forest industry research cooperatives

EUCALYPT COOPERATIVE

Eucalypt Cooperative

Report No.3

Modelling Eucalyptus Fastigata growth in New Zealand

by

M.van der Colff, M. Kimberley

THE JOINT FORCES OF CSIRO & SCION

ensis

CSIRO

SCION

THE JOINT FORCES OF CSIRO & SCION

THE JOINT FORCES OF CSIRO & SCION

ensis

Modelling Eucalyptus Fastigata growth in New Zealand
by
M. van der Colff, M. Kimberley

Date: June 2005
Client: Eucalypt Cooperative

CSIRO scion

THE JOINT FORCES OF CSIRO & SCION

ensis

Eucalypt Cooperative REPORT No: 3

Modelling Eucalyptus fastigata growth

in New Zealand

By

M. van der Colff, M. Kimberley

Introduction

Eucalyptus fastigata has been grown in New Zealand since the 1880s (Weston, 1957). It is currently being planted in the Central North Island for fibre production by Carter Holt Harvey Forests and GSL Capital, whose estates are managed by Hardwood Managers Ltd. This resource is around 8000 hectares. A further smaller, more scattered resource of E. fastigata has also been established mainly by the small growers, such as the NZ Farm Forestry Association, with the emphasis on the production of sawn timber. Currently there are very few decision support tools available to managers of Eucalypt stands to assist them in managing and maximising the growth and yield potential from their stands. Recently an E. nitens growth model was developed utilising New Zealand growth data for the Eucalypt Cooperative by S. Candy (Candy, 2003).

The development of a new stand growth model for Eucalyptus fastigata growing in New Zealand is described in this report. This growth model was constructed using Permanent Sample Plot (PSP) data collected before 2004.

Data

Two sources of data were used for developing the model:
1. *Ensis*' Permanent Sample Plot (PSP) system. Cooperative members have been collecting data and establishing trials for over 16 years. These valuable data were used in this study.
2. Carter Holt Harvey Forests (CHHF) have collected data for over 20 years from a range of sample plots located mainly in the Kinleith Forest estate. This data is now also located on the PSP system, administered by *Ensis*. Approval was given by CHHF to use their data in this study.

Scatter plots of stand variables were produced to check the compatability of the *Ensis* and CHHF *E. fastigata* data sets. Data sets were then combined and prepared for analysis. Growth measurement plots from the current *E. fastigata* database were screened for errors, and converted to a suitable format for modelling. Any anomalies were taken care before the decision was made to include or exclude data in subsequent analysis. From the combined data sets, 118 plots containing 854 measurements were found to be suitable for growth model development. A summary of the data is contained in Table 1.

Table 1. Summary of data

Variable	Mean	Std Dev	Minimum	Maximum
Age (years)	16.8	6.8	10	66
MTD (cm)	39.6	10.8	14.5	100
MTH (m)	29.5	8.4	10.7	58.4
Altitude (m, a.s.l)	422	190	15	820
Final stocking (stems/ha)	1259	747	220	4400

Model development

The growth model development was commenced in February 2004. A stand-level growth model which predicts growth based on height, basal area and stocking inputs was constructed. Various sigmoidal growth functions were tested for predicting these variables. Most even-aged forest stand growth models are based on sigmoidal functions in which yield (ie. Height, basal area, volume) is expressed as a function of age. In this study, the Chapman-Richards, Schumacher, Hossfeld, Weibull, and Logistic functions were considered. A comprehensive discussion on these models can be found in Kimberley and Ledgard (1998).

Height/age model

The Chapman-Richards model was found to best model height development. The model was fitted with an intercept of 0.25 m, representing height at planting. The coefficients were estimated using SAS macro NLINMIX version 8 (Little et al. 1996). The base age of the Site Index (*SI*) used in constructing the height/age model was set to 15 years since planting. *SI* is the height of the dominant trees in the stand at a predetermined age (Burkhart and Tennent 1977). The fit of each function was judged using the log likelihood statistics, with a smaller number indicating a better fitting model.

Table 2. Log likelihood values for height/age models

Function	-2 Log Likelihood
Anamorphic Chapman-Richards	1730.9
Polymorphic Chapman-Richards	1583.9

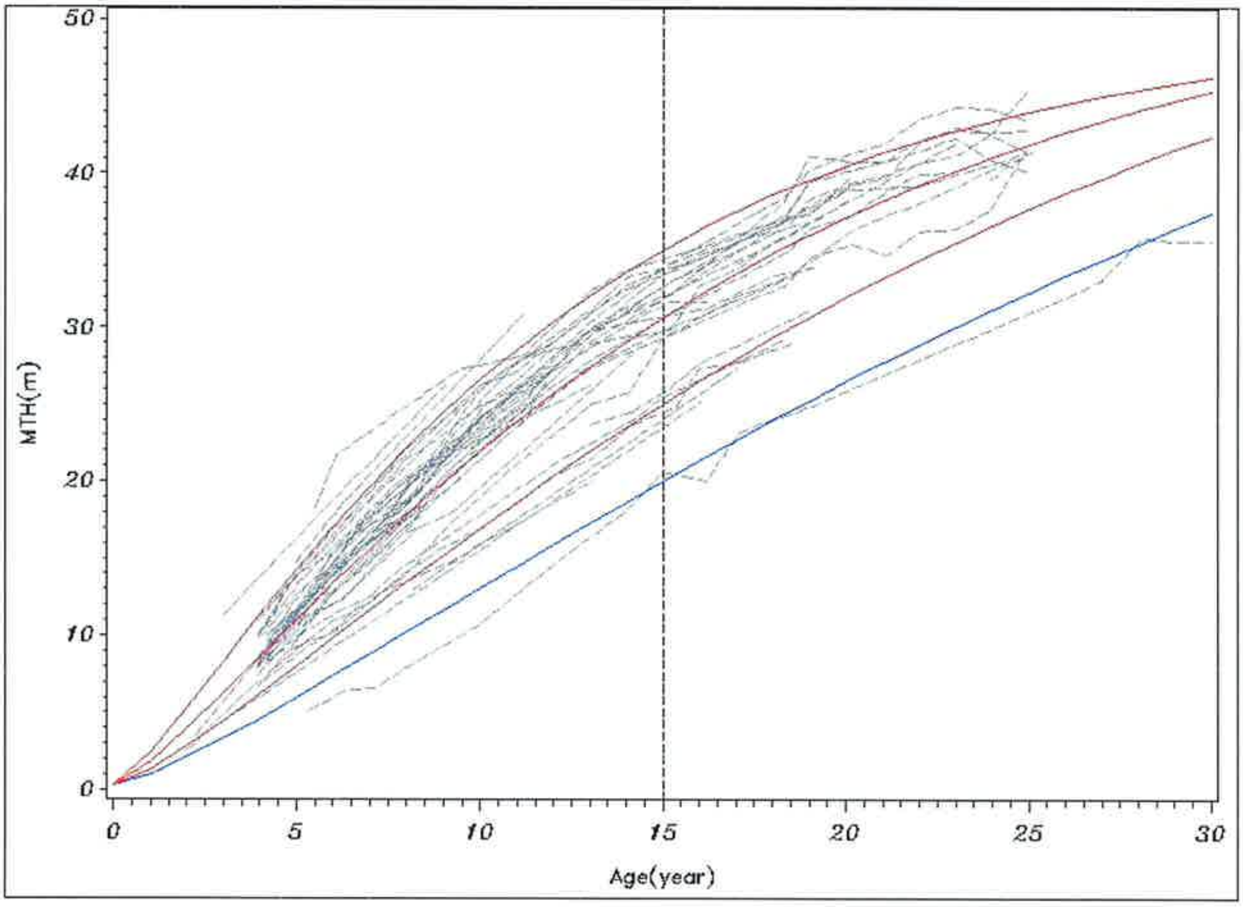
To determine whether any environmental or site variable could be used to improve the model, correlation were obtained between the fitted parameters of the model for each plot, and SI, stocking, altitude, latitude and longitude. The results showed a significant relationship between the slope parameter and SI. From Table 2, a polymorphic Chapman-Richards model in which the slope parameter was expressed as a function of SI, had the best fit.

