Heather McKenzie

FRI/INDUSTRY RESEARCH COOPERATIVES

MANAGEMENT OF EUCALYPTS COOPERATIVE

FOREST RESEARCH INSTITUTE
PRIVATE BAG
ROTORUA

A Volume and Taper Equation for New Zealand Grown *Eucalyptus nitens*

A. Gordon E. Hay P. Milne

Report No. 9 November 1990

Project Record No. 2605

Forest Research Institute
Rotorua
New Zealand

MINISTRY OF FORESTRY

A Volume and Taper Equation for New Zealand Grown *Eucalyptus nitens*

A. Gordon E. Hay P. Milne

Report No. 9 November 1990

FRI/INDUSTRY RESEARCH COOPERATIVES

EXECUTIVE SUMMARY

Tree sectional measurement data from various locations in New Zealand have been collected to derive a taper equation for *Eucalyptus nitens.*

Nearly three hundred trees spanning a wide range of tree sizes were suitable for use in the analysis.

Two equations were derived from the measurements. The first, a bark equation, is used to predict under bark diameter from over bark diameter, position on the stem and tree height. The second, a taper equation, is used to predict over bark diameter from position on the stem, tree height and breast height diameter.

These equations can be used to estimate the volume and dimensions of any stem section as required by many yield prediction systems.

INTRODUCTION

Tree volume and taper equations are used to determine under bark stem volume of whole trees given breast height diameter and tree height. They can also predict volume, diameters and taper of stem sections. These equations are basic components of all stand inventory, growth and yield, forest planning and product simulation systems.

Eucalyptus nitens has been planted successfully on a wide range of sites throughout New Zealand, from the Bay of Plenty southward. The growth of this species has been very good on many of these sites.

However until now the only equation used for volume assessment has been the multi-species Eucalyptus merchantable volume table derived in 1961, intended for use in "the National Exotic Forest Survey for estimates of merchantable volume of stands of mixed eucalypt species" (T38, Duff and Bary 1961). This table did not include any *E. nitens* trees.

Tree sectional measurement data have been collected to derive a taper equation for *E. nitens.* This equation is required for calculating basic yield information and predicting the volume and diameter of stem sections when making growth and yield assessments.

Modern yield and pre-harvest assessment systems need equations that can estimate the volume and dimensions of any stem section, hence the approach used here was to develop a general taper equation which met these requirements.

Taper equations are widely used within the MARVL (and MicroMARVL (1989)) pre-harvest inventory system (Deadman and Goulding (1978)) where the flexibility of equations that can estimate volumes and diameters of any stem section is fully exploited. The ability to apply these inventory techniques to *E. nitens* stands will benefit the forest manager.

The data collection and derivation of the *E. nitens* equations has been undertaken as a project forming part of the work programme of the Management of Eucalypts Cooperative.

NOTATION

Vub	stem volume under bark in m^3
Dbh	breast height (1.4m) diameter over bark in cm
H	total tree height in m
h	level above ground of a point on the stem in m
l	$H - h$ (length from the tip of the tree in m)
Dob	diameter over bark in cm
Dub	diameter under bark in cm

DATA

Data were collected from a wide range of sites in both the North and South Islands, from the Bay of Plenty through to Southland. The main limiting factor in gathering a wide-ranging data set has been the scarcity of stands over fifteen years old. Table 1 gives some details of the stands the sample sectional measurements were drawn from.

