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AN INITIAL GROWTH AND YIELD MODEL FOR *EUCALYPTUS SALIGNA* SM. IN NEW ZEALAND

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REPORT NO. 16 OCTOBER 1991

Project Record 3061

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A report prepared on contract for the Management of Eucalypts Cooperative

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EXECUTIVE SUMMARY

AN INITIAL STAND GROWTH & YIELD MODEL FOR *EUCALYPTUS SALIGNA* SM. IN NEW ZEALAND

This report describes the construction of a growth model for *E. saligna*, formulated from permanent sample plot data, in Auckland and Rotorua Regional forests.

Eighty-three permanent sample plots were available for model building.

Site index was assigned to each plot, by either:

(a) interpolation of top heights around age 20; or

(b) fitting a difference equation.

The top height function $H = S \exp(\beta(1/20^{\gamma} - 1/T^{\gamma}))$ is utilised in the growth model.

Of the 4 models tested a variant of the Schumacher yield-age function had the greatest precision of the site index variable being highly significant.

$$G_2 = G_1^{(T_1/T_2)\beta} \exp(\alpha(1-(T_1/T_2)^{\beta})) \exp(\gamma(1 - (T_1/T_2)^{\beta}))$$

where G_2, G_1 = net basal area/ha at times T_1 and T_2 and S = site index.

Essentially, no mortality occurs in thinned stands.

A model for unthinned stands was assayed, but it must be regarded as provisional; variation is high, and little data runs over an appreciable time period. A plausible model is:

$$N_2 = N_1 \exp[-\beta(T_2^{\gamma} - T_1^{\gamma})]$$

For volume production many models were assayed, but none performed better than

$$V = \beta_0 + \beta_1 H + \beta_2 GH$$

The constructed model represents a very plausible initial simulator for *E. saligna*. Testing to data shows logical growth projections, but it is inevitable that further data will compromise predictions to some extent, particularly if used in areas not covered by current data. Specifically, much data used here is procured from immature sample plots; site index is difficult to estimate accurately, and these may need to be revised with more measures available.

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INTRODUCTION

This report describes the construction of a growth model for *E. saligna*, formulated from permanent sample plot data, in Auckland and Rotorua Regional forests. [Geographically, the model is limited to frost-free or very low frost-rated areas.]

Eighty-three permanent sample plots were available for model building, with stand characteristics:

	Mean	s.d.	Min	Max	
Basal area/ha	14.95	±11.23	0.22	47.27	(m^2/ha)
Stocking	663	±775	40	3133	(stems/ha)
Age	14	±9.9	2	35	(years)
Top-height	23.2	±13.65	3.2	50.8	(m)

A minority of plots experienced coppice growth over and above original establishment stockings; these data were ignored in the model construction.

SITE INDEX

Site index was assigned to each plot, by either:

(a) interpolation of top-heights around age 20;

or

(b) fitting a difference equation, of form

$$H_2 = H_1 \exp[-\beta(1/T_2 - 1/T_1)] \tag{1}$$

where H_2, H_1 = top height at ages T_2 and T_1, respectively.

By definition, when T = 20, H = S (site index), so we have:

$$S = H \exp[-\beta(1/20 - 1/T)] \tag{2}$$

and a site index value was assigned to plots by choosing (H, T) nearest to age 20. [In some instances, this amounted to extrapolating over 15 years; while this represents the best available estimate at present, it is inevitable some estimates will be revised at a later date.]

The plot site indices have summary statistics:

	Mean	s.d.	Min.	Max.	(Site index)
Auckland	33.1	±5.0	18.3	39.8	(m)
Rotorua	24.3	±10.2	11.0	42.0	

TOP-HEIGHT PROJECTION

Several models were assayed:

$$H_2 = \alpha - \beta((\alpha - H_1)/\beta)^{(T_1/T_2)^\gamma} \tag{3a}$$

$$H_2 = H_1^{(T_1/T_2)^\beta} \exp\,[\alpha(1 - (T_1/T_2)^\beta)] \tag{3b}$$

$$H_2 = H_1 \left[\frac{1 - \exp\,(-\beta T_2)}{1 - \exp\,(-\beta T_1)}\right]^\gamma \tag{3c}$$

which represent different equations constructed from the Weibull (Ratkowsky, 1990), log-reciprocal (Schumacher, 1939), and Chapman-Richards (Clutter *et al.*, 1983) yield equations, respectively.

