DEVELOPMENT OF NEW STAND-LEVEL HEIGHT-AGE CURVES FOR DOUGLAS-FIR IN NEW ZEALAND

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Abstract

Data from a total of 9,487 measurement occasions in 1,930 sample plots aged from 4 to 130 years were used to fit new height/age curves for Douglas-fir in New Zealand, using a base age of 40 years. A range of six model forms were fitted to the data, with Chapman-Richards, Hossfeld and Schumacher functions showing general superiority based on the Schwarz's Bayesian Criterion (SBC). The best model form was found to be the Chapman-Richards. The results showed significant relationships between all parameters and latitude and longitude, but weaker relationships with altitude and stocking. Latitude was strongly linked to the shape parameter, which could be expressed as a linear combination of latitude and site index. Hence, a polymorphic form of the preferred function (Chapman-Richards) was applied that included a linear combination of site index and latitude. These new curves are recommended for use throughout New Zealand.

Table of Contents

Introduction

A height/age equation is a basic component of stand growth and yield models, and is used to predict height development of a stand given the growth conditions and the stand age. Previously, height/age functions for Douglas-fir in New Zealand have been constructed at the regional level. For example, Table 1 shows a list of the tables/functions available for Douglas fir in New Zealand.

Table 1 - Existing Douglas-fir height/age tables in STANDPAK

Standpak Table	Investigator(s)	Year	Region
12	Burkhart&Tennent	1977	Rotorua
17	Leitch	1983	Canterbury
18	Leitch	1979	Nelson
28	Leitch	1982	Southland-Otago
31	Tennent	1984	Nelson
39	Law	1990	South Island
41	Liu Xu	1990	Rotorua

There has been no previous attempt to build a national height/age model for Douglas fir in New Zealand. Furthermore, most current stand-level height/age curves available for Douglas-fir in New Zealand as implemented in STANDPAK (Whiteside and Sutton, 1983) were developed pre-1990s. New data have been continuously collected since then and are available in the Permanent Sample Plot (PSP) system (Ellis and Hayes 1997). This report describes the derivation of a new stand-level height/age equation for Douglas-fir in New Zealand that includes environmental parameters.

Data

Data from sample plots throughout New Zealand aged from 4 to 130 years were extracted from the Permanent Sample Plot (PSP) system. The data consisted of height measurements taken at yearly or two-yearly intervals, and was systematically screened for obvious anomalies, i.e. suspect measurements, and data from stands with high mortality, were omitted from further analysis. Data from a total of 9,487 measurement occasions in 1,930 sample plots were used. Descriptive statistics of the data are shown in

Table 2.

Table 2 - Descriptive statistics of the data

Variable	Mean	Std Dev	Min	Max
MTH (m)	23.35	9.90	2.00	57.80
Age (yrs)	29.52	13.78	4.00	130.13
Site Index (m)	32.09	3.98	12.90	45.50
Latitude (S)	40.87	2.56	37.90	46.20
Altitude (m, a.s.l.)	441.54	157.51	12.00	960.00
Longitude (E)	173.99	2.71	167.70	177.80
Stocking (Stems/ha)	791.52	614.47	87.00	4025.00

Methods

The stand-level aggregate height used is mean top height (MTH), which is the predicted mean height of the 100 largest diameter well-formed stems per hectare (Burkhart and Tennent 1977). Likewise, most height/age curves include a site-specific parameter that reflects a site's ability to sustain height growth, i.e. site index (*SI*). Site index is traditionally defined as the height of the dominant trees in the stand at a predetermined age (Burkhart and Tennent 1977 - in this study the base age was set to 40 years from planting. The remaining parameters of the model are all 'global' parameters as they have fixed coefficients for all stands.

A range of equations used for modelling height/age was described by Kimberley and Ledgard (1998). In this study, six models were fitted to the data, using the SAS macro NLINMIX version 8 (Littell *et al.*, 1996). Three models showed general superiority, i.e. Chapman-Richards, Hossfeld and Schumacher functions:

$$MTH = 0.25 + (MTH_{40} - 0.25)\left\{\frac{1 - \ell^{-\ell^{a}T}}{1 - \ell^{-\ell^{a}40}}\right\}^{1/(b)} \quad (1)$$

The three selected models were then compared using Log Likelihood ($\chi 2$) and Schwarz's Bayesian Criterion (SBC) (Schwarz 1978). The model that yields the SBC value closest to zero is considered the best. SBC is the preferred criteria compared to the Log Likelihood or $\chi 2$-test, as SBC is useful for comparing models with different numbers of parameters. Log Likelihood statistics do not account for the number of terms in the model and the number of observations used.