Table 1. Sample Details

Location	No. of trees	Age	Stocking	Mean *Dbh* (cm)	Mean *H* (m)
Nelson ^{B}	44	12	200	27	20
WestCoast ^{B}	19	10	600	27	18
Goudies	38	7	600 *	24	16
GlenTunnel20	23	20	1000	30	25
GlenTunnel30 ^{B}	13	30	200	45	28
Kaingaroa					
0181 ^{B}	8	6	1667	13	12
0481 ^{B}	8	6	1667	14	14
0781 ^{B}	7	6	1667	13	14
1080 ^{B}	9	6	1667	13	11
1082 ^{B}	9	6	1667	16	16
Longwood A ^{B}	20	12	200	24	20
Longwood ^{B}	19	12	200	25	19
Rotoaira	20	11	100	27	20
Rotoehu ^{B}	2	6	6470	11	12
Slab Hut	5	5	1500	10	11
Kaingaroa (Coppice)	52	6	4873	9	10

At locations marked ^{B} bark thickness was measured directly.
* from Shelterbelt

Bark Subsample

In order to determine an accurate relationship between over and under bark diameter, a subset of the trees described in table 1 were measured more intensively. At each sectional point on the stem, diameters were measured over bark, then the bark peeled and the diameter under bark measured. These direct measurements of under bark diameters were made on trees from the locations marked ^{B} in Table 1.

Data Edits

All sectional measurements were run through a comprehensive set of computer edits to screen out possible measurement and recording errors. Trees with extreme or inconsistent measurements were removed. Graphical displays of tree profiles were compared with sample averages to select outliers and atypical trees for more detailed checking. A total of two hundred and ninety six trees were considered suitable for inclusion in the main data set.

Data Ranges

The range of *Dbh* and *H* is shown in Table 2.

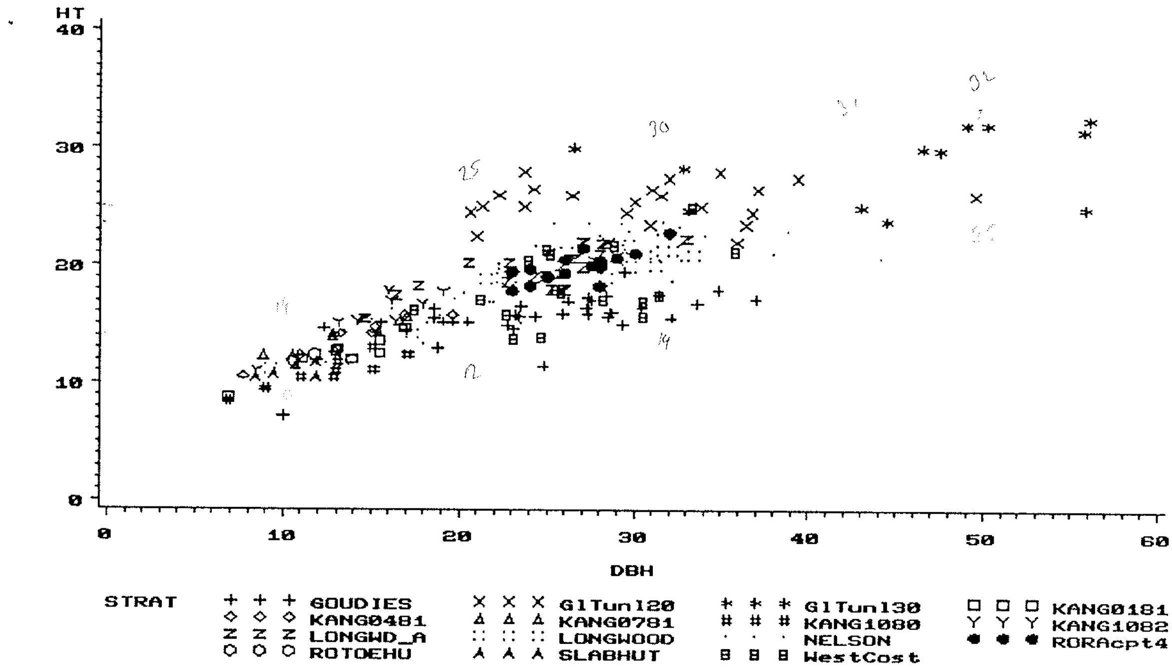
Table 2. Data Range

Variable		minimum	mean	maximum
Dbh (cm)	Bark sample	7	29	56
	Whole sample	6	28	56
H (m)	Bark sample	8	20	33
	Whole sample	6	20	33
Vub (m^3)	Bark sample	0.014	0.480	2.650
(approximate)	Whole sample	0.009	0.447	2.650

The relationship of *H* to *Dbh* by location is shown in figure 1.