The height/age model for *E.fastigata* in New Zealand has the following form:

$$MTH = 0.25 + (SI - 0.25)\left\{\frac{1-\exp(-T\exp(a+p.SI))}{1-\exp(-15\exp(a+p.SI))}\right\}^{1/b}$$

The coefficient estimates were *a* = –4.904 (s.e 0.246), *p* = 0.073 (s.e 0.007), *b* = 0.761 (s.e 0.021). All parameters were statistically significant.

Figure 1 shows the height/age model for a range of SI (25,30,35 and 40 m). Actual MTH data for all plots are superimposed using dotted lines.



Mean bias of the model (mean of actual-predicted in m^2/ha) and RMSE (squared root of mean quadratic bias in m^2/ha) were obtained for three different starting age classes and two projected age classes (Table 3), and indicate no trend of bias with starting or projection age..

Table 3. Mean Bias and RMSE for height predictions using three starting age groups.

Starting age group	Projection age group	Statistics	
		Bias	RMSE
7.5<age<=12.5	12.5<age<=17.5	0.02	0.54
	17.5<age<=22.5	0.00	1.59
12.5<age<=17.5	17.5<age<=22.5	0.10	0.54
	22.5<age<=27.5	-0.04	1.43
17.5<age<=22.5	22.5<age<=27.5	0.23	0.57

To calculate SI from a given height and age, an iterative procedure is required. The equation is rearranged to:

$$SI = 0.25 + (MTH - 0.25)\left\{\frac{1-\exp(-T\exp(a+p.SI))}{1-\exp(-15\exp(a+p.SI))}\right\}^{1/b}$$

A starting point (e.g., 20 m) is assigned to SI and the equation used iteratively until SI is predicted to adequate precision.

Basal Area model

Several forms of the Weibull, Schumacher and Chapman-Richards growth functions were tested for predicting Basal Area (BA). In addition, several modifications of the local asymptote Schumacher function were tested. Table 4 shows the fit statistics of the tested functions. The modified Schumacher function has the lowest –2 log likelihood value and hence was chosen as the function to use for BA model.

Table 4. Log likelihood values for BA models

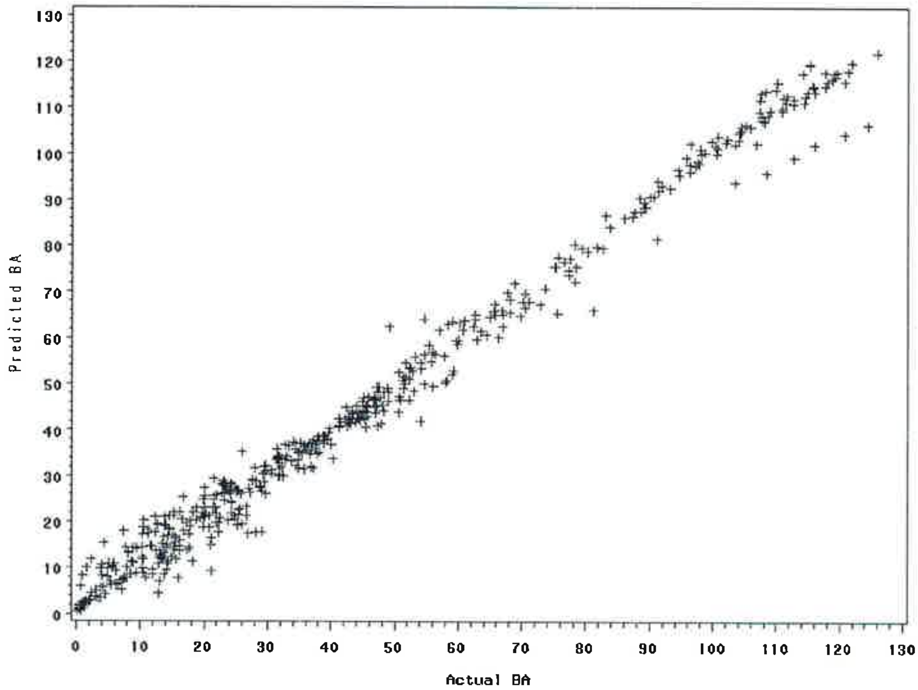
Function	-2 Log Likelihood
Anamorphic Chapman-Richards	3438.1
Anamorphic Schumacher	3453.6
Anamorphic Weibull	3457.5
Polymorphic Chapman-Richards	3389.0
Polymorphic Weibull	3408.3
Polymorphic Schumacher	3397.5
Modified Anamorphic Schumacher	3353.5
Modified Anamorphic Chapman-Richards	3385.5

The equation for the modified Schumacher function with local asymptote has the following form:

$$BA = c.\ln(SPH).\exp\left(-a\left(T^{b} - 15^{b}\right)\right)$$

where BA is the BA, SPH is stocking (stems/hectare), T is age (years), *a* and *b* are coefficients. The estimates of the coefficients were a = 11.110 (se. 0.737) and b = –0.981 (se. 0.048). The average values of the local parameter *c* were 10.212 (se. 0.499). The performance of the BA model is shown in Fig. 2, and mean BA predictions for a range of stockings at an average site are shown in Fig. 3.