After extensive modelling, a model

$$H_2 = H_1 \exp\,(-\beta(1/T_2^\gamma - 1/T_1^\gamma)) \tag{4}$$

was adopted, which models the data very satisfactorily. Parameter estimates were obtained from the non-linear routine available from SAS, PROC NLIN, using the Marquardt convergence routine (SAS Institute, 1985; Draper & Smith, 1981).

$$\hat{\beta} = 3.868\ 692\ 694$$
$$\hat{\gamma} = 0.374\ 510\ 987$$

Equation (4) can be manipulated to accommodate site index, through:

$$S = H_1 \exp\,(-\beta(1/20^\gamma - 1/T_1^\gamma))$$

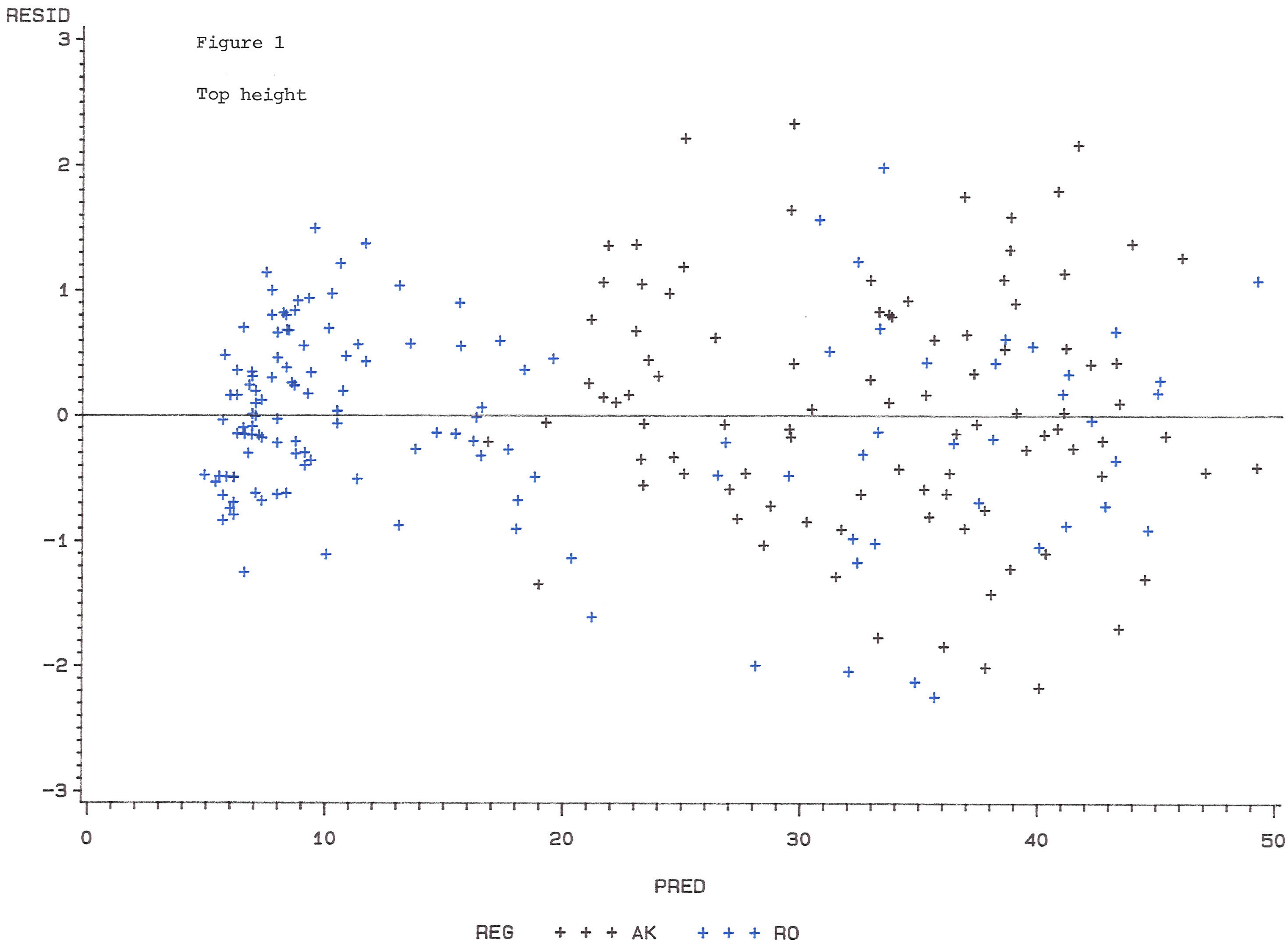