Figure 1.

Tree Height Dbh Range by Location



ANALYSIS

The analysis proceeded in two stages. First the bark subsample was examined to determine how the ratio of sectional area over bark to under bark varied with tree size and position on the stem. An equation was derived for predicting under bark diameter.

The whole sample was then used to derive a taper equation for predicting over bark diameter.

Bark Equation

The sectional area ratio, $\left(\frac{Dub}{Dob}\right)^2$, varied little with tree size but showed a clear relationship with the position up the stem. When plotted over proportion of tree height, $\frac{l}{H}$, a clear relationship was discernible. Relative bark thickness decreased quite rapidly from ground level up to twenty five percent of tree height where it reached a minimum. From this point to the top of the tree the relative bark thickness increased slowly, approaching the same proportion as at ground level.

Quite a large amount of variation was present and some part of this may be attributable to location. However tests of the Dub residuals from equation 1 showed the worst bias to be an over estimate of 0.4 cm on the 88 observations from WestCoast.

The equation fitted to predict *Dub* from *Dob* is:

$$Dub = \sqrt{Dob^2\left(\alpha_0 + \alpha_1 \frac{l}{H} + \alpha_2\left(\frac{l}{H}\right)^8\right)} \quad \ldots 1$$

where $\alpha_0 = 0.8066$ (se. 0.0042)

$\alpha_1 = 0.1309$ (se. 0.0079)

$\alpha_2 = -0.1231$ (se. 0.0063)

Table 3. *Dub* residuals from equation 1.

No. of observations	Residual Mean (cm)	Residual Std. deviation (cm)
1051	0.018	0.516

Taper Equation

Plots of $\left(\frac{Dob}{Dbh}\right)^2$ over $\frac{l}{H}$ showed some anomalous points below breast height. By making a simple adjustment to *Dbh* to account for tree height (and hence the relative position of breast height), the variation in this region was reduced. A variable $\left(\frac{Dob}{Dbh\frac{H}{H-1.4}}\right)^2$ was constructed and a number of simple models based on two terms of a polynomial in $\frac{l}{H}$ were fitted.

Using only two terms no solution could be found which fitted very well. Trends in the residual errors were clearly visible even after fitting some very flexible models. A third term in $\frac{l}{H}$ was added and an adequate fit was obtained.

The taper equation is:

$$Dob = \sqrt{\left(Dbh\frac{H}{H-1.4}\right)^2 . \left(\beta_1\left(\frac{l}{H}\right)^2 + \beta_2\left(\frac{l}{H}\right)^{\gamma_2} + \beta_3\left(\frac{l}{H}\right)^{32}\right)} \qquad \ldots 2$$

where $\beta_2 = -0.2957$ (se. 0.0177)

$\gamma_2 = 5.7928$ (se. 0.8199)

$\beta_3 = 0.4963$ (se. 0.0188)

$$\beta_1 = 1 - \frac{\beta_2\left(1-\frac{1.4}{H}\right)^{\gamma_2} + \beta_3\left(1-\frac{1.4}{H}\right)^{32}}{\left(1-\frac{1.4}{H}\right)^2}$$

The taper equation shows very little bias. The mean of the *Dob* residuals is -0.11 cm with a standard deviation of 1.38 cm.

