMEAN

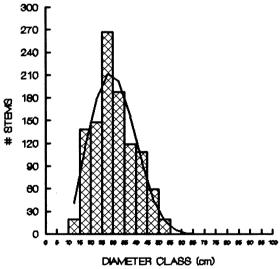




WN1150/1/85/3 (504sph)

WN313/1/201/4 (1067sph)





#### CANTERBURY

The Canterbury growth model (CANT) was developed in 1988, based on approximately 200 plots from Eyrewell, Balmoral, Burnham, Bottle Lake, Ashley, Hanmer, Omihi, Waimate and Geraldine Forests. The model covers a wide range of sites and conditions found on both the Canterbury plains and foothills. (See Table 5).

Table 5
RANGE OF DATA

Variable	Min	Max	Mean
Age (yrs)	4.0	33.2	
Site index (m)	15.4	31.3	22.4
Basal area (m2/ha)	0.6	73.1	
Stocking (sph)	124	5377	
Top height (m)	2.8	34.1	

This data was used in conjunction with the "best" method identified in Part I to construct equations (below) for deriving the Weibull coefficients for Canterbury. Examples of their application are given on the next page; plot details are contained in Appendix 1.

 $R^2 = 0.81$ 

DMIN = 
$$-1.675 + 0.8399 * DMEAN - 0.19437 * BAREA + 0.1195 * AGE$$

$$R^2 = 0.87$$

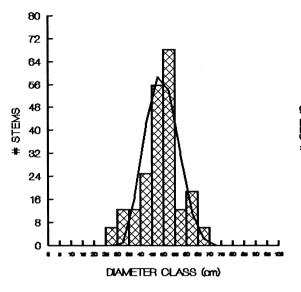
$$DMAX = 0.9745 * DMEAN + 0.08466 * BAREA + 0.352 * MTH$$

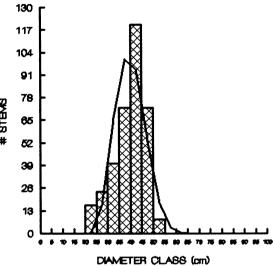
$$R^2 = 0.98$$

$$DVAR = 0.8976 * (((DMIN^2 - DMAX^2) * 0.00007854)/4)^2$$

$$R^2 = 0.94$$

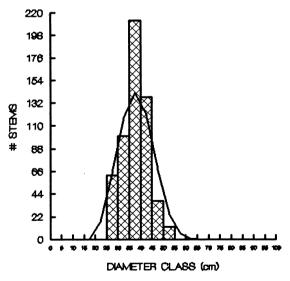
$$a = (1.0055 * lnA + 0.01877 * BAREA + 0.06723 * DMEAN) / MTH$$

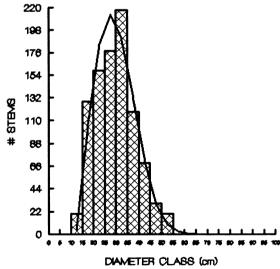




CY425/0/16/0 (563sph)

CY101/0/1/0 (939sph)





#### NELSON

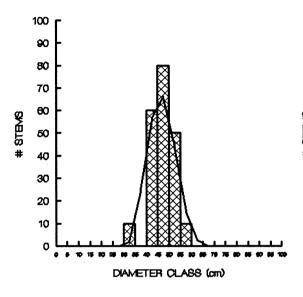
The Nelson growth model was developed in 1989 as a replacement for the earlier Golden downs model (GDNS81). It is based on approximately 410 plots from Golden Downs, Motueka, Wairau, Rai, Hira, and Tutaki Forests, and those of Baigent Forest Industries. The model covers the wide range of soil types and conditions found in the Nelson region. (See Table 6).