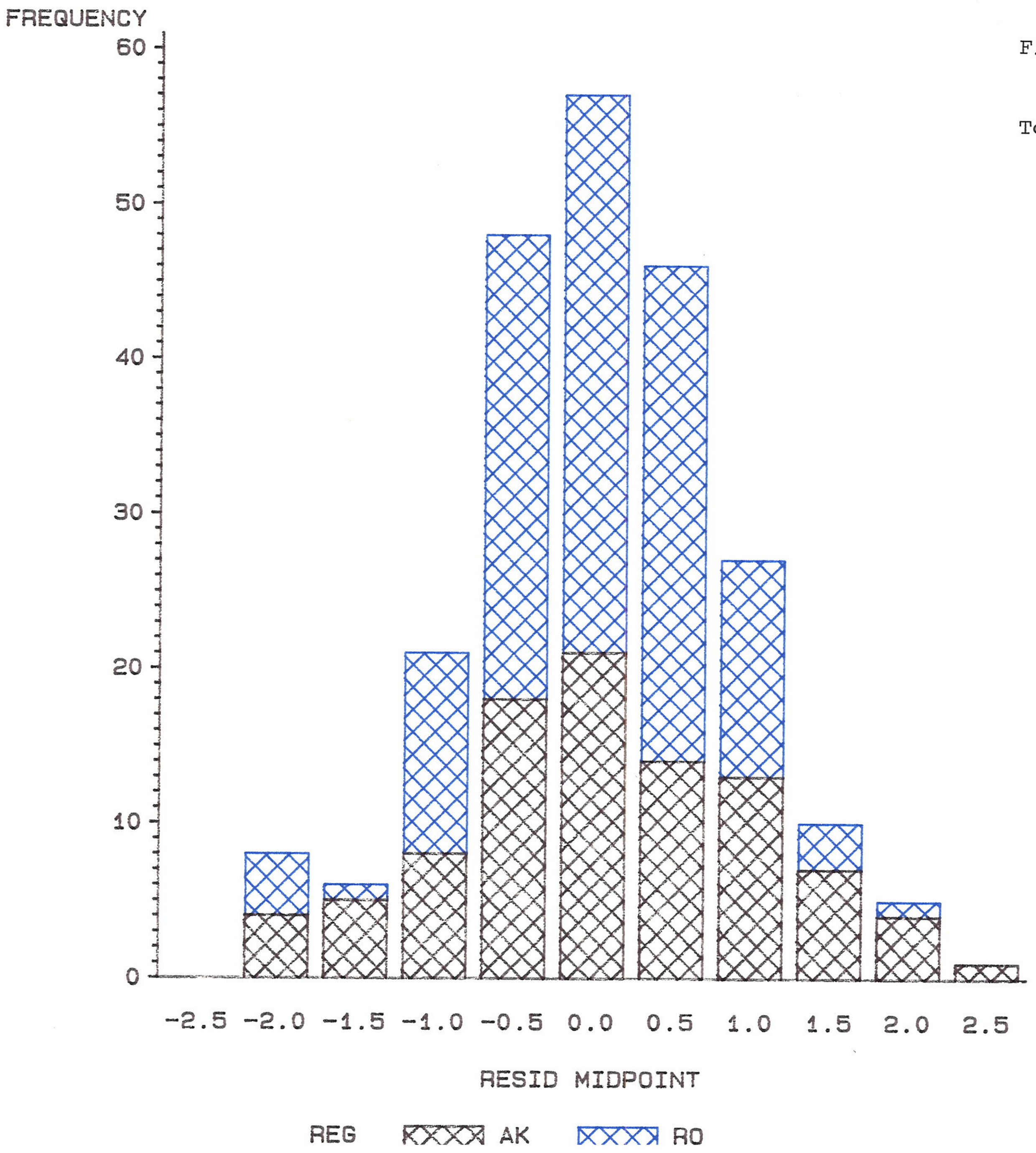

RESID
Figure 1
Top height
3
2
1
0
-1
-2
-3
0
10
20
30
40
50
PRED
REG + + + AK + + + RO

Figure 2

Top height



giving

$$H = S \exp(\beta(1/20^{\gamma} - 1/T^{\gamma})) \tag{5}$$

Formulation (5) is the top height function utilised in the growth model. Figures 1 and 2 depict, respectively:

(1) a plot of residual vs predicted values for the function, with data distinguished between the Auckland and Rotorua regions. Overtly, the equation is unbiassed by region;

(2) a histogram of residuals, subdivided by regions, which substantiate conclusion (1).