DISCUSSION

Derived Volume Equation

Stem volume under bark from the tip of the tree to a point l_1 is calculated by summing the sectional area from $l=0$ to $l=l_1$. Equations 1 and 2 can be combined and integrated to give an expression for stem volume.

$$Vub_{l_1} = \int_0^{l_1}\left(\frac{\pi Dub^2}{40000}\right)dl$$

$$= \frac{\pi}{40000}\int_0^{l_1}\left(\left(\alpha_0+\alpha_1\frac{l}{H}+\alpha_2\left(\frac{l}{H}\right)^8\right)Dob^2\right)dl \qquad \text{from 1}$$

$$= \frac{\pi Dbh^2H^2}{40000(H-1.4)^2}$$

$$\int_0^{l_1}\left(\left(\alpha_0+\alpha_1\frac{l}{H}+\alpha_2\left(\frac{l}{H}\right)^8\right)\left(\beta_1\left(\frac{l}{H}\right)^{\gamma_1}+\beta_2\left(\frac{l}{H}\right)^{\gamma_2}+\beta_3\left(\frac{l}{H}\right)^{\gamma_3}\right)\right)dl \qquad \text{from 2}$$

integrating

$$= \frac{\pi Dbh^2H^2}{40000(H-1.4)^2}$$

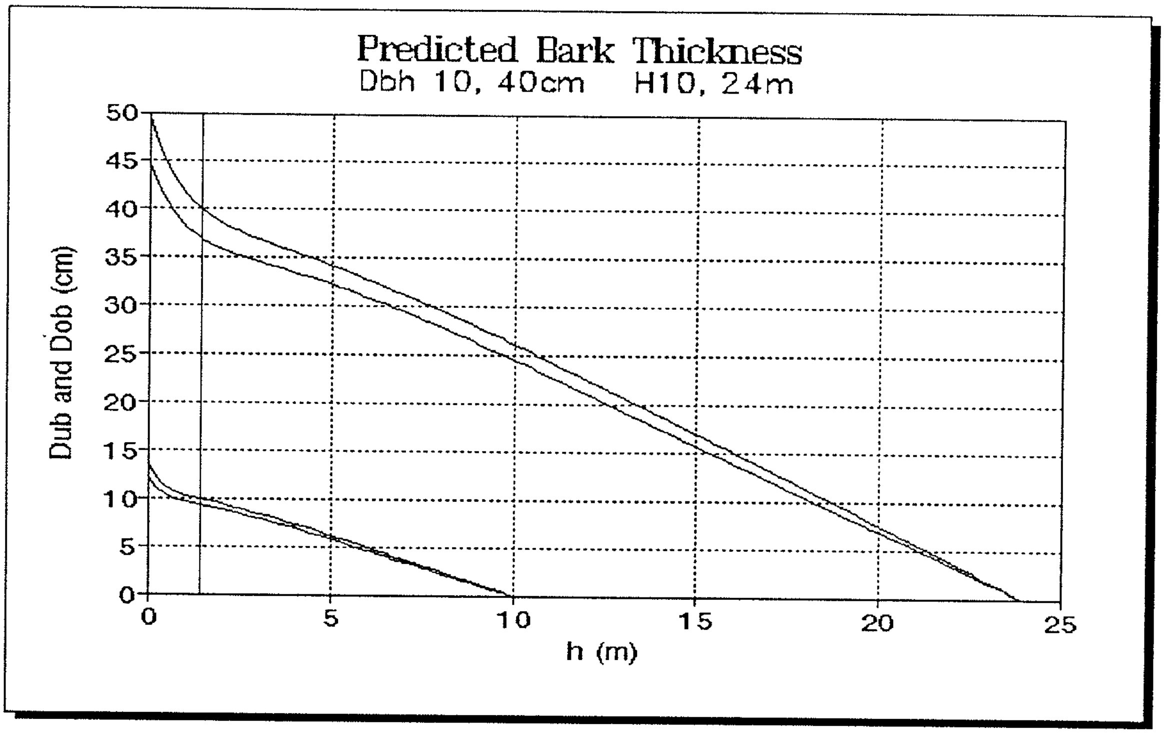
$$\Bigg[\ \frac{\alpha_0\beta_1 l_1^{\gamma_1+1}}{H^{\gamma_1}(\gamma_1+1)}+\frac{\alpha_0\beta_2 l_1^{\gamma_2+1}}{H^{\gamma_2}(\gamma_2+1)}+\frac{\alpha_0\beta_3 l_1^{\gamma_3+1}}{H^{\gamma_3}(\gamma_3+1)}$$