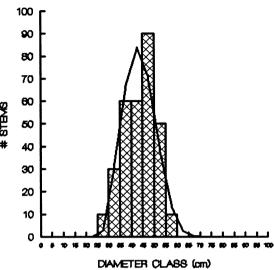
Table 6
RANGE OF DATA

Variable	Min	Max	Mean
Age (yrs)	3.0	46.7	27.4
Site index (m)	18.7	35.7	28.5
Basal area (m2/ha)	0.3	86.8	24.3
Stocking (sph)	110	4075	597
Top height (m)	2.0	49.0	38.2

This data was used in conjunction with the "best" method identified in Part I to construct equations (below) for deriving the Weibull coefficients for this region. Examples of their application are given on the next page; plot details are contained in Appendix 1.

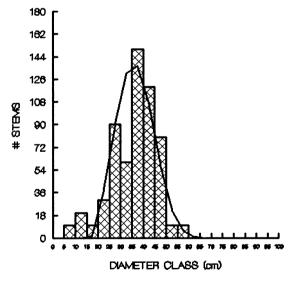
DMIN = 
$$-2.578 + 1.0686 * DMEAN - 0.17184 * BAREA - 0.1961 * MTH R2 = 0.87$$
  
DMAX =  $2.49 + 0.8433 * DMEAN + 0.12475 * BAREA + 0.3158 * MTH R2 = 0.96$   
DVAR =  $0.96574 * (((DMIN2 - DMAX2) * 0.00007854)/4)^2$   
R<sup>2</sup> = 0.90  
a =  $(-1.623 + 1.5215 * lnA + 0.5701 * lnB + 0.05071 * DMEAN) / MTH R2 = 0.82$ 

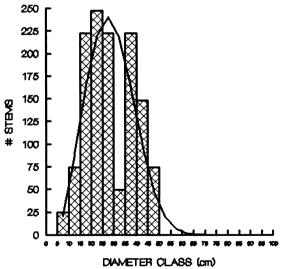




NN462/0/78/3 (590sph)

NN462/0/69/5 (1284sph)





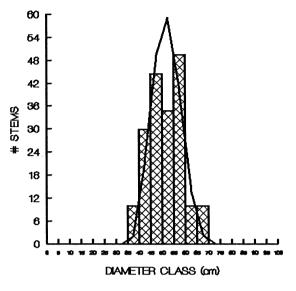
#### NORTH ISLAND SANDS

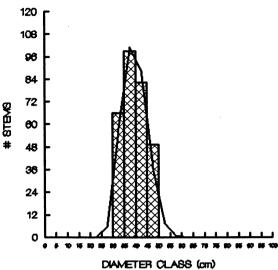
The North Island sands growth model (SANDS) was developed in 1984, based on approximately 510 plots from Aupouri, Mangawhai, Woodhill, Waiuku, Santoft, Tangimoana and Waitarere Forests. The model covers a reasonable range of sites and conditions (see Table 7), although the very poor (protection) sites and unthinned stands are not well represented.

Table 7
RANGE OF DATA

Variable	Min	Max	Mean
Age (yrs)	3.8	46.1	
Site index (m)	12.8	35.6	25.0
Basal area (m2/ha)	0.8	70.7	
Stocking (sph)	99	2536	
Top height (m)	3.1	42.3	

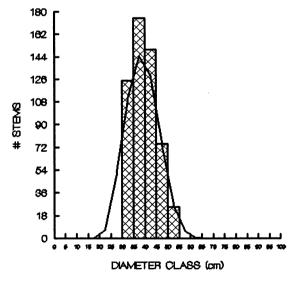
This data was used in conjunction with the "best" method identified in Part I to construct equations (below) for deriving the Weibull coefficients for the region. Examples of their application are given on the next page; plot details are contained in Appendix 1.