RESIDUAL STATISTICS

Mean	s.d.	Skewness	Kurtosis
0.022	0.647	-0.06	0.28

which also indicate a well-behaved equation. A test for non-normality is non-significant at the 5% level.

NET BASAL AREA/HA

Four models were assayed:

$$G_2 = G_1^{(T_1/T_2)} \exp(\alpha(1 - (T_1/T_2))) \tag{6}$$

$$G_2 = G_1^{(T_1/T_2)^{\beta}} \exp(\alpha(1 - (T_1/T_2)^{\beta})) \tag{7}$$

$$G_2 = \alpha - \beta((G_1 - \alpha)/\beta)^{(T_1/T_2)^{\beta}} \tag{8}$$

$$G_2 = G_1^{(T_1/T_2)^{\beta}} \exp(\alpha(1 - (T_1/T_2)^{\beta})) \exp(\gamma S(1 - (T_1/T_2)^{\beta})) \tag{9}$$

where G_2, G_1 = net basal area/ha at times T_1 and T_2. S = site index.

Models (6), (7) and (9) are variants of the Schumacher (1939) yield-age function used by Clutter (1963) and generalised later by Clutter & Jones (1980). Such functional forms have been utilised by numerous modellers for many years (see Clutter *et al.* (1983), Chapter 4). Model (8) is a Weibull formulation.