$$+\frac{\alpha_1\beta_1 l_1^{\gamma_1+2}}{H^{\gamma_1+1}(\gamma_1+2)}+\frac{\alpha_1\beta_2 l_1^{\gamma_2+2}}{H^{\gamma_2+1}(\gamma_2+2)}+\frac{\alpha_1\beta_3 l_1^{\gamma_3+2}}{H^{\gamma_3+1}(\gamma_3+2)}$$

$$+\frac{\alpha_2\beta_1 l_1^{\gamma_1+9}}{H^{\gamma_1+8}(\gamma_1+9)}+\frac{\alpha_2\beta_2 l_1^{\gamma_2+9}}{H^{\gamma_2+8}(\gamma_2+9)}+\frac{\alpha_2\beta_3 l_1^{\gamma_3+9}}{H^{\gamma_3+8}(\gamma_3+9)}\ \Bigg] \qquad \ldots 3$$

Interpreting Equation 1

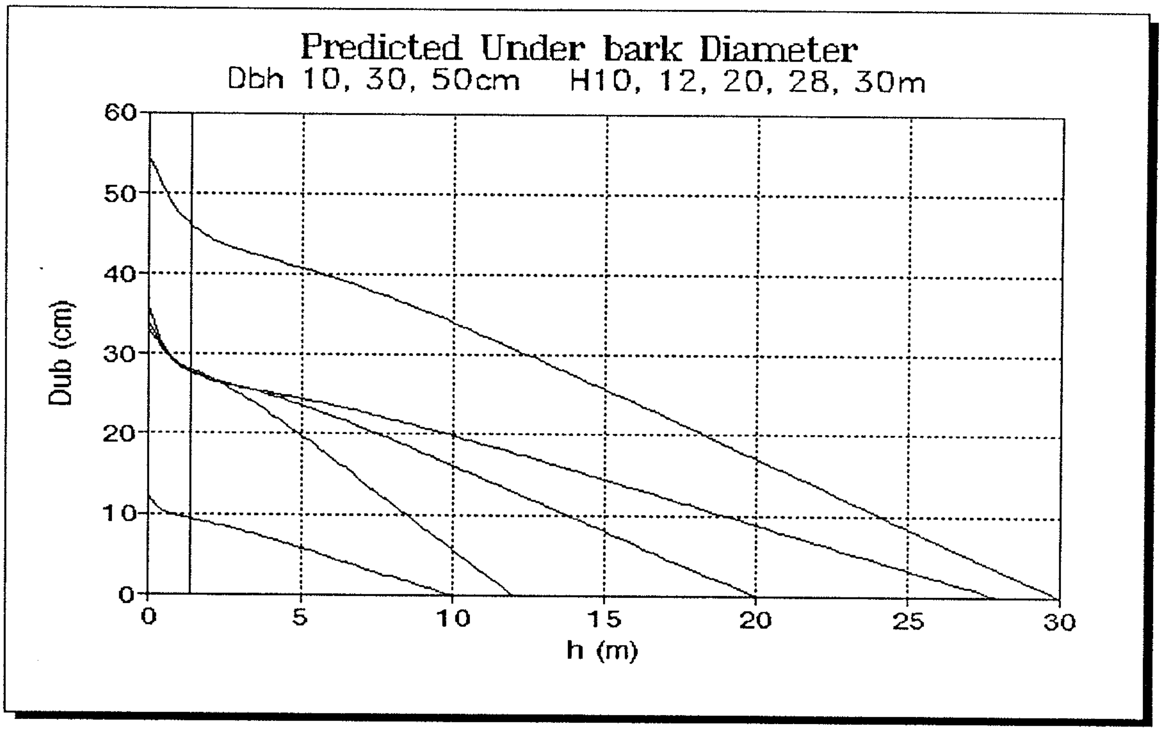
Figure 2.



Interpreting Equation 2

Figure 3 shows some taper curves predicted by equation 2.

