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A Volume and Taper Equation for New Zealand Grown *Eucalyptus saligna*

A. Gordon E. Hay

Report No. 8 November 1990

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MINISTRY OF
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Forest Research Institute
Rotorua
New Zealand

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EXECUTIVE SUMMARY

Tree sectional measurement data from various locations in the North Island have been collected to derive a taper equation for *Eucalyptus saligna.*

Over two hundred trees spanning a wide range of tree sizes were suitable for use in the analysis.

Two equations were derived from the measurements. The first, a bark equation, is used to predict under bark diameter from over bark diameter, position on the stem and tree height. The second, a taper equation, is used to predict over bark diameter from position on the stem, tree height and breast height diameter.

These equations can be used to estimate the volume and dimensions of any stem section as required by many yield prediction systems.

INTRODUCTION

Tree volume and taper equations are used to determine under bark stem volume of whole trees given breast height diameter and tree height. They can also predict volume, diameters and taper of stem sections. These equations are basic components of all stand inventory, growth and yield, forest planning and product simulation systems.

Eucalyptus saligna has been planted in New Zealand on a wide range of sites. It is most suited to the warm temperate areas of the North Island, from Northland to Taranaki/Wanganui and Bay of Plenty/Hawkes Bay.

Currently the only equation applicable to *E saligna,* derived from New Zealand grown trees, is a multi-species Eucalyptus merchantable volume table derived in 1961, intended for use in "the National Exotic Forest Survey for estimates of merchantable volume of stands of mixed eucalypt species" (T38, Duff and Bary 1961). This table included only 10 *E saligna* trees and under estimated the sample volume of these trees by 12.3 percent.

Tree sectional measurement data from various locations in the North Island have been collected to derive a taper equation for *E saligna.* This equation is required for calculating basic yield information and predicting the volume and diameter of stem sections when making growth and yield assessments.

Modern yield and pre-harvest assessment systems need equations that can estimate the volume and dimensions of any stem section, hence the approach used here was to develop a general taper equation which met these requirements.

Taper equations are widely used within the MARVL (and MicroMARVL (1989)) pre-harvest inventory system (Deadman and Goulding (1978)) where the flexibility of equations that can estimate volumes and diameters of any stem section is fully exploited. The ability to apply these inventory techniques to *E saligna* stands will benefit the forest manager.

The data collection and derivation of the *E saligna* equations has been undertaken as a project forming part of the work programme of the Management of Eucalypts Cooperative.

NOTATION

Vub	stem volume under bark in m^3
Dbh	breast height (1.4m) diameter over bark in cm
H	total tree height in m
h	level above ground of a point on the stem in m
l	*H* - *h* (length from the tip of the tree in m)
Dob	diameter over bark in cm
Dub	diameter under bark in cm

DATA

Historical data from twelve sites were collated and data collected from three new sites to give a better sample coverage from the climatic range of *E saligna.* The sectionally measured sample trees came from a number of locations. Table 1 gives some sample details.

Table 1. Sample Details

Location	No. of trees	Age	Stocking	Mean *Dbh* (cm)	Mean *H* (m)
Rotoehu A	11	14	1700	23	27
Rotoehu B	5	14	1700	27	29
Rotoehu C	5	11	2240	13	17
Rotoehu D	9	23	1400	40	41
Rotoehu E	19	11	1400	19	21
Silverdale A	8	25	400	42	29
Silverdale B	9	25	400	44	28
HawkesBay	48	3	6000	6	7
Frankton	10	5	6666	9	13
Athenree A	6	33	200	61	38
Athenree B	10	28	200	56	34
Athenree C	9	20	1500	18	21
Tairua	9	18	1000	23	23
Taheke	39	17	800	26	25
Kawerau	34	9	1200 *	16	16
Warkworth	14	13	1000	35	26

* The sample from Kawerau was taken from a Nelder spacing trial.

Bark Subsample

In order to determine an accurate relationship between over and under bark diameter, a subset of the trees described in Table 1 were measured more intensively. At each sectional point on the stem, diameters were measured over bark, then the bark peeled and the diameter under bark measured. These direct measurements of under bark diameters were made on trees from Taheke, Kawerau and Warkworth, a total of forty three trees.