Figure 2. Predicted against actual BA measurements

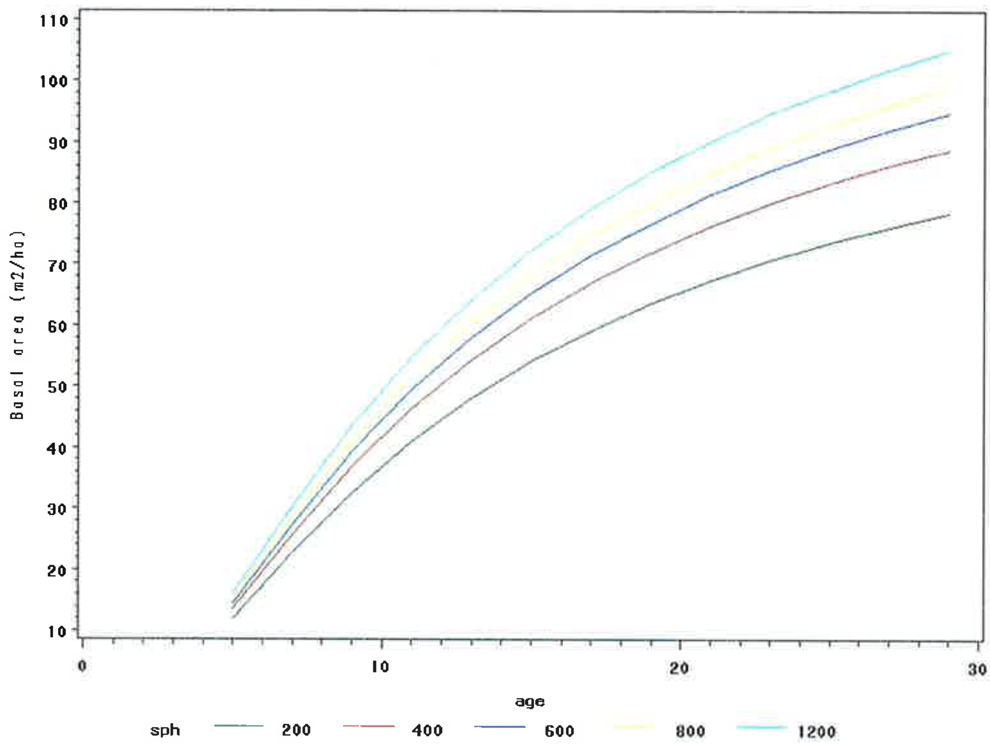


The performance of the equations was judged in the same manner as the height equations. The predicted BA for each PSP measurement was categorised into three age ranges used as the starting values. The bias (mean of actual-predicted in m^2/ha) and RMSE (squared root of mean quadratic bias in m^2/ha) were obtained for the projected age in each PSP sample.

Table 5. Mean bias and RMSE for BA prediction

Starting age group	Projection age group	Statistics	
		Bias (m)	RMSE
7.5<age<=12.5	12.5<age<=17.5	-0.85	3.67
	17.5<age<=22.5	-1.03	5.01
12.5<age<=17.5	17.5<age<=22.5	-0.50	3.92
	22.5<age<=27.5	0.37	4.72
17.5<age<=22.5	22.5<age<=27.5	1.39	6.55

Figure 3. BA growth predicted using the new Schumacher function for a range of stockings at an average site.



Mortality function

Two commonly used mortality functions were tested (Woollons 1998):

Function 1: $N_2 = N_1.e^{a(T_2 - T_1)}$

Function 2: $N_2 = N_1 - a\left(T_2^b - T_1^b\right)$

A modification of function 1 in which *a* parameter was expressed as a function of stocking was also tested. Overall, Function 1 performed better than the other two. The mortality function chosen was therefore:

$$N_2 = N_1.e^{-0.0115(T_2 - T_1)}$$

where N_1= stocking (stems/ha) at age T_1, and N_2= stocking (stems/ha) at age T_2. Actual and predicted stockings are shown in the Figs. 4 and 5.

