

A validation of the 300 index growth model

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Plantation Management Cooperative

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EXECUTIVE SUMMARY

A VALIDATION OF THE 300 INDEX GROWTH MODEL

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The stability of the 300 Index Growth Model was tested against more than 5,000 sample plots located throughout the country. An approach that tests for drift in the index over time was used with plots classified on the basis of site and regime. A stable index indicates that the model is performing well while a positive drift is evidence of model under-prediction and a negative drift indicates over-prediction.

For most sites and regimes, the model performs well up to age 30 years and at final crop stockings below 800 stems/ha. However, it under-predicts yield above age 30 years and over-predicts at stockings above 800 stems/ha. The effects of pruning and thinning are accurately modelled. A slight tendency to over-predict yield on low-productivity sites was detected. There was also a slight tendency to under-predict yield on traditional Forest sites but to be unbiased on Farm sites.

There was some regional variation in the performance of the model. It over-predicts yield on Coastal Sand sites, and also has a slight tendency to over-predict in Northland, Auckland, the South Island West Coast and Canterbury. It under-predicts yield in Otago and Southland and also in Gisborne and Wellington (based on limited data). There is also a slight tendency to under-predict yield in Bay of Plenty, Nelson and Marlborough.

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INTRODUCTION

Model validation is an important step in the development of a forest growth model. According to Goulding (1979), validation should be a formal, independent process concerned with evaluating the model as a whole. Validation is not a process of proving whether the model is ‘correct’ as no model will ever perfectly reflect reality and will always at best only present an approximation of the true situation. Goulding quotes Van Horn (1969) as defining validation as “The process of building an acceptable level of confidence that an inference about a simulated process is a correct or valid inference about the actual process”. Thus rather than proving that the model is absolutely correct, the validation step should aim to ensure that forest management decisions made on the basis of model predictions are valid and can be made with confidence.

Development of a model involves several stages that will generally include collection and processing of data, model formulation, parameter estimation, testing of model hypotheses and assumptions, programme coding and verification, and finally a formal validation. The validation should preferably be carried out using independent data, but sometimes model development data is used. However, Goulding (1979) emphasises that the development of a forestry growth model is generally cyclic in nature. If the validation identifies shortcomings in the model, these will generally be corrected and a further validation performed to determine whether they have been rectified. This process may continue for several cycles before the model becomes widely accepted as able to provide reliable inferences about the system.

For a forest growth model, the validation procedure should answer several questions such as the following:

- How good the model is at predicting levels of growing stock?
- Over what site types are predictions accurate?
- How close is the model’s behaviour to reality at predicting the effects of management treatments such as tree spacing, timing of thinning, and pruning?

The recently developed 300 Index Growth Model (Kimberley et al., 2005) predicts radiata pine BA for a wide range of site types and silvicultural treatments. It is currently implemented in FORCASTER and the Radiata Pine Calculator. In a recent Plantation Management Cooperative project, the stability of the 300 Index Growth Model was tested on three contrasting site types: Dry East Coast, Fertile ex-farmland, and Coastal sand-dunes (ref.). The 300 Index predictions were relatively stable over a wide range of age classes and stockings and the model generally performed well on dry and fertile sites. However, there was a negative drift in the 300 Index over time on coastal sand sites north of Auckland and on a few dry sites (Taradale, Marlborough and Eyrewell). This means that on these sites, growth projections from young ages will tend to be over-predicted. There was also a small positive drift on fertile sites in Otago/Southland indicating that some under-prediction is likely on these sites.

There is a need to extend this validation to a wider range of sites, especially typical forest sites from throughout the country, and this report describes such a validation.

OBJECTIVE

Validate the 300 Index Growth Model for a wide variety of sites and regimes from throughout New Zealand.

METHODS

The 300 Index Model is calibrated for any site using two measures of site productivity: Site Index, a measure of height growth productivity, and the 300 Index, a measure of volume productivity. Both Indices can be supplied directly by a user based on knowledge of expected productivity for a particular site. Alternatively, the indices can be estimated from growth measurement data from an existing stand. The model assumes that Site Index and 300 Index do not vary over time and do not change with stocking.