Data Edits

All sectional measurements were run through a comprehensive set of computer edits to screen out possible measurement and recording errors. Trees with extreme or inconsistent measurements were removed. Graphical displays of tree profiles were compared with sample averages to select outliers and atypical trees for more detailed checking. A total of two hundred and forty trees were considered suitable for inclusion in the main data set.

Data Ranges

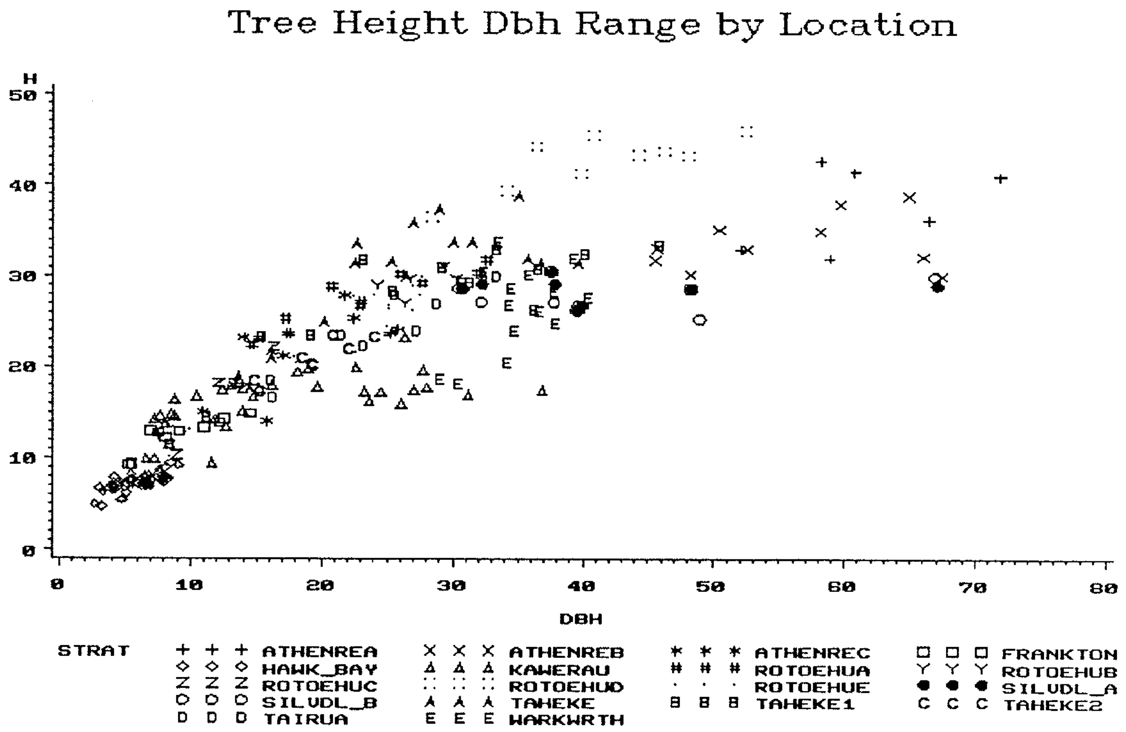
The range of *Dbh* and *H* is shown in table 2.

Table 2. Data Range

Variable		minimum	mean	maximum
Dbh (cm)	Bark sample	4	22	46
	Whole sample	3	26	72
H (m)	Bark sample	7	22	34
	Whole sample	4	27	47
Vub (m^3)	Bark sample	0.004	0.301	1.945
(approximate)	Whole sample	0.002	0.500	6.399

The relationship of *H* to *Dbh* by location is shown in figure 1.

Figure 1.



ANALYSIS

The analysis proceeded in two stages. First the bark subsample was examined to determine how the ratio of sectional area over bark to under bark varied with tree size and position on the stem. An equation was derived for predicting under bark diameter.

The whole sample was then used to derive a taper equation for predicting over bark diameter.