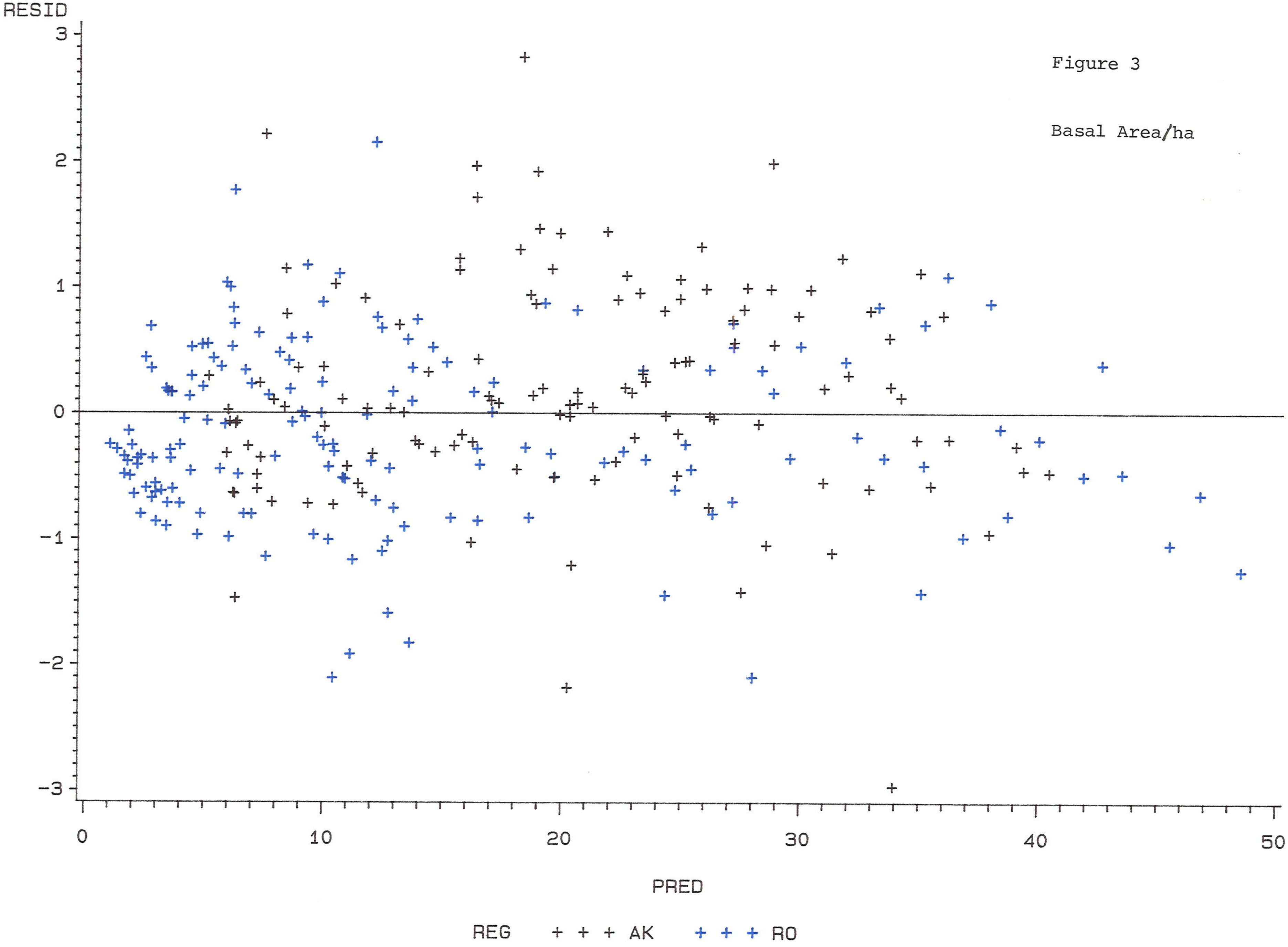
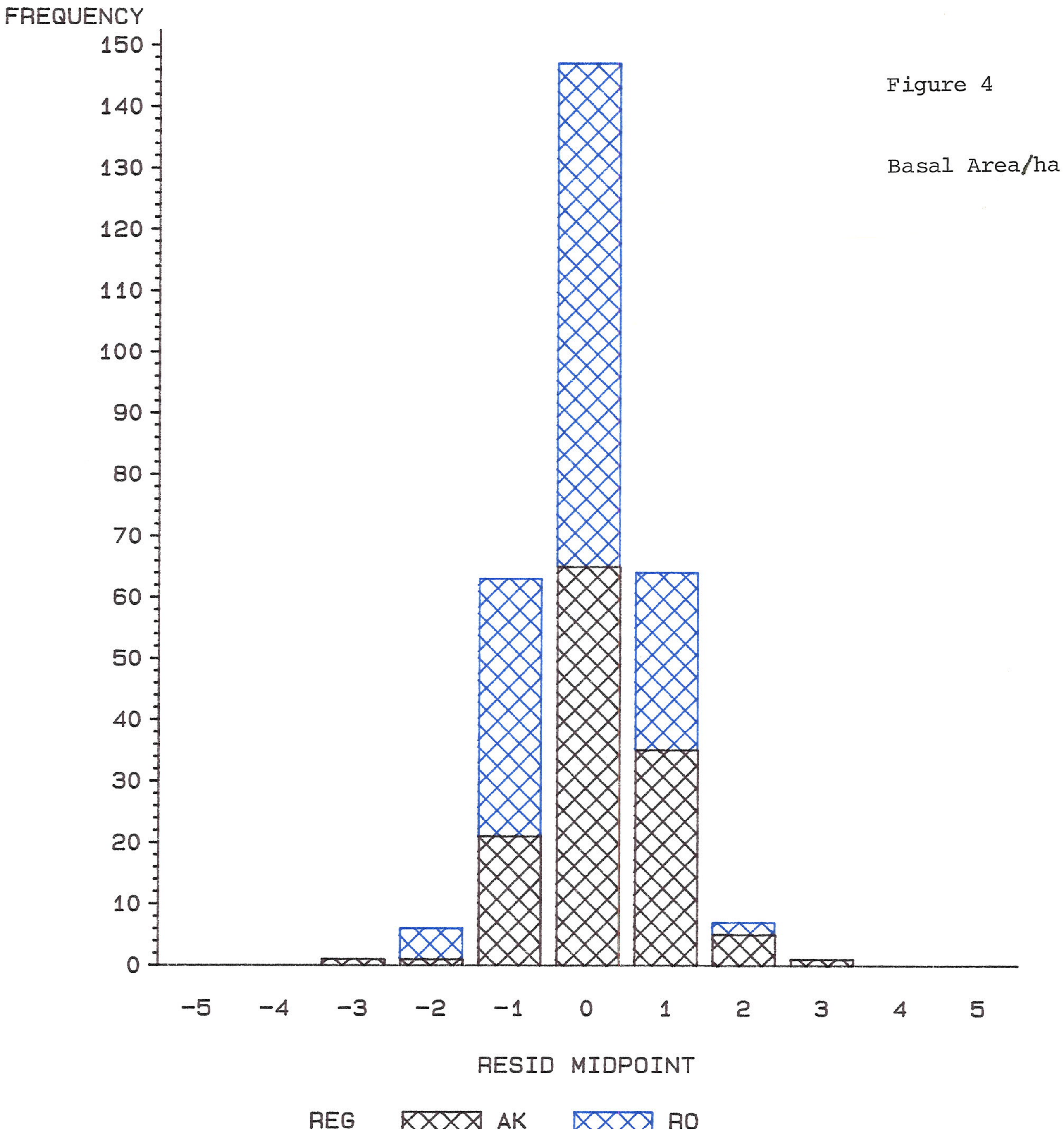