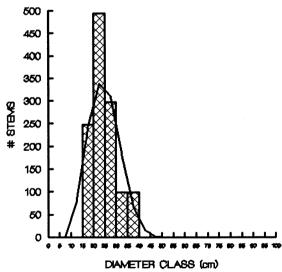




AK520/9/4/0 (550sph)

AK517/0/7/0 (1238sph)





#### AUCKLAND CLAYS

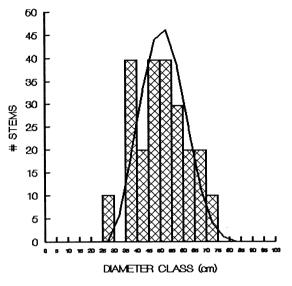
The Auckland Clays growth model (CLAYSFERT) was developed in 1987, based on plots from Glenbervie, Riverhead, Whangapoua and Maramarua Forests. The model is derived from the earlier model (CLAYS), which was restricted to adequately fertilised stands from the above forests plus Tairua, Waipoua, Puhipuhi and Waitangi Forests. The range of data incorporated in the original model is given in Table 8.

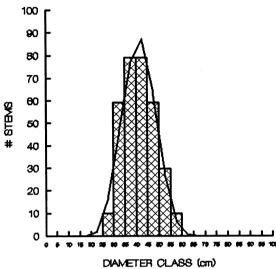
Table 8
RANGE OF DATA

Variable	Min	Max	Mean
Age (yrs)	4.9	31.9	14.8
Site index (m)	23.5	38.7	30.2
Basal area (m2/ha)	1.9	76.4	29.9
Stocking (sph)	100	2247	711
Top height (m)	4.1	43.9	22.5

This data was used in conjunction with the "best" method identified in Part I to construct equations (below) for deriving the Weibull coefficients. Examples of their application are given on the next page; plot details are contained in Appendix 1.

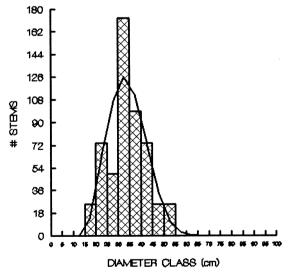
DMIN = 
$$-4.631 + 0.9145 *$$
 DMEAN -  $0.0902 *$  BAREA -  $0.00853 *$  AGE<sup>2</sup> R<sup>2</sup> =  $0.80$  DMAX =  $0.9072 *$  DMEAN +  $0.5315 *$  MTH +  $0.001584 *$  SPH R<sup>2</sup> =  $0.94$  DVAR =  $0.91563 *$  (((DMIN<sup>2</sup> - DMAX<sup>2</sup>) \*  $0.00007854$ )/4)<sup>2</sup> R<sup>2</sup> =  $0.93$  a =  $(-2.751 + 0.01739 *$  BAREA +  $1.417 *$  InA +  $1.356 *$  InD) / MTH R<sup>2</sup> =  $0.84$ 

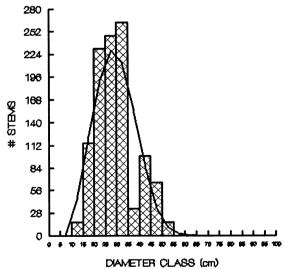




AK439/0/4/0 (543sph)

AK401/0/11/0 (1087sph)





#### SOUTHLAND

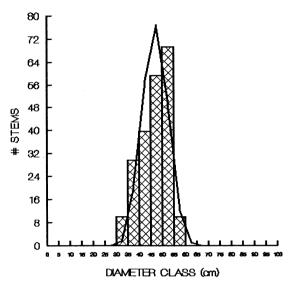
The Southland growth model (SGM3) was developed in 1988. The model has an improved height/age function, but is otherwise very similar to the earlier model SGM2/SOUTH (1982).

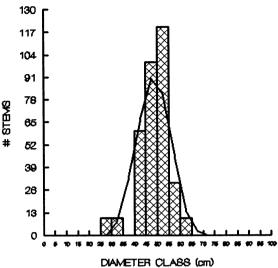
Table 8
RANGE OF DATA

Variable	Min	Max	Mean
Age (yrs)	10.0	48.0	
Site index (m)	15.4	32.8	24.5
Basal area (m2/ha)	11.9	124.6	
Stocking (sph)	136	3249	
Top height (m)	0.1	56.6	

This data was used in conjunction with the "best" method identified in Part I to construct equations (below) for deriving the Weibull coefficients for the model. Examples of their application are given on the next page; plot details are contained in Appendix 1.