Bark Equation

The sectional area ratio, $\left(\frac{Dub}{Dob}\right)^2$, varied little with tree size but showed a clear relationship with the position up the stem. When plotted over proportion of tree height, $\frac{l}{H}$, two groups of data emerged.

The measurements from Taheke showed slightly thinner bark than those from Warkworth and Kawerau. The differences were found to be statistically significant.

A single bark equation was fitted and the bias with respect to the two groups calculated. This proved to be too small to justify separate equations. The bias is shown in table 3.

The equation fitted to predict *Dub* from *Dob* is:

$$Dub = \sqrt{Dob^2\left(\alpha_0 + \alpha_1\frac{l}{H} + \alpha_2\left(\frac{l}{H}\right)^8\right)} \qquad \ldots 1$$

where $\alpha_0 = 0.8161$ (se. 0.0091)

$\alpha_1 = 0.09528$ (se. 0.01829)

$\alpha_2 = -0.2312$ (se. 0.01524)

Table 3. *Dub* residuals from equation 1.

Group	No. of observations	Residual Mean (cm)	Residual Std. deviation (cm)
Warkworth, Kawerau	173	-0.24	0.62
Taheke	186	0.33	0.59
Pooled	359	0.05	0.69

Taper Equation

Initial plots of $\left(\frac{Dob}{Dbh}\right)^2$ over $\frac{l}{H}$ showed a considerable amount of variation not directly related to the position of the diameter on the stem.

Tree size was clearly shown to be a related factor when the observations were labelled by location. The data from Hawkes Bay which included the smallest trees (mean *Dbh* 6 cm) were on one edge of the band of points.

The coefficients of a simple polynomial function in $\frac{l}{H}$ fitted to $\left(\frac{Dob}{Dbh}\right)^2$, were estimated for each tree and examined for relationships with tree size and total tree taper, $\frac{Dbh}{H}$. Some of the coefficients were associated with *Dbh* and *H*, none with $\frac{Dbh}{H}$.

These relationships provided the basis for constructing a new, more general, model which was fitted to the data and further refined and simplified. The final version was conditioned (using the β_1 coefficient) to ensure that the taper equation passed through *Dbh* at breast height.

The data set includes a preponderance of measurements from small trees. As the most important use of the equation will be in predicting volumes of pre-harvest trees, the observations were weighted by approximate tree volume when the model coefficients were estimated to produce a balanced solution. The taper equation is:

$$Dob = \sqrt{Dbh^2\left(\beta_1\left(\frac{l}{H}\right)^{\frac{\gamma_1}{H^{.2}}} + \frac{\beta_2}{(DbhH)^{.3}}\left(\frac{l}{H}\right)^{\gamma_2}\right)} \qquad \ldots 2$$

where $\beta_2 = 4.298$ (se. 0.061)

$\gamma_1 = 2.610$ (se. 0.122)

$\gamma_2 = 30.72$ (se. 3.09)

$$\beta_1 = \frac{1 - \frac{\beta_2}{(DbhH)^{.3}}\left(1 - \frac{1.4}{H}\right)^{\gamma_2}}{\left(1 - \frac{1.4}{H}\right)^{\frac{\gamma_1}{H^{.2}}}}$$

Analysis of the residuals showed no error trends except for a tendency to over estimate *Dob* above sixty percent of *H* on six of the larger trees. These trees all came from the AthenreeA and AthenreeB samples which were drawn from low stocking stands. It is likely that the small *Dob* measurements were due to heavy branching in the crown resulting in rapid diameter reduction in the 'main' leader.

The taper equation shows very little bias. The mean of the *Dob* residuals is 0.08 cm with a standard deviation of 1.91 cm.