One way of testing the stability of the model is to predict the 300 Index from permanent sample plot (PSP) growth measurement data, and determine whether it remains constant over time, or shows evidence of drift. For example, suppose the Index is found to drift downward over time for a particular site type. On such sites, if the 300 Index Model is run using an early measurement as a starting point, it will tend to increasingly over-predict the basal area. This will occur because the model assumes a constant 300 Index based on the initial measurement, when in practice the Index should be drifting downward.

The following procedure was used to validate the model. Firstly, permission was sought and granted by the majority of data controllers for the Ensis PSP database, to use data for this project. Growth data were then extracted from a wide range of sites throughout the country. Data from the previous validation study (of fertile, dry and sand sites) were included. Data used in developing the model were not included in this study which thus provides a completely independent test of the model. The data were then checked for suitability. The most important requirement for accepting a plot was a full and accurate stocking and pruning history. The 300 Index was then estimated for each measurement using a Microsoft Excel VBA (Visual Basic for Applications) implementation of the model. These 300 Index values were then analysed to determine whether they remained stable over time within each plot, or showed evidence of drift.

An example of this validation approach applied to a single example plot is shown in Fig. 1. In this plot, when projecting yield from age 11 years, the model over-predicts. However, when projecting from age 17 years, the model is accurate. This behaviour is reflected in the 300 Index values for ages 11, 17 and 27 years. The index drifts downwards between ages 11 and 17 years and remains constant from 17 to 27 years.