Figure 4. Actual PSP stocking against log(Basal Area) measurements

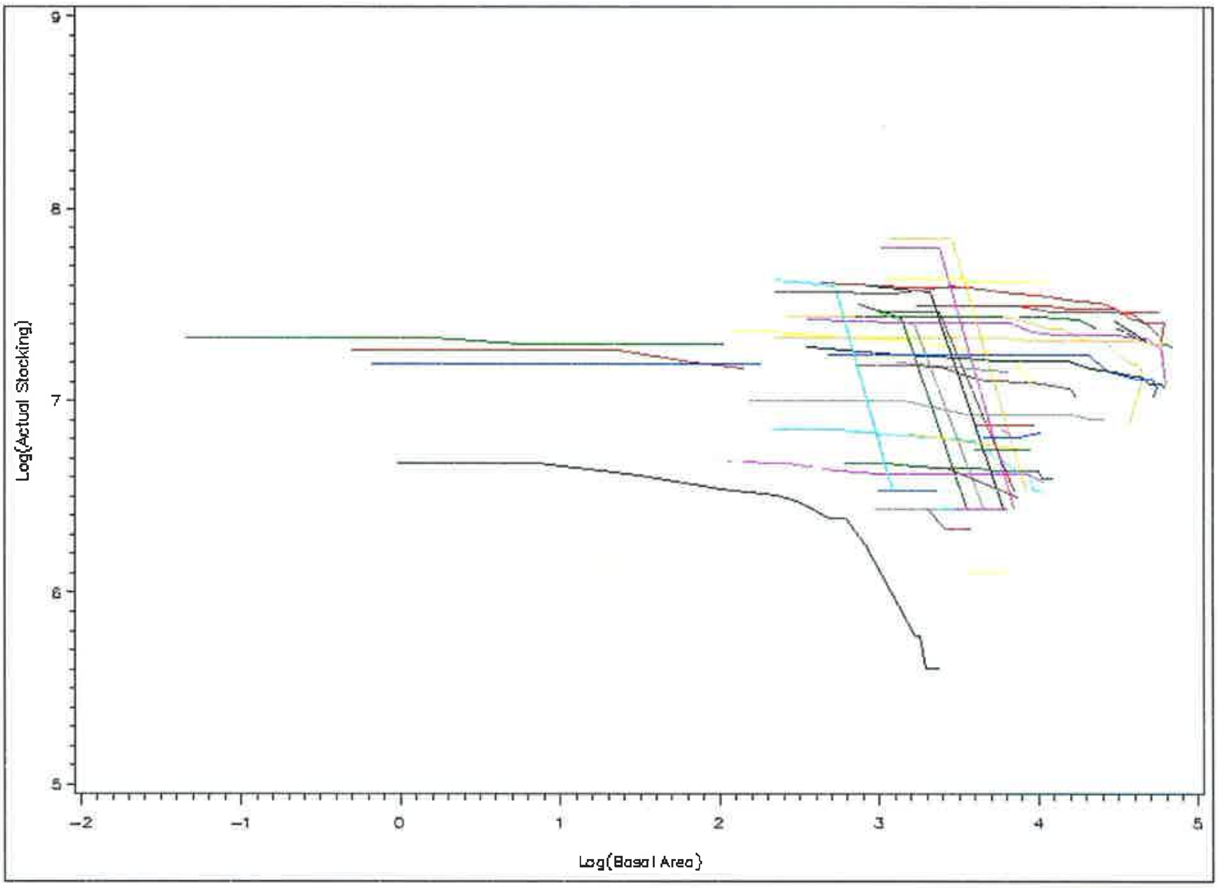
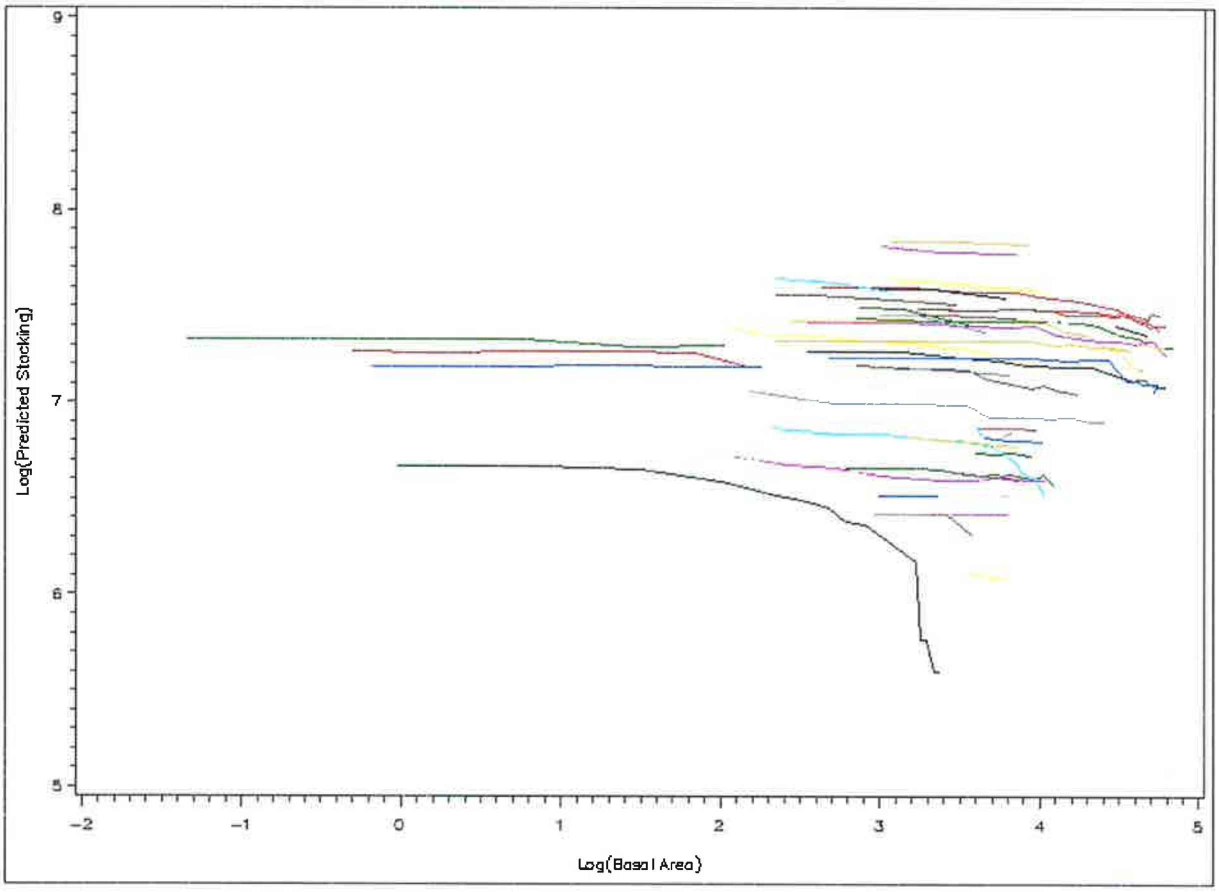

Log(Actual Stocking)
9
8
7
6
5
−2
−1
0
1
2
3
4
5
Log(Basal Area)

Figure 5. Predicted PSP stocking against log(Basal Area) measurements



Thinning function

A thinning function was required to predict the BA following thinning. It needed to account for the fact that generally, smaller trees are removed during thinning (ie. thinning occurs from below). A commonly used function that reflects this was fitted to the data:

$$B_2 = B_1 \left(\frac{N_2}{N_1} \right)^a$$

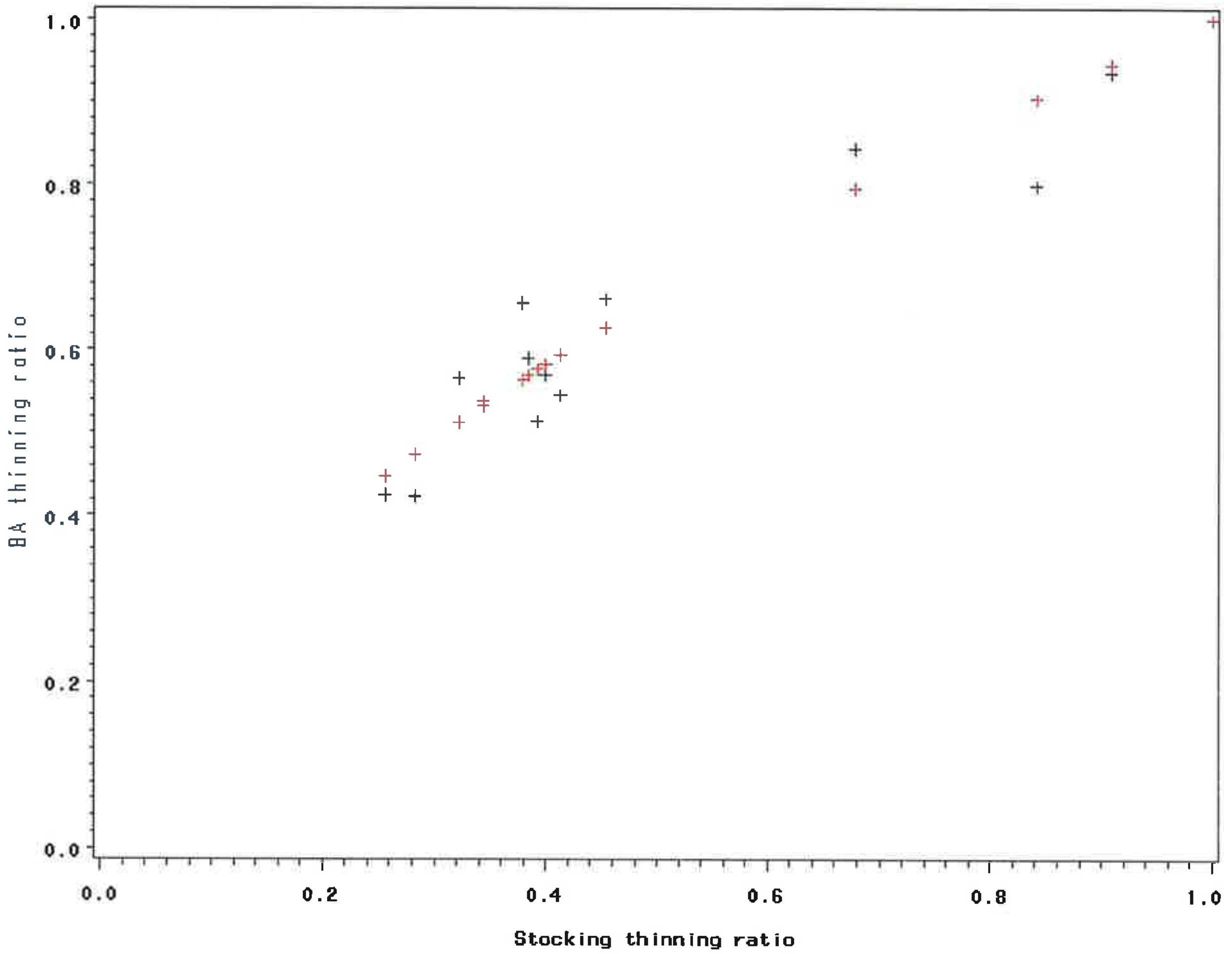
where B_1 and N_1 are Basal Area (m^2/ha) and Stocking (stems/ha) before thinning. B_2 and N_2 are Basal Area (m^2/ha) and Stocking (stems/ha) after thinning.

A modified version of thinning function that expresses parameter *a* as a function of pre-thinning stocking or age was also tested. However, this modified function did not give significant improvement or coefficients. The simpler model is chosen as the thinning function and is expressed as:

$$B_2 = B_1 \left(\frac{N_2}{N_1} \right)^{0.5909}$$

and illustrated against the data in Fig. 6.

Figure 6. Actual measurements (black symbols) and predicted values (red symbols) of BA thinning ratio against Stocking thinning ratio



Diameter distribution

For any given plot an individual tree basal area was assumed to have a 3 parameter Weibull distribution (Weibull, 1939). The density and cumulative distribution functions for this distribution are (eg. Johnson et al, 1994; Goulding et al, 1979):

$$f(b) = \frac{c}{a}\left(\frac{b - b_{\min}}{a}\right)^{c-1} e^{-\left(\frac{b-b_{\min}}{a}\right)^{c}}$$

and

$$F(b) = 1 - e^{-\left(\frac{b-b_{\min}}{a}\right)^{c}} \qquad , b>b_{\min}$$

where b is the individual tree basal area in m^2, and b_{min} (the threshold parameter or minimum basal area), a and c are parameters defining the location and shape of the distribution.

The mean and variance of this distribution are (Johnson et al, 1994):

$$E(b) = a\Gamma(1/c + 1) + b_{\min}$$

$$\mathrm{var}(b) = a^2\left[\Gamma(2/c + 1) - (\Gamma(1/c + 1))^2\right]$$

The density and cumulative functions for dbh can be derived from the basal area distribution:

$$f(d) = \frac{2cd}{ka}\left(\frac{d^2 - d^2{}_{\min}}{ka}\right)^{c-1} e^{-\left(\frac{d^2-d^2{}_{\min}}{ka}\right)^{c}} \qquad , d>d_{\min}$$

$$F(d) = 1 - e^{-\left(\frac{d^2-d^2{}_{\min}}{ka}\right)^{c}} \qquad , \qquad d>d_{\min}$$

where d is the individual tree diameter in cm and $k=40000/\pi$.