To link environmental variables and height growth, correlations were obtained between the fitted parameters of the Chapman-Richards model for each plot to stocking, altitude, latitude and longitude using polymorphic forms of the Chapman-Richards function, e.g.

$$MTH = 0.25 + (MTH_{40} - 0.25)\left\{\frac{1 - \ell^{-\ell^{a}T}}{1 - \ell^{-\ell^{a}40}}\right\}^{\frac{1}{[b+((p+q.latitude).MTH_{40})]}} \quad (2)$$

The goodness-of-fit statistics for each of the models were evaluated and the best model was chosen based on SBC and log-likelihood. Stocking was included in the analysis as it has been shown to significantly affect the height/age relationships of planted Douglas-fir in the Pacific North West (Flewelling et al, 2001)

The best performing model (Equation 2) was applied to the data, and the predictions were compared to data by calculating the residuals. The residual mean top height was calculated as the actual value minus the estimated value. Predicted site index for each plot was estimated by re-arranging the algebraic format of Equation 2.

Actual site index for each plot was calculated in the PSP system by using the height measurement nearest to the base age of 40 years. Site index can be predicted using the height and age observations of any measurement, but the risk of error is least when using measurements closest to age 40 years.

The accuracy of the mean top height predictions across a range of ages was examined by cross validation using the Root Mean Square Error (RMSE) for different ages.

$$RMSE = \sqrt{\frac{\sum_i \left(MTH_i - \Pr edictedMTH_i\right)^2}{N}} ,$$

where N is the total number of measurements.

Finally, data from the Bay of Plenty, Waikato, Malborough, Canterbury, Otago and Southland were used to test the new model against existing height/age models H39 developed for Douglas-fir in the South Island (Law, 1990) and H41 developed for Douglas-fir in the Central North Island (Xu, 1990)

Results

Table 3 lists the final maximized value of the Log Likelihood (χ^2) and Schwarz's Bayesian Criterion (*SBC*) (Schwarz 1978) for the three non-linear models. From this it is evident that the Chapman-Richards equation provides the best fit.

Table 3 - Fit statistics of the three types of non-linear mixed models.

Model	Criteria	Statistics
Chapman-Richards	$-2\chi^2$	30,267.5
	SBC	-15,141.3
Hossfeld	$-2\chi^2$	30,319.9
	SBC	-15,167.5
Schumacher	$-2\chi^2$	30,633.7
	SBC	-15,324.4

The correlation between environmental variables and the local variable in a polymorphic Chapman-Richards function is presented in

Table 4. These results show significant relationships between all parameters, and latitude and longitude, but not with altitude and stocking. Latitude was strongly linked to the shape parameter (*b*), which could be expressed as a linear combination of latitude and site index (i.e. mean top height at age 40 years). Hence, a polymorphic form of the Chapman-Richards equation was applied that included a linear combination of site index and latitude for the shape parameter.

Table 4 – Correlation between environmental variables and the parameters of the anamorphic Chapman-Richards equation (* = significant at P<0.05)

Parameter	Latitude	Altitude	Longitude	Stocking
MTH_{40}	-0.21812*	-0.04385	0.16196*	-0.07915*
a	-0.14383*	-0.00674	0.11452*	-0.02317
b	-0.18265*	0.01484	0.15292*	-0.04521