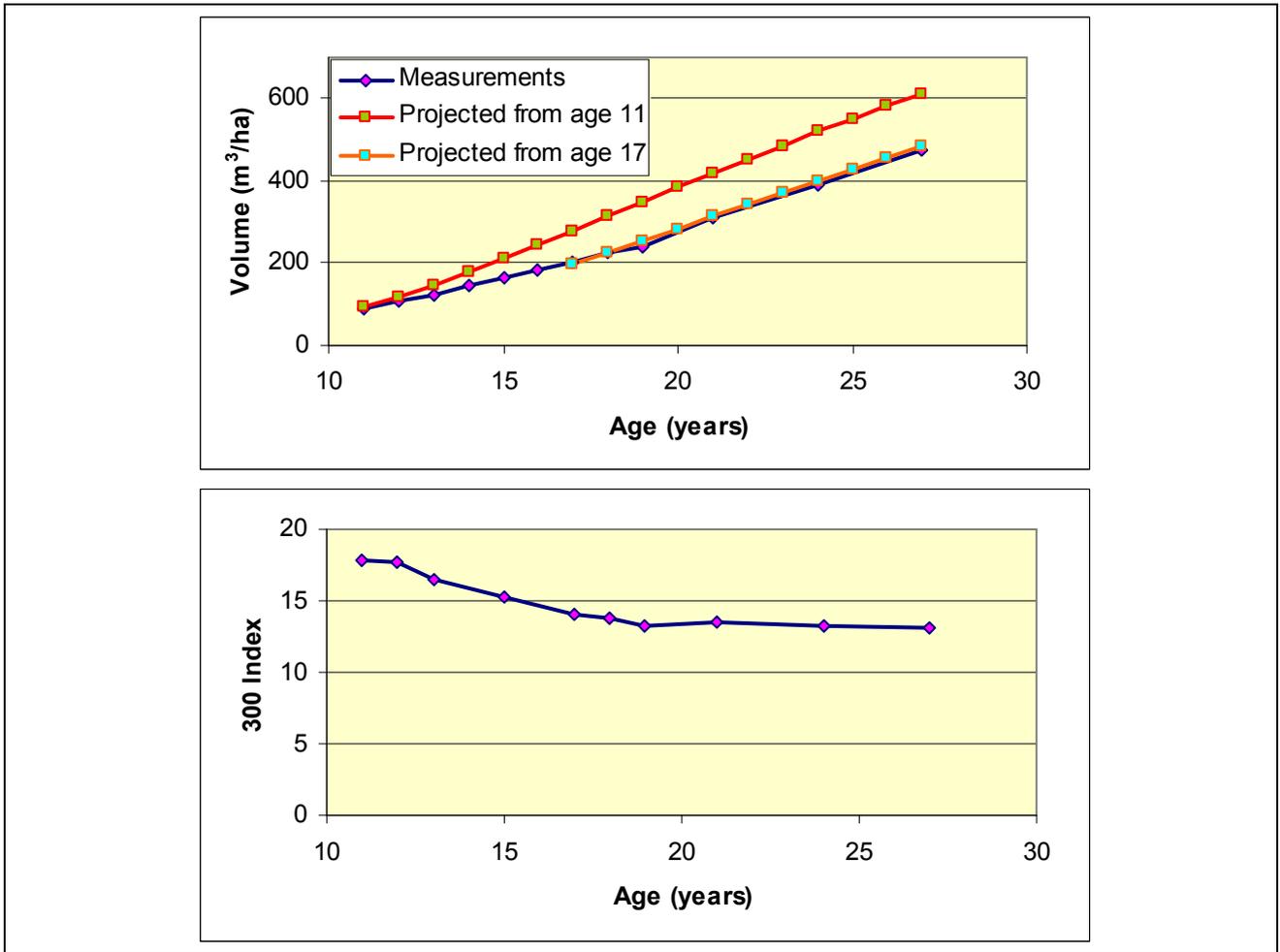


Fig. 1. Example of the validation method applied to an example plot.

The above example illustrates how a drift in the index within a plot provides clear evidence of model over- or under-prediction. However, in practice, the validation procedure utilised data from many plots simultaneously. To simplify interpretation, age was classified into 5-year classes (5-10 years, 10-15 years, etc.). It was important when analysing the data that corrections were made to account for mean differences in the 300 Index between plots. For example, more recently planted stands generally have higher 300 Indices than earlier plantings, and because they are over-represented in the younger age classes, an apparent strong negative drift in the index occurs when a simple mean is plotted against age. To overcome this problem, a linear mixed modelling approach was used. The following mixed model was fitted using the SAS procedure PROC MIXED:

$$[1] \quad I300_{ijk} = a + p_i + t_j + e_{ijk}$$

where $I300_{ijk}$ is the 300 Index estimated for the k th measurement in the j th age class in the i th plot, a is the overall mean, p_i is a random term representing the i th plot, t_j is a fixed effect representing the j th age class, and e_{ijk} is the residual error term. Using this model, the age class fixed effects provide precise measures of any trend or drift in the index over time within plots.

An alternative model, which assumed that the drift is linear with age, was also used to quantify the average drift and test whether it was significantly different from zero. This model was a random coefficient regression model, also fitted using PROC MIXED:

$$[2] \quad I300_{ij} = a_i + b \times Age + e_{ij}$$

where $I300_{ij}$ is the 300 Index estimated for the j th measurement in the i th plot, a_i is a random intercept term for the i th plot, b is a fixed slope term, Age is the age of the measurement, and e_{ij} is the residual error term. This provides a measure of the expected over- or under-prediction of volume MAI projected from a stand measurement over a given prediction period. In most cases, only data less than age 30 years was included.

These two models were applied to plots grouped in different ways, to determine the performance of the model for different site types. The following site type groupings were used:

- 300 Index: <19, 19-23, 23-27, 27-31, >31
- Geographic location – regional authority boundaries were used but coastal sand sites were treated as a separate region
- Previous land use: ‘Farm’ (ex-pasture sites), ‘Forest’ (ex-forest or scrub sites), and ‘Sand’ (coastal sand sites)
- Site Index: <23, 23-27, 27-31, 31-35, >35
- Planting date: pre-1940, 1940-1959, 1960-1979, post-1979

Similarly, to determine the performance of the model for different management regimes, the following classes based on regime were analysed:

- Stocking (stems/ha): <200, 200-300, 300-500, 500-800, >800
- Pruning: unpruned, pruned height < 5m, pruned height > 5m
- Thinning age (years): <12, 12-15, >15

RESULTS AND DISCUSSION

Most of the validation was restricted to plots in stands established after 1959, with final stockings less than 3000 stems/ha, and plot measurements of more than age 6 years. There were 5,089 plots satisfying these conditions. A further 602 plots in stands established prior to 1960 were used to test the model against historic data, but these were not included in the main validation. Plots were classified according to previous vegetation cover into ex-pasture ‘Farm’ sites, traditional ‘Forest’ sites, and coastal ‘Sand’ sites. The numbers of plots provided by each PSP database controller is listed in Table 1 and their distribution by region (based on Regional Authority boundaries) is shown in Table 2.

Table 1. Numbers of plots contributed by each PSP database controller summarised by land use prior to establishment.

Controller	Total	Farm	Forest	Sand
BLAK	6	0	6	0
CFL	1	1	0	0
ERNS	146	27	12	107
FFPM	871	514	275	82
FRIC	35	23	12	0
FRIE	237	82	120	35
FRIM	429	18	349	62
FRIP	1	0	1	0
FRIS	658	86	519	53
FRIU	2	0	2	0
HFML	35	0	35	0
JNL	123	0	31	92
KTML	570	2	568	0
MAF	25	0	0	25
MLDC	9	0	9	0
NTFM	27	0	0	27
NZFM	5	1	4	0
PFOL	81	27	40	14
RAYN	94	35	53	6
SGMC	1352	391	837	124
TLWC	90	1	89	0
WEYH	292	9	283	0
Total	5089	1217	3245	627

Table 2. Distribution of validation plots by region and land use prior to establishment. Only plots in stands planted from 1960 are included.

Region	Total	Farm	Forest	Sand
Northland	566	183	124	259
Auckland	238	12	74	152
Waikato	492	144	304	44
Bay of Plenty	1851	184	1667	0
Gisborne	59	56	3	0
Hawkes Bay	417	286	131	2
Taranaki	7	6	0	1
Wanganui/Manawatu	219	31	17	171
Wellington	60	6	54	0
Nelson	326	10	316	0
Marlborough	185	66	119	0
West Coast	97	1	96	0
Canterbury	229	40	189	0
Otago	243	135	108	0
Southland	100	57	43	0
Total	5089	1217	3245	629

The means, standard deviations and correlations of the 300 Index, Site Index, and BA Index (BA Index = $50 \times 300\text{Index} / \text{SI}$ approximates the BA at age 20 years for a '300 Index' stand) for all plots in stands established since 1959 are shown in Table 3. Stem volumes of trees must be a function of height and BA and the 300 Index which measures the stem volume productivity is therefore correlated with both Site Index (from Table 3, $r = 0.71$; see also Fig. 2) and BA Index ($r = 0.79$). However, the BA Index is only very weakly correlated with Site Index (from Table 3, $r = 0.15$; see also Fig. 3) showing that the site propensity for tree diameter growth is almost unrelated to the propensity for height growth.

Table 3. Means, standard deviations and correlations for productivity index for plots in stands established from 1960.

Productivity Index	Mean	s.d.	Correlations		
			300 Index	BA Index	Site Index
300 Index	24.9	6.5	1	0.79	0.71
BA Index	29.3	8.5	0.79	1	0.15
Site Index	42.3	4.7	0.71	0.15	1