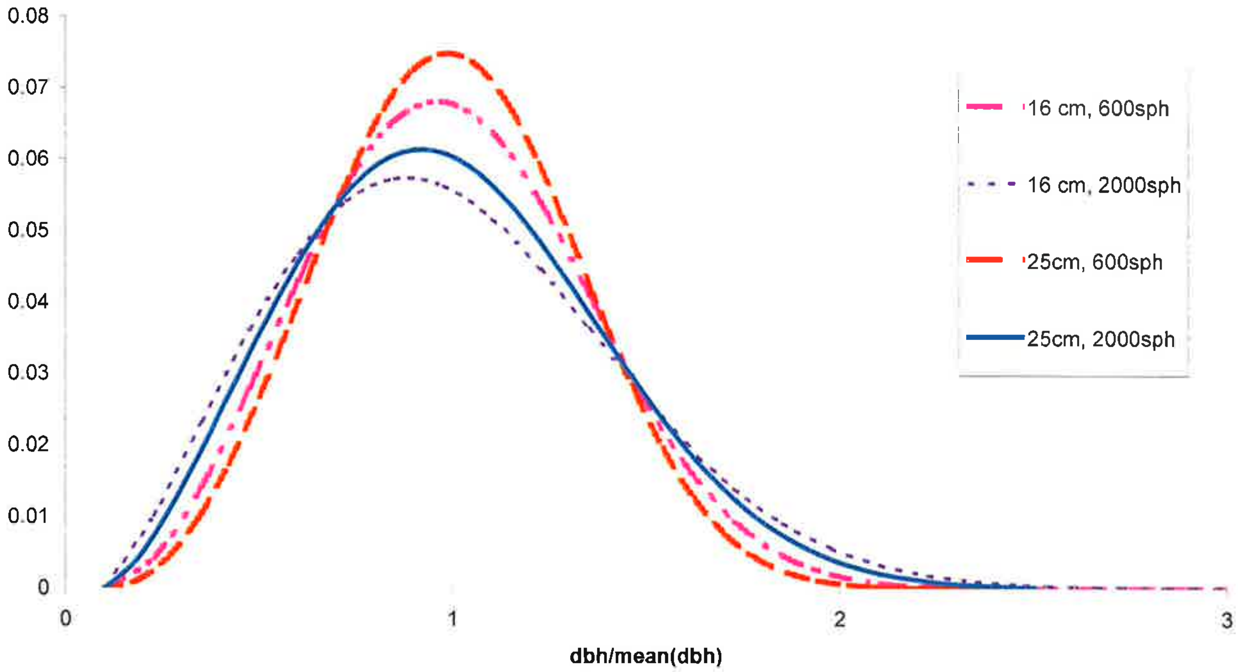
These parameters were derived by fitting Weibull distribution to the sample plots using PROC UNIVARIATE (SAS Institute, 2004). The threshold parameter was estimated by minimising the Kolmogorov-Smirnov goodness-of-fit criteria, and the other two parameters were estimated by maximum likelihood. The mean and variance of the distribution for each plot were obtained using the above equations. The threshold parameter (d_{min}) and variance were then modelled using $d_{min}/E(d)$ and sqrt(var(d))/E(d) as dependent variables with E(d) as a fixed covariate. The following equations were derived:

$$d_{min}=a_0 . d$$

$$\mathrm{var}= d^2 (a_1+a_2.d+a_3.N)^2$$

Parameter	Coefficients	Std Error
a_0	0.1021	0.0128
a_1	0.3752	0.0305
a_2	-3.5E-3	9.5E-4
a_3	5.2E-5	1.2E-5

where d_{min} is the minimum individual tree diameter (cm), *var* is the variance among individual tree diameter, *d* is the mean individual tree diameter (cm) and N is stocking (stems/ha). Using the equations presented earlier and method described by Garcia (1981) of moments estimation for Weibull distribution, *a* and *c* parameters as described in the density function were calculated. Figure 7 shows a predicted diameter distribution for three different mean diameter values: 16cm, 19cm and 22cm. The spread of diameters is seen to increase at higher stocking while relative to mean diameter, the spread reduces for larger mean diameters.



Height/Diameter curves

The Patterson equation was used as a height/diameter equation for predicting height across the range of diameters within a stand (Goulding & Shirley, 1979). It is expressed in the following form:

$$h = 1.4 + \left(a + \frac{b}{d}\right)^{-2.5}$$

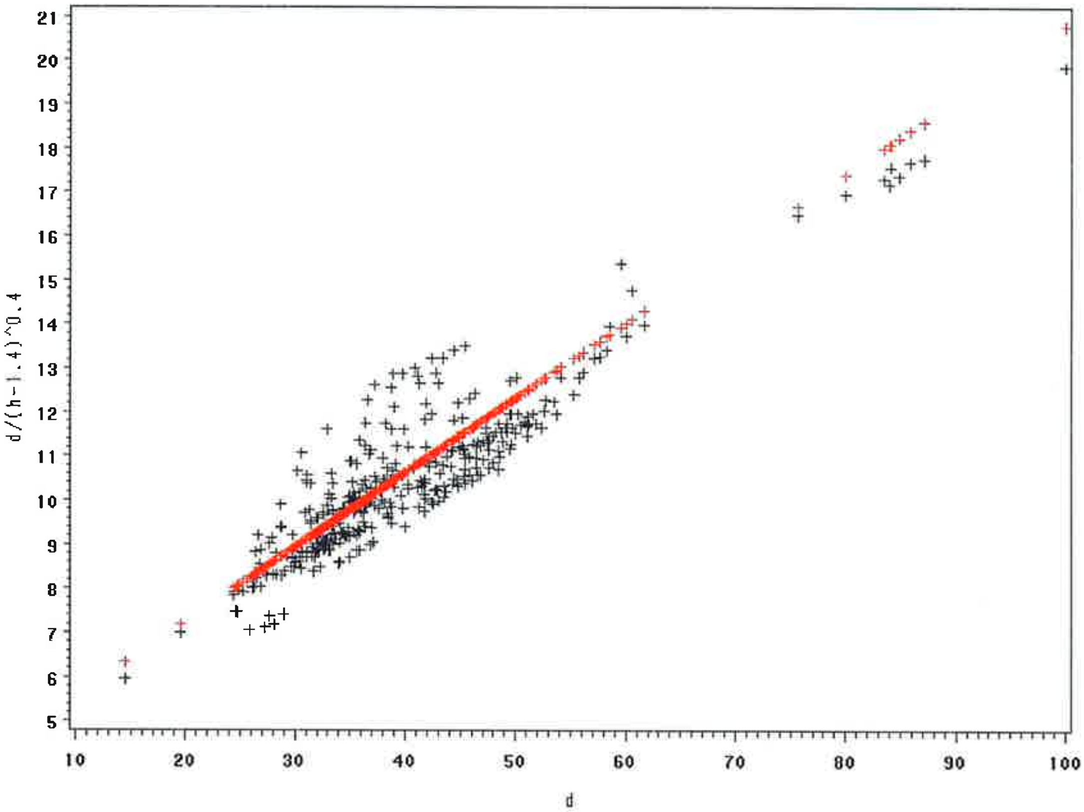
where *h* is mean top height (m), *d* is mean top diameter (the quadratic mean diameter of the 100 largest stems per hectare in cm), *a* and *b* are the coefficients to be estimated.

This function can be more conveniently expressed as a linear function in the following form:

$$\frac{d}{(h-1.4)^{0.4}} = a.d + b$$

The coefficient estimates were a=4.330 (se 0.154) and b=0.154 (se.3.7E-3). Actual and predicted values are shown in Fig. 8.