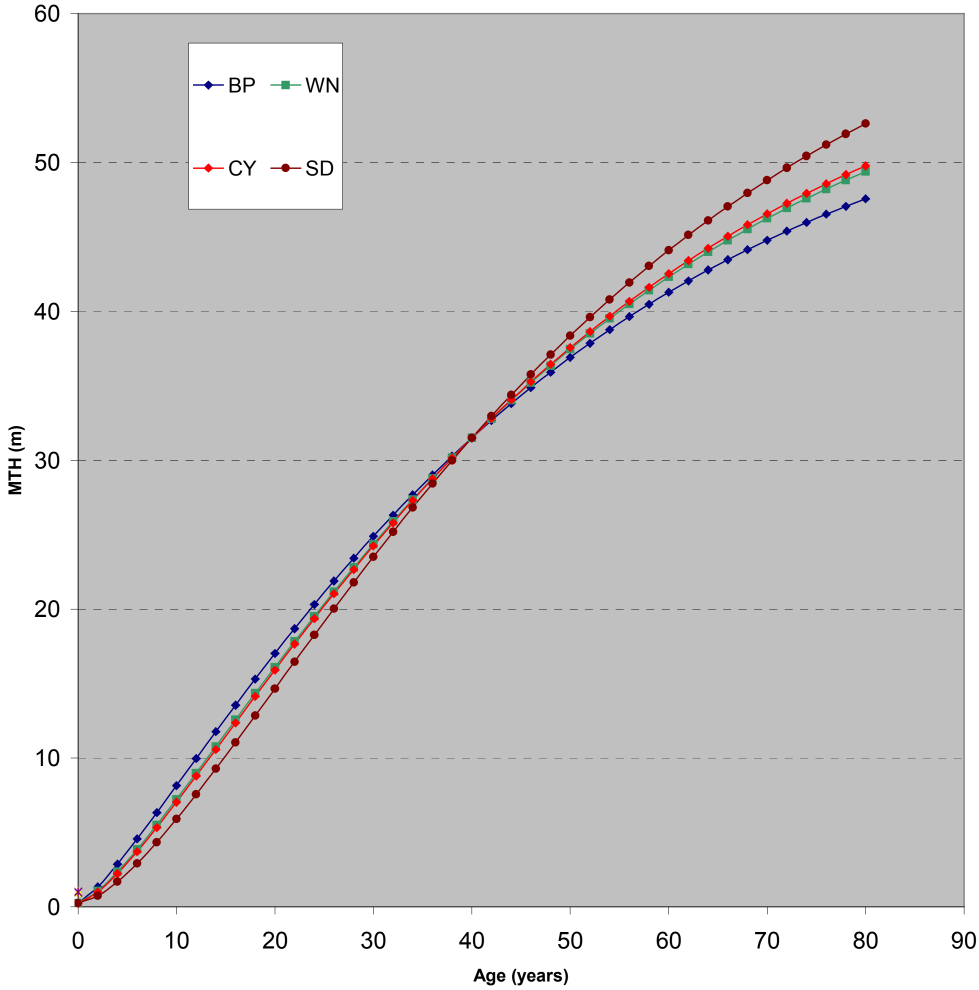
Although the correlation of stocking with MTH_{40} was just significant at the 95% level, the relationship was rather weak in comparison to latitude with MTH_{40} and its presence in the model would complicate the function without improving the fit at a practical level. Excluding any site effect from the slope parameter (*a*) did not penalise the overall fit of the height/age curve. Hence only the shape parameter was modified by latitude. Parameter estimates are shown in Table 5.

SBC of this function was –14,651.5 and $-2\chi^2$ was 29,288, representing a substantial improvement on the fit statistics shown in table 3. Figure 1 shows the Chapman-Richards height/age curves for a fixed site index (31.5m) and a range of latitudes .

Table 5 – Parameter estimates for the polymorphic form of the Chapman-Richards equation that includes latitude and site index in the shape parameter

Coefficient	Estimate	Std. error
a	-3.7082	0.01375
b	0.3844	0.01835
p	0.0338	0.00087
q	-0.00057	0.00002

Figure 1 - The Polymorphic Chapman-Richards height/age curves. Solid lines represent the height growth from Bay of Plenty to Southland for SI=31.5m, i.e. it shows the effect of latitude