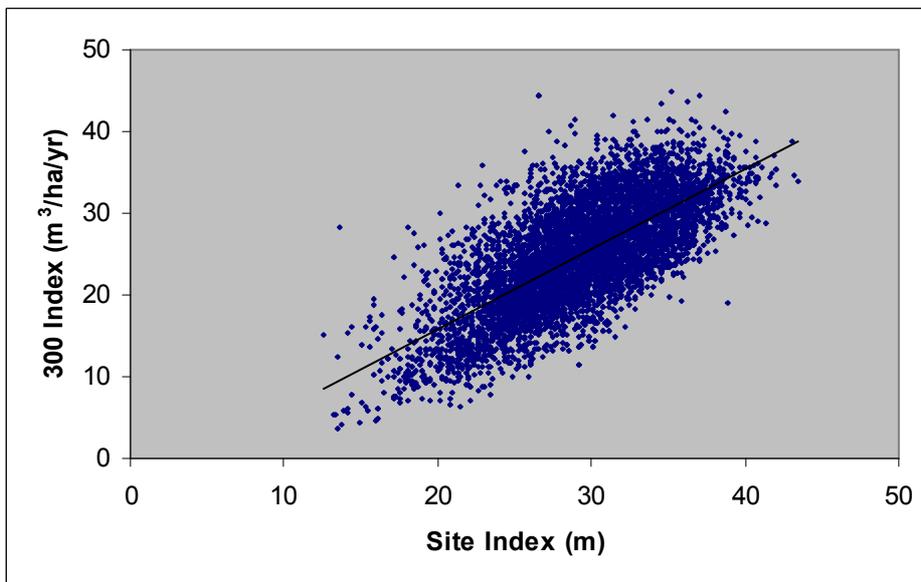


Fig. 2. The 300 Index versus Site Index for all PSPs in stands established from 1960.

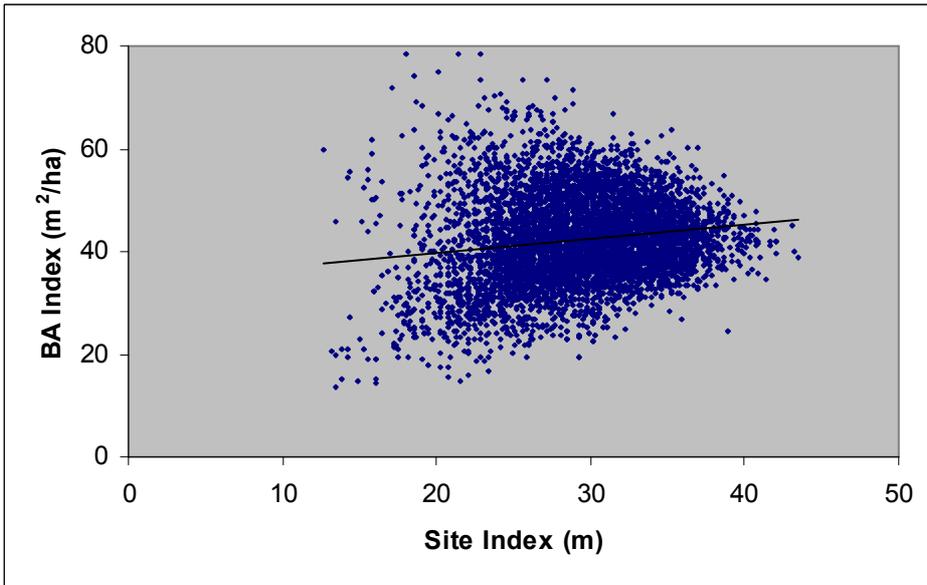


Fig. 3. The BA Index versus Site Index for all PSPs in stands established from 1960.

Applying Model [1] to the validation data produces the mean 300 Indices in 5-year age classes shown in Fig. 4. The index remained nearly constant or slightly increasing up to about age 25 or 30 years. Beyond this age, there is a steady increase in the index with age. This implies that the model will project yield from an early measurement up to age 30 years well, but will under-predict yield beyond age 30 years. A clearer picture of the performance of the model is shown in Fig. 5 which shows the results of fitting Model [1] to the validation data split into four groups on the basis of planting date. This indicates that the index has a positive drift even below age 30 years for earlier plantings, especially for stands planted before 1940. However, stands planted since 1980 show no index trend for ages up to 30 years. Data from stands aged much greater than 30 years were only available from the earlier plantings, but the pronounced bias for older ages would still presumably apply to recent plantings if they were allowed to grow beyond age 30 years.