DISCUSSION

Derived Volume Equation

Stem volume under bark from the tip of the tree to a point l_1 is calculated by summing the sectional area from $l=0$ to $l=l_1$. Equations 1 and 2 can be combined and integrated to give an expression for stem volume.

$$Vub_{l_1} = \frac{\pi}{40000}\int_0^{l_1} (Dub^2)\,dl$$

$$= \frac{\pi}{40000}\int_0^{l_1}\left(\left(\alpha_0 + \alpha_1\frac{l}{H} + \alpha_2\left(\frac{l}{H}\right)^8\right)Dob^2\right)dl \qquad \text{from 1}$$

$$= \frac{\pi Dbh^2}{40000}\int_0^{l_1}\left(\left(\alpha_0 + \alpha_1\frac{l}{H} + \alpha_2\left(\frac{l}{H}\right)^8\right)\left(\beta_1\left(\frac{l}{H}\right)^{\frac{\gamma_1}{h^{\cdot 2}}} + \frac{\beta_2}{(DbhH)^{\cdot 3}}\left(\frac{l}{H}\right)^{\gamma_2}\right)\right)dl \qquad \text{from 2}$$

integrating

$$= \frac{\pi Dbh^2}{40000}\Bigg[\ \frac{\alpha_0\beta_1}{H^{\frac{\gamma_1}{H^{\cdot 2}}}\left(\frac{\gamma_1}{H^{\cdot 2}}+1\right)}\, l_1^{\frac{\gamma_1}{H^{\cdot 2}}+1} + \frac{\alpha_0\beta_2}{(DbhH)^{\cdot 3}H^{\gamma_2}(\gamma_2+1)}\, l_1^{\gamma_2+1}$$

$$+ \frac{\alpha_1\beta_1}{H^{\frac{\gamma_1}{H^{\cdot 2}}+1}\left(\frac{\gamma_1}{H^{\cdot 2}}+2\right)}\, l_1^{\frac{\gamma_1}{H^{\cdot 2}}+2} + \frac{\alpha_1\beta_2}{(DbhH)^{\cdot 3}H^{\gamma_2+1}(\gamma_2+2)}\, l_1^{\gamma_2+2}$$

$$+ \frac{\alpha_2\beta_1}{H^{\frac{\gamma_1}{H^{\cdot 2}}+8}\left(\frac{\gamma_1}{H^{\cdot 2}}+9\right)}\, l_1^{\frac{\gamma_1}{H^{\cdot 2}}+9} + \frac{\alpha_2\beta_2}{(DbhH)^{\cdot 3}H^{\gamma_2+8}(\gamma_2+9)}\, l_1^{\gamma_2+9}\ \Bigg] \qquad \ldots 3$$

Interpreting the Equations

Figure 2. Bark thickness predicted by equation 1.