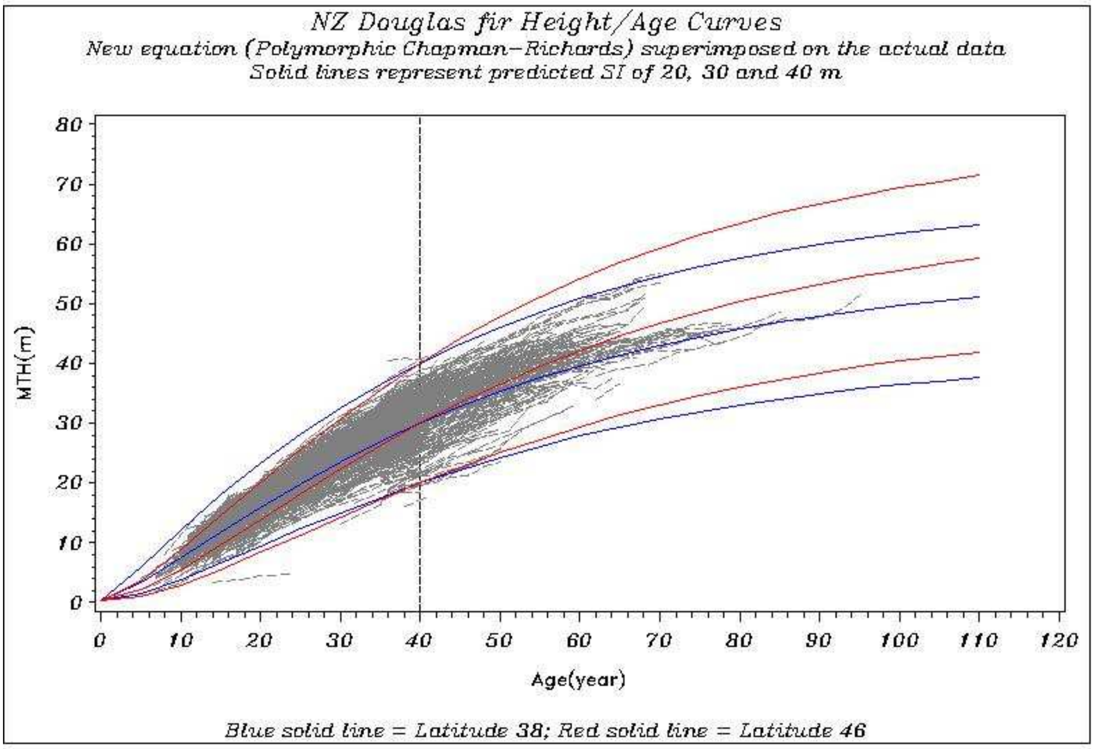


Figure 2. Fitted height/age curves for Site Indices of 20m, 30m, and 40m (MTH at age 40 years), showing effects of latitudes 38S and 46S, together with the data.

Table 6 shows the data distribution across 5 regions used on the cross validation test. Results from the cross validation are presented in Table 7- Table 13.

Table 6. Descriptive statistics for regions used in cross validation

Region (no. plots)	Variable	Mean	Std Dev	Min	Max
Bay of Plenty (420 plots)	MTH	24.91588	10.24129	4.9	51.6
	Age	30.77157	15.0133	8	95.04
	SI	34.41524	3.38675	19.3	41.8
	Latitude	38.44381	0.224161	37.9	38.9
	Altitude	476.5929	140.807	195	800
	Longitude	176.4848	0.170234	176.1	176.8
	Stocking	772.031	569.9809	138	2700
Canterbury (200 plots)	MTH	18.4719	8.529048	6.8	39.3
	Age	23.55443	12.22416	10.13	61.02
	SI	29.764	3.215728	16	38.8
	Latitude	43.105	0.934312	42	44.4
	Altitude	555.52	124.4453	210	960
	Longitude	171.549	1.25678	169.5	173
	Stocking	621.535	457.7892	206	1950
Marlborough (14 plots)	MTH	18.15747	6.090913	6.1	31.4
	Age	23.70126	9.133579	8	46
	SI	31.34286	3.961296	25.1	41.1
	Latitude	41.29286	0.120667	41	41.5
	Altitude	230.4286	105.1341	60	480
	Longitude	173.5	0.161722	173.1	173.7
	Stocking	1839.57	819.8765	463	3383
Otago (134 plots)	MTH	20.61077	10.74043	4.3	47.4
	Age	28.13412	14.5211	7.02	78
	SI	30.10075	4.50808	19.8	44.2
	Latitude	45.64179	0.765742	42.5	46.1
	Altitude	311.6194	145.2859	60	625
	Longitude	169.7746	0.840487	169.1	172.9
	Stocking	1180.48	702.8659	207	2822
Southland (48 plots)	MTH	21.30087	7.671401	5.9	38.9
	Age	29.76065	11.66418	11.02	59.01
	SI	29.58542	3.162815	20.8	34.7
	Latitude	45.96667	0.266045	45.5	46.2
	Altitude	227.25	185.2152	50	500
	Longitude	168.0979	0.289893	167.7	168.6
	Stocking	1234.08	773.6365	200	2716
Waikato (53 plots)	MTH	26.27536	8.5418	5.8	41.4
	Age	31.39478	8.830971	7.03	41
	SI	33.38302	3.820881	20.3	41.7
	Latitude	38.33962	0.100651	38.2	38.6
	Altitude	518.0566	72.42992	470	714
	Longitude	175.7132	0.162951	175.5	176.3
	Stocking	705.9623	470.5798	321	2954