Figure 8. Patterson's height and diameter function (actual values superimposed by predicted values in red)

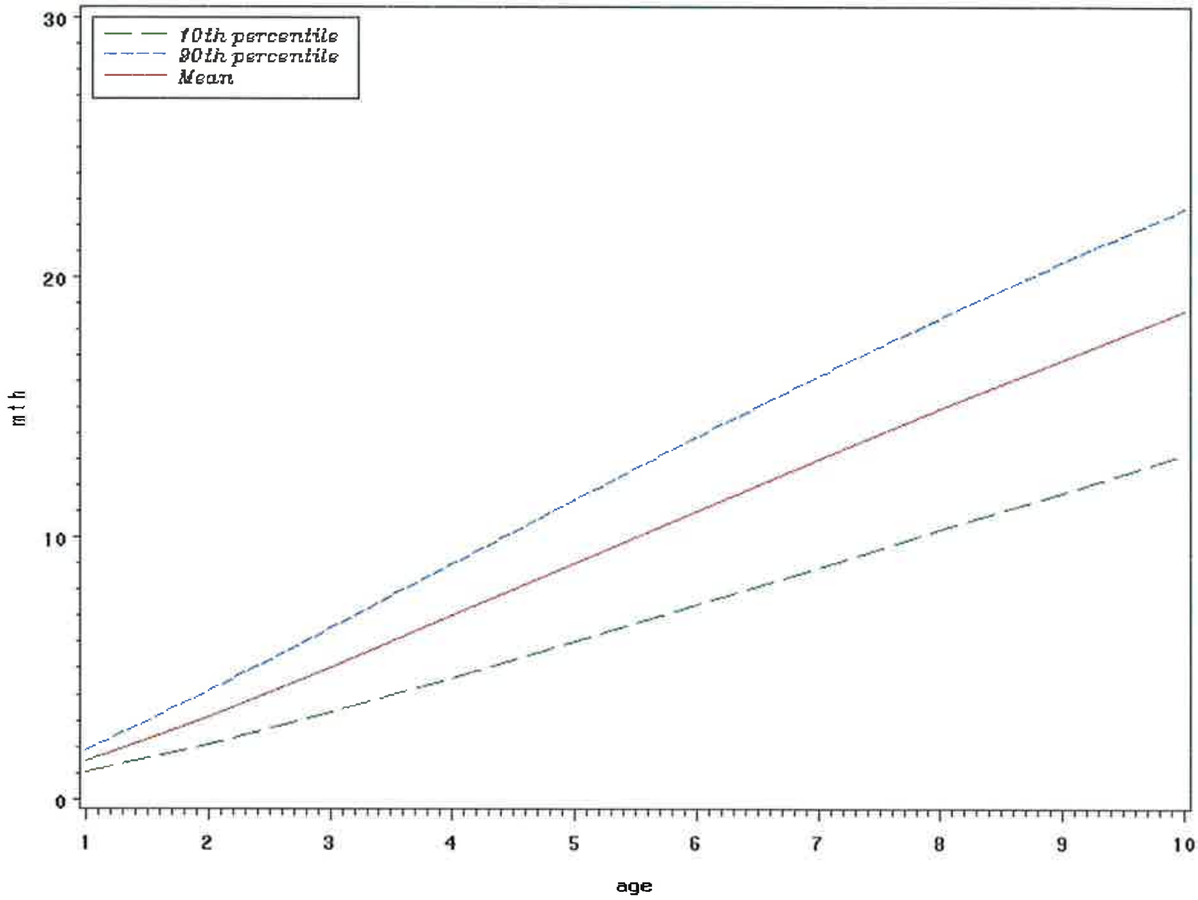


Starting values

In order to use the modelling system, the user must supply starting values which ares generally available from plot measurements. Occasionally this type of data are not easily obtained. To assist the forester in this case, BA and

MTH were obtained for a range of ages from the PSP system. The mean, 10th percentile and 90th percentile were then calculated for MTH and plotted against age (Fig. 9). The three lines can be used to find suitable starting values of MTH for average, poor and good sites.

Figure 9. Mean, 10th percentile, and 90th percentile of MTH in the PSP.