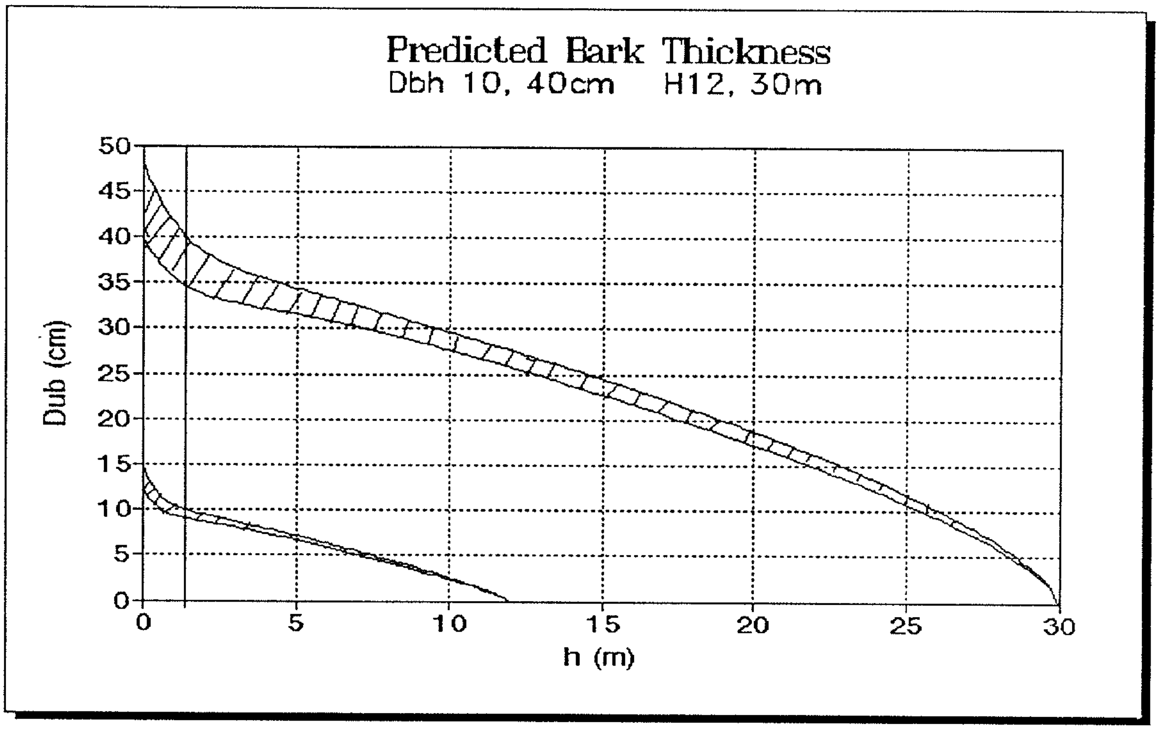
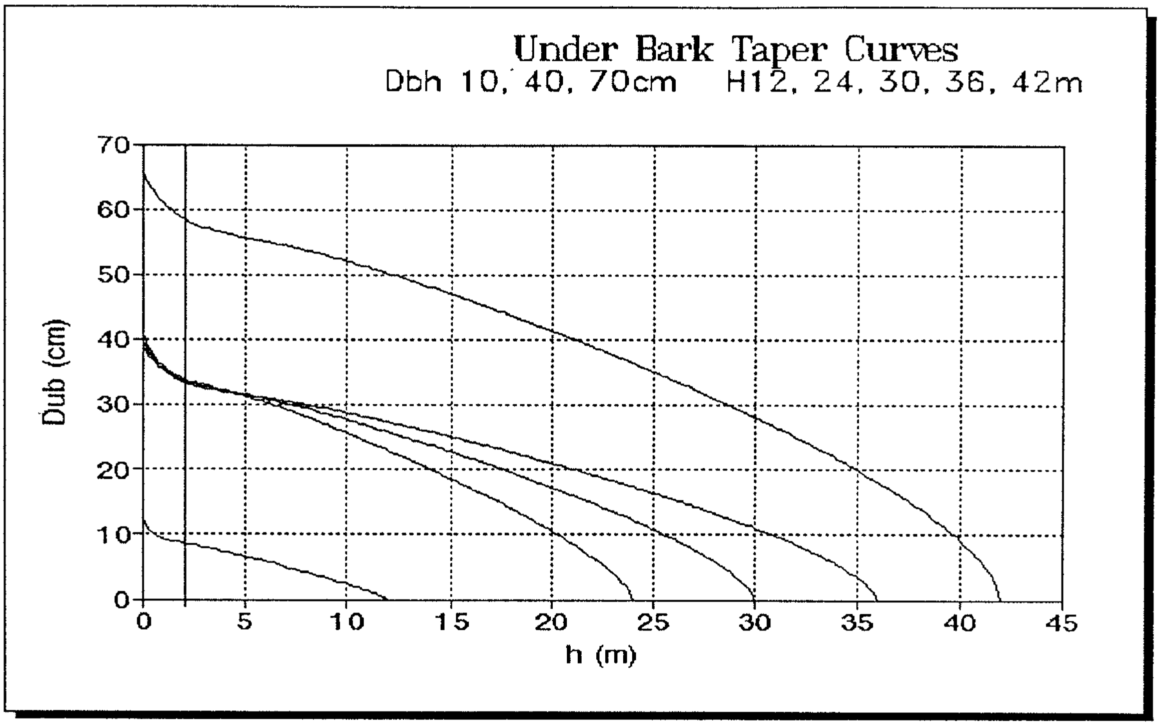


Figure 3. Some taper curves predicted by equation 2.



Equations 1, 2 and 3 should be used to predict the volume and taper of *E saligna* over the range of *Dbh* and *H* shown in figure 1. If the equations are applied outside the range of the sample data the results should be treated with caution.

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