Table 7 - Cross validation results for all data

Age		RMSE	
Measurement	Prediction	Anamorphic	Polymorphic
10	20	*1.19*	1.28
	30	*2.67*	2.82
	40	*3.58*	4.53
20	30	*1.43*	1.52
	40	*2.47*	2.59
	50	*3.56*	3.65
	60	*3.03*	3.64
30	40	*1.32*	1.33
	50	2.65	*2.34*
	60	2.69	*2.36*
	70	*2.22*	2.57
40	50	1.33	*1.23*
	60	2.12	*1.79*
	70	*1.48*	1.76
50	60	1.48	*1.34*
	70	1.91	*1.76*
	80	3.04	*2.68*
60	70	0.99	*0.97*
	80	1.46	*1.35*
70	80	1.27	*1.16*

Table 8 - Cross validation results against data from the Bay of Plenty

Age		RMSE	
Measurement	Prediction	National	H41 CNI
10	10	0.47	0.39
	20	1.34	0.86
	30	2.57	1.63
20	20	0.44	0.48
	30	1.14	1.04
	40	2.34	2.29
30	30	0.43	0.44
	40	1.40	1.46
	50	1.77	1.93
40	40	0.49	0.51
	50	1.07	1.06
	60	1.68	1.49
50	50	0.53	0.50
	60	1.32	1.22
60	60	0.48	0.43
	70	1	0.96
70	70	0.46	0.47
	80	1.27	1.23

Table 9 - Cross validation results for Waikato data

Age		RMSE	
Measurement	Prediction	National	H41 CNI
10	10	0.28	0.17
	20	1.51	1.10
	30	1.96	1.62
20	20	0.42	0.38
	30	1.76	1.64
	40	2.35	1.95
30	30	0.42	0.42
	40	1.07	1.06
40	40	0.85	0.86

Table 10 - Cross validation results for Canterbury data

Age		RMSE	
Measurement	Prediction	National	H39 SI
10	10	0.25	0.24
	20	1.09	1.06
	30	6.08	6.71
20	20	0.23	0.20
	30	1.44	1.22
	40	0.82	0.85
30	30	0.47	0.50
	40	0.48	0.54
	50	0.70	1.03
40	40	0.41	0.43
	50	1.15	1.06
	60	2.02	2.77
50	50	0.53	0.55
	60	2.10	2.56
60	60	0.71	0.79

Table 11 - Cross validation results for Marlborough data

Age		RMSE	
Measurement	Prediction	National	H39 SI
10	10	0.37	0.18
	20	0.92	0.60
	30	1.80	1.58
20	20	0.30	0.30
	30	0.94	0.94
30	30	0.59	0.65
	40	0.48	0.80
	50	0.56	1.40
40	40	0.45	0.46
	50	0.13	0.23
50	50	0.04	0.02

Table 12 - Cross validation results for Otago data

Age		RMSE	
Measurement	Prediction	National	H39 SI
10	10	0.65	0.96
	20	1.39	1.49
	30	2.73	2.80
20	20	0.80	1.03
	30	2.06	1.69
	40	3.23	2.36
30	30	0.53	0.48
	40	1.59	1.43
	50	2.67	2.10
40	40	0.59	0.57
	50	1.41	1.26
	60	1.64	1.88
50	50	0.36	0.35
	60	1.28	1.40
60	60	0.47	0.49
	70	0.96	1.10

Table 13 - Cross validation results for Southland data

Age		RMSE	
Measurement	Prediction	National	H39 SI
10	10	0.28	0.25
	20	1.98	4.15
	30	4.01	7.63
20	20	1.22	2.19
	30	2.58	3.87
30	30	0.51	0.48
	40	1.72	1.54
40	40	1.06	1.04
	50	1.29	1.30
	60	1.35	1.22
50	50	0.59	0.58
	60	0.73	0.68

Discussion

Of the various functions tested, the Chapman-Richards model performed the best. Both anamorphic and polymorphic forms of Chapman-Richards model were explored and tested against available data from across New Zealand. Regional differences were explored and compared to the previous models. Generally, the polymorphic form outperformed the anamorphic form, especially in prediction above 30 years of age. The polymorphic form permitted environmental variables such as latitude and site index to be included in the formulation to account for site differences.

Initial height growth in the southern part of the country tended to be slower than the northern parts, but this reversed at later ages. Although the new national model did not consistently produce lower RMSE at all ranges of ages, the difference in performance when compared to existing regional models was minimal. The preferred model is therefore the new polymorphic Chapman-Richards form, as it provides a simpler nation-wide model without any significant loss of fit to the data.

Furthermore, height prediction is normally calculated according to the growth model region assigned to the plot. Some confusion can occur when the area of interest is on the border between different growth regions. This new national model overcomes this problem, as it is borderless. Adjustment in height prediction for various locations is modelled by including a latitude effect in this new national model. This can be set to the nearest degree of latitude without any loss in precision.

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