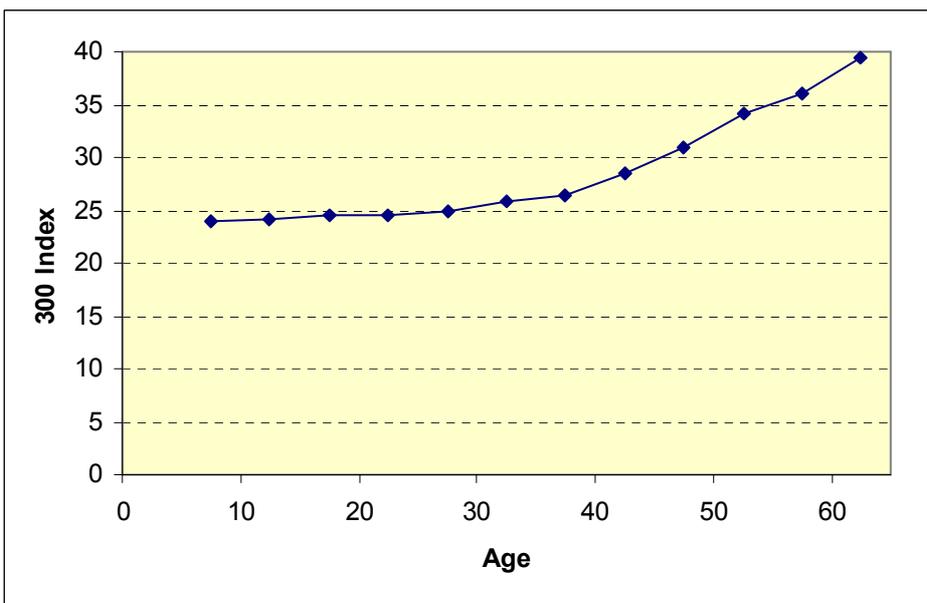


Fig. 4. Drift in the mean 300 Index over time calculated using the complete validation data set.

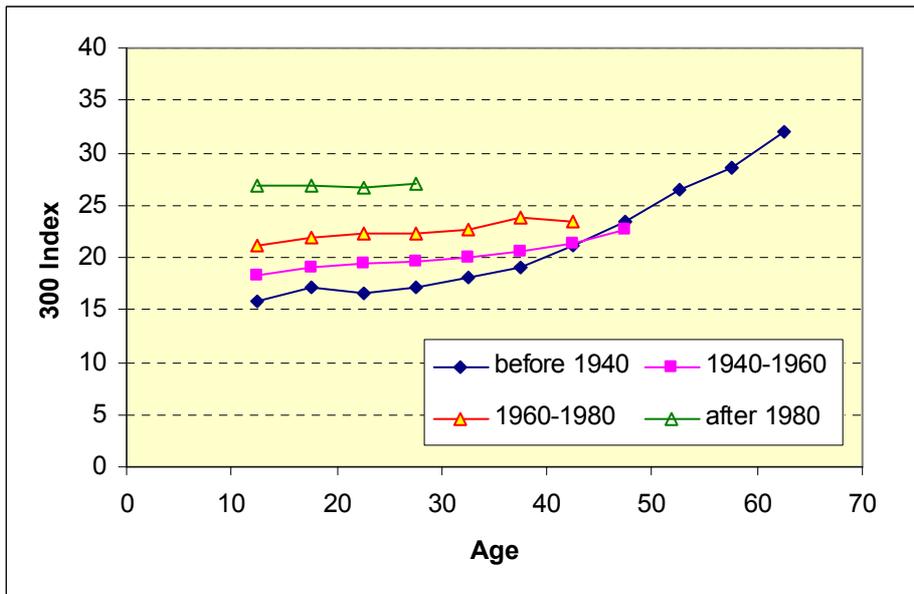


Fig. 5. Drift in the mean 300 Index over time for different planting eras.