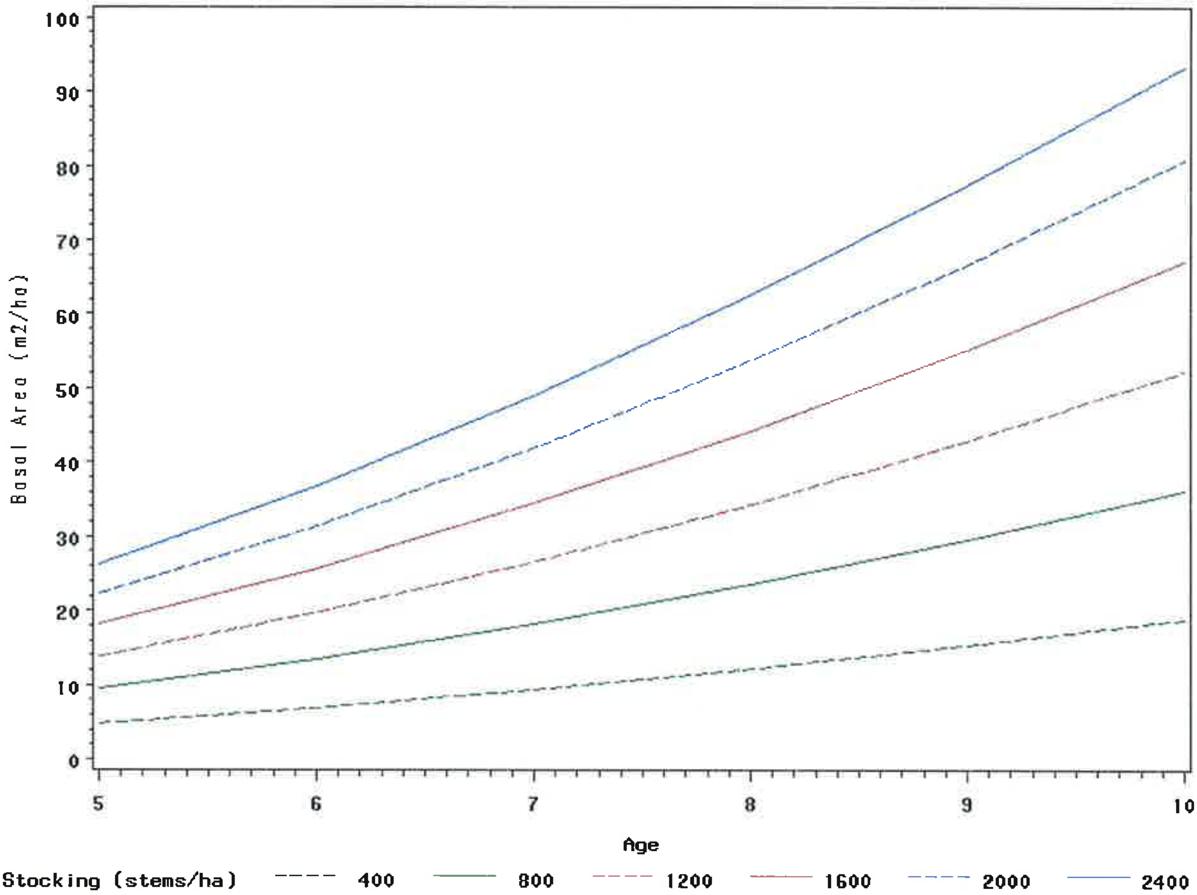


A function for predicting mean BA from age and stocking was derived using all measurements 10 years and younger:

$$B = (aT)\left(1 - e^{bNT}\right)$$

where B is basal area (m^2/ha) at age T and stocking N (stems/ha). The coefficient estimates were a=24.504 (se. 8.36) and b=-0.2E-4 (se. 7.0E-6).This function (Fig. 10) can be used for obtaining BA starting values for average sites for a ranges of stockings and is suited to ages younger than 10 years.

Figure 10. Model for predicting a Basal Area starting value for an average site in NZ



Model validation

Due to limited amount of data available, all clean data was used to fit the growth model with no independent validation data. Cross validation could be performed with new data to check the precision of the fit shown in this report. Forward predictions of each growth component at various age classes can be calculated using actual PSP measurements as starting points. These predictions are then compared back to actual measurements in PSP. Prediction errors can be shown as percentages of mean predicted values and tabulated across a range of site characteristics.

Eucalypt Cooperative REPORT No: 3

Appendix 1

Four main components (Height/age curves, BA growth, Volume, thinning and mortality function) of the growth models were programmed into EXCEL. The following graph shows the main interface of the calculator.

Draft Fastigata Growth Model - February 2005
Property of Eucalypt Cooperative and Ensis
Programmed by M. van der Colff

Enter stand data and charts will update growth projection automatically

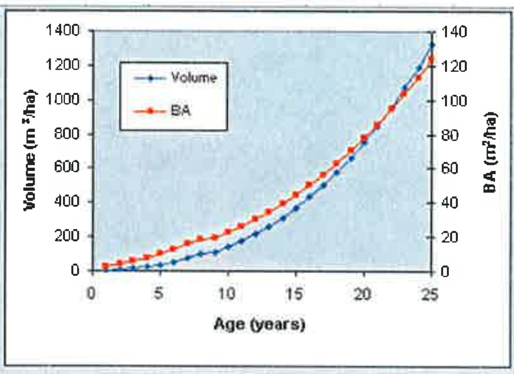
Stand Info	
Age (years)	5
BA (m2/ha)	10
Stocking (stems/ha)	2000
SI (m)	32

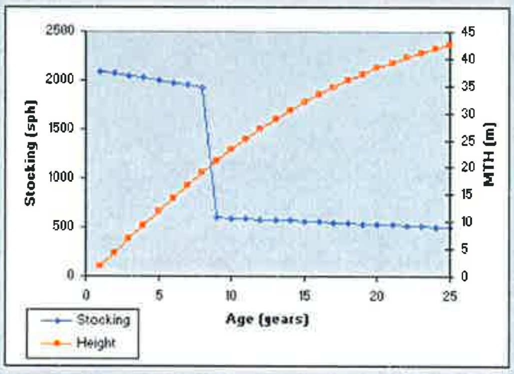
Thinning info	
Age (years)	9
Sph after thinning (s/ha)	600

Calibrate Site Index	
Age (years)	10
MTH (m)	25
Estimated SI (m)	33.6

Simulated Growth

Age	MTH	BA	Volume	Stocking
1	2.0	2.2	3.3	2094
2	4.3	3.8	7.6	2070
3	6.8	5.6	14.5	2047
4	9.4	7.7	24.2	2023
5	11.9	10.0	37.4	2000
6	14.4	12.6	54.2	1977
7	16.8	15.6	75.3	1955
8	19.1	18.8	100.9	1932
9	21.3	19.0	111.2	600
10	23.3	22.4	141.5	593
11	25.3	26.1	176.5	586
12	27.1	30.2	216.5	580
13	28.9	34.6	261.8	573
14	30.5	39.5	312.8	566
15	**32.0**	**44.7**	**369.6**	**560**
16	33.4	50.3	432.6	554
17	34.8	56.4	502.0	547
18	36.0	63.0	578.1	541
19	37.2	70.1	661.3	535
20	38.3	77.6	751.8	529
21	39.3	85.7	849.8	523
22	40.2	94.4	955.8	517
23	41.1	103.6	1069.9	511
24	41.9	113.5	1192.5	505
25	42.6	124.0	1324.0	499
26	43.4	135.1	1464.5	493
27	44.0	147.0	1614.5	488
28	44.6	159.5	1774.3	482
29	45.2	172.8	1944.3	477
30	45.7	186.9	2124.7	471





Cooperative confidentiality

The data, growth model, report and software prepared are confidential to members of the Eucalypt Cooperative.

Acknowledgments

We gratefully acknowledge assistance provided by C. Andersen, the Permanent Sample Plot Administrator to extract data from the system.

References

Garcia, O. 1981: Simplified method-of-moments estimation for the Weibull distribution. New Zealand Journal of Forestry Science 11:304-306.

Goulding, C. J and Shirley, J.W. 1979: A method to predict the yield of log assortments for long term planning. Pp 301-304. in Elliot, D.A (ed.) "Mensuration for Management of Exotic Forest Plantations". NZ Forest Service, FRI Symposium no. 20.

Kimberley, M.O and Ledgard, N.J.1998: Site Index curves for Pinus nigra grown in the south island high country, New Zealand. New Zealand Journal of Forestry Science 28(3): 389-399

Weibull, W. 1939: A statistical theory of the strength of material. Ing Vetensk Akad Handl 153.

Woollons, R.C. 1998: Even-aged stand mortality estimation through a two-step regression process. Forest Ecology and Management 105: 189-195