Figure 3

Basal Area/ha


FREQUENCY
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
RESID MIDPOINT
REG
AK
RO
Figure 4
Basal Area/ha

Model (6) quickly showed to be unsuitable while model (8) proved inferior to the Schumacher variants, (models 7 and 9). Model (7) however, increased precision by 30%, and residual patterns produced no overt systematic plottings, indicative of a plausible model. Model (9), on the other hand, increased precision by a further 12%, with the site index variable being highly significant. Residual plottings are very satisfactory (see Figure 3 and Figure 4), with summary statistics:

Residuals	Mean	σ^2	Skewness	Kurtosis
(G/ha)	-0.0043	0.63	0.103	1.02

A Kolmogorov-Smirnov test for normality is totally accepted ($p < 0.91$). Parameter estimates for model 9 are:

$\alpha = 3.574\,256\,067$

$\beta = 0.823\,549\,750$

$\gamma = 0.027\,146\,286$

(The parameter associated with site index, is logically positive.)

LIVE STEMS/HA

Overtly, for thinned stands, essentially no mortality occurs, hence a model

$$N_2 = N_1 \text{ [irrespective of time period]} \tag{10}$$

suffices.

A model for unthinned stands was assayed, but it must be regarded as provisional; variation is high, and little data runs over an appreciable time-period. A plausible model is:

$$N_2 = N_1 \exp\left[-\beta(T_2^{\gamma} - T_1^{\gamma})\right] \tag{11}$$

which seems to fit available data adequately. Parameter estimates are:

$\beta = 0.007\,690\,840$

$\gamma = 1.286\,896\,444$

For completeness, and given models (10) and (11) do not contribute to any other stand variable estimates, they are included in the growth-model.

VOLUME/HA

Many models were assayed, but none performed better than

$$V = \beta_0 + \beta_1 H + \beta_2 GH \qquad (12)$$

where in (12)

V = volume/ha
G = basal area/ha (at equivalent time)
H = top-height (at equivalent time).

Model (12) gave an ANOVA

Source	d.f.	SS	MS
Regr.	2	7195999	3597999***
Error	340	5883	17.30

R2 = 0.9992 c.v. = 3.23%

Parameter estimates		t-value
β_0	1.278988	2.402
β_1	0.212858	5.399
β_2	0.322518	273.776

Residual patterns were satisfactory, with no overt systematic trends.

Models including regional dummy variables were explored to an extent, but then abandoned. Although statistically significant, predictions were extremely close to those of model 12, and may well be confounded with grouped proximity, rather than a true regional difference.

GROWTH MODEL

Models (5), (9) and (10) to (12) were utilised to construct a growth and yield model. Two programs were written, in BASIC and FORTRAN, the latter acting as an arithmetic check.

The source code for the BASIC program is given in the Appendix.

Mean stand diameter ($\bar{d}$) is estimated through the relationship

$$\sqrt{\left(\frac{40000 \times G}{N \times \pi}\right)} = \bar{d}_q$$

The following convention was adopted for top height (H_1) and site index (S) inputs:

(1) Where S is given, but H_1 is not, then H_1 is estimated, and H at age 20 = S.

(2) Where H_1 is given, but S is not, then S is estimated, and H at age 20 = estimated (S).

(3) When BOTH H_1 and S are given, then H at age 20 = estimated (S), NOT the given S. Other conventions lead to illogical or inconsistent growth values.

GENERAL

The constructed growth model represents a very plausible initial simulator for *E. saligna*. Testing to date shows logical growth projections, but it is inevitable that further data will compromise predictions a little, particularly if used in areas not covered by current data. Specifically, much data used here is procured from imature sample plots; site index is difficult to estimate accurately, and these may need to be revised with more measures available.

ACKNOWLEDGEMENTS

This model and report are a consequence of an agreement between the writer, the School of Forestry, University of Canterbury, and the Management of Eucalypts Cooperative.

Mrs Heather McKenzie prepared the data for modelling purposes. The data utilised are the property of the Management of Eucalypts Cooperative.

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