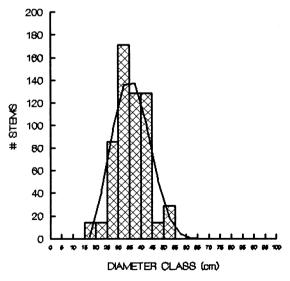
DMIN = 
$$-2.307 + 0.9437 * DMEAN - 0.16583 * BAREA - 0.003365 * AGE2 R2 = 0.86
DMAX =  $1.780 + 0.8534 * DMEAN + 0.08649 * BAREA + 0.4399 * MTH$  R<sup>2</sup> = 0.97  
DVAR =  $0.85971 * (((DMIN2 - DMAX2) * 0.00007854)/4)2$  R<sup>2</sup> = 0.91  
a =  $(-3.02 + 2.1955 * lnA + 0.3828 * lnB + 0.03244 * DMEAN) / MTH$  R<sup>2</sup> = 0.93$$

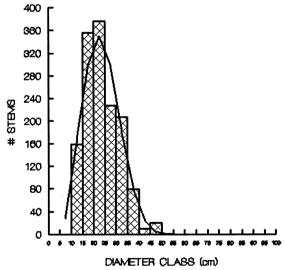




SD681/2/4/0 (586sph)

SD448/2/1/0 (1436sph)





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APPENDIX 1

## EXAMPLE PLOT STATISTICS

## Hawkes Bay:

Forest	Plot	Age	Barea	Sph	Top Ht
Mohaka Kaweka Esk	WN 1320/1/109/1 WN 2100/1/107/1 WN 1150/1/85/3	21.0 26.0	36.44 50.15 75.44	208 310 504	28.5 26.3 36.7
Gwavas	WN 313/1/201/4	25.0	83.93	1067	36.4

## Canterbury:

Forest	Plot	Age	Barea	Sph	Top Ht
Ashley	CY 560/1/5/0	24.0	42.43	217	32.5
Hanmer	CY 560/2/1/0	25.0	45.25	352	29.2
Geraldine	CY 425/0/16/0	26.1	66.24	563	28.7
Bot. Lake	CY 101/0/1/0	33.0	70.81	939	37.5

## Nelson:

Forest	Plot	Age	Barea	Sph	Top Ht
G. Downs	NN 446/0/76/7	26.1	36.67	210	36.2
G. Downs	NN 379/0/5/0	27.0	47.25	310	37.8
Rai	NN 462/0/78/3	27.1	64.52	590	33.6
Rai	NN 462/0/69/5	30.0	92.90	1248	34.6

## North Island Sands:

Forest	Plot	Age	Barea	Sph	Top Ht
Woodhill	AK 321/0/30/0	31.1	40.33	188	33.8
Woodhill	AK 434/0/19/0	25.0	37.11	297	29.7
Aupouri	AK 520/9/4/0	28.0	67.29	550	27.8
Woodhill	AK 517/0/7/0	23.1	63.75	1238	34.8

## Auckland Clays:

Forest	Plot	Age	Barea	Sph	Top Ht
Maramarua	AK 570/2/3/0	29.3	48.73	227	42.9
Waipoua	AK 711/3/2/0	25.1	44.68	326	30.5
Riverhead	AK 439/0/4/0	25.0	51.72	543	35.9
Whangapoua	AK 401/0/11/0	27.1	80.34	1087	36.1

## Southland:

Forest	Plot	Age	Barea	Sph	Top Ht
Rankleburn	SD 170/0/19/0	21.2	38.03	217	27.4
Longwood	SD 588/0/4/0	24.2	64.01	340	30.3
Rankleburn	SD 681/2/4/0	27.0	62.04	586	33.4
Rankleburn	SD 448/2/1/0	23.0	69.21	1436	28.5

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