Figure 3.

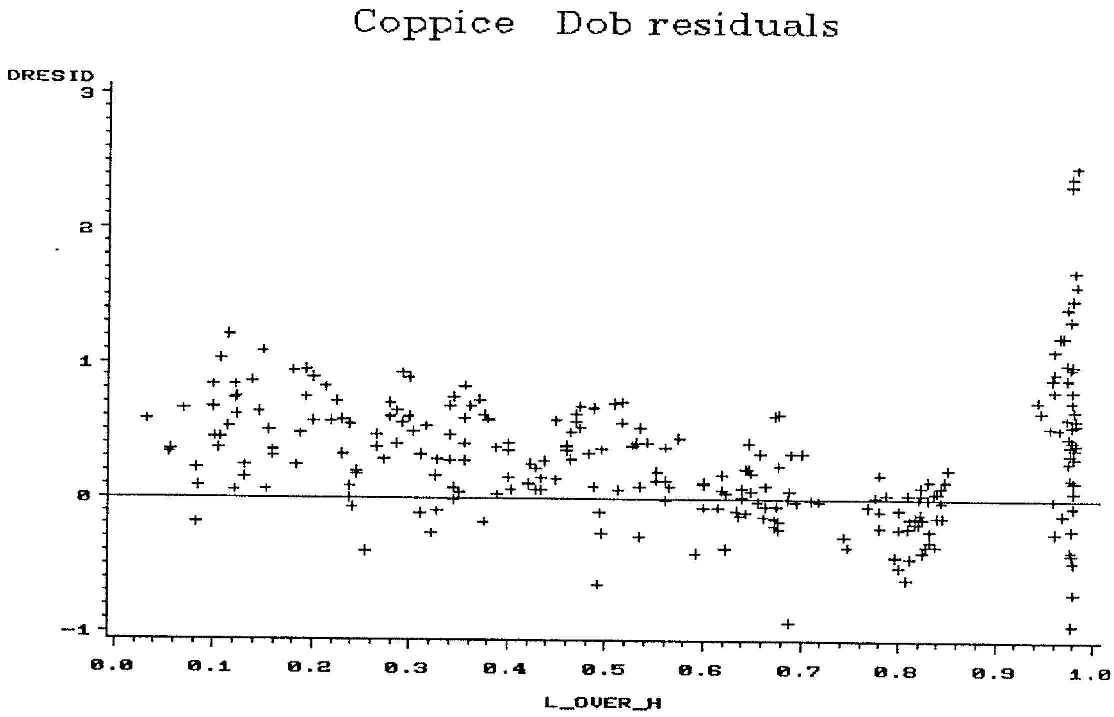


Application to Coppice

Sectional measurement data of six year old coppice stems from Kaingaroa were available but not used in fitting the taper equation. These stems were standing at 4873 stems per hectare when measured and it was considered likely that they would have a more cylindrical shape than the bulk of the sample data.

Equation 2 was used to predict the over bark diameters of the coppice stems and the errors, $Dob - \hat{Dob}$, were calculated. The predictions of *Dob* were reasonably accurate from ground level up to approximately fifty percent of tree height. Above this point equation 2 under estimated *Dob*. The errors are shown in figure 4.

Figure 4.



This result was as expected. But the low taper shown by these stems cannot be wholly attributed to the density of the stand. Comparison of actual and predicted *Dob* of the two sample trees from Rotoehu, which were from a higher stocked stand (table 1), showed the predictions to be very accurate, most of the errors being no more than 0.2 cm.

Coppice stems error plots on *Dbh* and *H* showed no trends in the error with tree size.

The bark thickness of the coppice stems could not be compared with equation 1 but, assuming no large differences in the proportion of bark, equation 3 should predict volume adequately up to half tree height. Above this point the volume will be under estimated.

Equations 1, 2 and 3 should be used to predict the volume and taper of *E. nitens* over the range of *Dbh* and *H* shown in figure 1. If the equations are applied outside the range of the sample data the results should be treated with caution.

ACKNOWLEDGEMENTS

The sectional measurement data used here have been collected over a number of years. The assistance of G.Oliver and H.McKenzie in assembling some of the data sets is gratefully acknowledged.

REFERENCES

GOULDING, C.J. and MURRAY J.C. 1976. Polynomial taper equations that are compatible with tree volume equations. N.Z. Journal of Forestry Science 5: 131-22

DEADMAN, M.W. and GOULDING, C.J. 1978. A Method for the Assessment of Recoverable Volume by Log type. New Zealand Journal of Forestry Science, 8(2) 225-39

DEMAERSCHALK, J. P. 1972: Converting volume equations to compatible taper equations. Forest Science 18(3): 241-5.

DUFF, G. and BARY, G.A.V. 1961 "1961 Volume Table. Eucalyptus Species. All Conservancies". New Zealand Forest Service. Forest Research Institute. Rotorua.

GORDON, A. 1983: Comparison of Compatible Polynomial Taper Equations. New Zealand Journal of Forestry Science, 13(2) 146-55.

LOETSCH,F.; ZOHRER,F.; HALLER,K.E. 1973. Forest Inventory Volume II BLV Verlagsgesellschaft, Munich.

MicroMARVL User Guide. Version 2.1. 1989 Forest Research Institute, Rotorua, New Zealand.

Table derived from Management of Eucalypts Report No 9
A Volume and Taper Equation for New Zealand Grown Eucalyptus nitens

Tree Volumes (cubic metres)

Height (m) \ Diameter (cm)	8	10	15	20	25	30	35	40	45	50	55	60
32										2.0623	2.4954	2.9698
30						0.7060	0.9610	1.2552	1.5886	1.9612	2.3731	2.8242
28					0.4646	0.6690	0.9106	1.1894	1.5054	1.8585	2.2488	2.6762
26					0.4384	0.6314	0.8594	1.1224	1.4206	1.7538	2.1222	2.5256
24				0.2635	0.4117	0.5929	0.8071	1.0541	1.3342	1.6471	1.9930	2.3719
22			0.1384	0.2461	0.3845	0.5537	0.7537	0.9844	1.2459			
20			0.1284	0.2282	0.3566	0.5135	0.6990	0.9130	1.1555			
18			0.1181	0.2099	0.3280	0.4724	0.6430	0.8398	1.0629			
16			0.1075	0.1912	0.2987	0.4302	0.5856					
14		0.0430	0.0967	0.1720	0.2687	0.3870	0.5268					
12	0.0243	0.0381	0.0857	0.1524	0.2381							
10	0.0212	0.0331	0.0746									
8	0.0181											

Table derived from Management of Eucalypts Report No 9
A Volume and Taper Equation for New Zealand Grown Eucalyptus nitens

Tree Volumes (cubic metres)

Height (m)												
32										2.0623	2.4954	2.9698
30						0.7060	0.9610	1.2552	1.5886	1.9612	2.3731	2.8242
28					0.4646	0.6690	0.9106	1.1894	1.5054	1.8585	2.2488	2.6762
26					0.4384	0.6314	0.8594	1.1224	1.4206	1.7538	2.1222	2.5256
24				0.2635	0.4117	0.5929	0.8071	1.0541	1.3342	1.6471	1.9930	2.3719
22			0.1384	0.2461	0.3845	0.5537	0.7537	0.9844	1.2459			
20			0.1284	0.2282	0.3566	0.5135	0.6990	0.9130	1.1555			
18			0.1181	0.2099	0.3280	0.4724	0.6430	0.8398	1.0629			
16			0.1075	0.1912	0.2987	0.4302	0.5856					
14		0.0430	0.0967	0.1720	0.2687	0.3870	0.5268					
12	0.0243	0.0381	0.0857	0.1524	0.2381							
10	0.0212	0.0331	0.0746									
8	0.0181											
Diameter(cm)	8	10	15	20	25	30	35	40	45	50	55	60