Table 4 obtained using Model [2], shows that the 300 Index drift prior to age 30 years in stands planted before 1980 averages about 0.07 units per year. Thus, for these historic stands, if yield at age 27 years is projected from an age 7 year measurement, the model will tend to under-predict volume MAI by about a 20×0.07 or $1.4 \text{ m}^3/\text{ha}/\text{yr}$, a negative bias of about 5-6% for a typical site. Table 4 also shows that this bias will increase markedly at ages greater than 30 years. However, there is no significant drift, and the model is therefore likely to be unbiased overall, for stands younger than 30 years planted since 1980. Because of the bias in historic plantings the remainder of the validation utilized only data from stands planted since 1960.

Table 4. Mean 300 Index drift for stands established in different eras.

Planting era	Annual drift ($\text{m}^3/\text{ha}/\text{yr}^2$)	
	Age < 30	Age > 30
Pre-1940	0.077	0.410
1940-1959	0.078	0.198
1960-1979	0.071	0.161
Post-1979	0.005	-

In Fig. 6, the result of fitting Model [1] to stands classified according to stocking is shown. Trends for stockings less than 800 stems/ha are almost flat or slightly increasing indicating little bias. However, at stockings greater than 800 stems/ha, the index has a negative drift, indicating that the model will tend to over-predict yield at high final crop stockings. Table 5 confirms that at stockings below 500 stems/ha, there is on average a slight positive drift, but that above 800 stems/ha, the drift became strongly negative. The mean index is also markedly lower at these high stockings. This could imply that yield predicted for very high stockings based on an index obtained from measurements in lower stocked stands, could be over-estimated even at young ages, and become increasingly so with age. However, it is also possible that the lower index at high stockings occurred because higher stocked stands tended to be on lower productivity sites. An examination of final-crop stocking trials with very high stockings might elucidate this point.

As noted in Kimberly et al. (2005), the modelling data set which consisted mainly of fairly recently established plantations, contained little data beyond age 30 years and for stockings of more than 800 stems/ha. Therefore, the inferior performance of the model outside these modelling dataset boundaries is not surprising. The bias at ages greater than 30 years and final stockings higher than 800 stems/ha, will be of no concern for most managers as these limits are beyond the normal range for New Zealand radiata pine plantations.

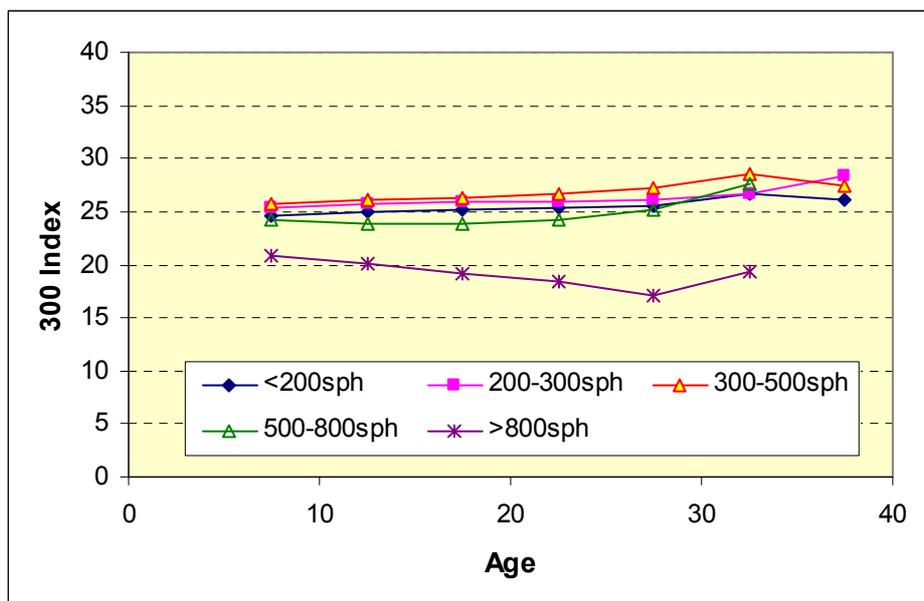


Fig. 6. Drift in the mean 300 Index over time in post-1959 stands classified by stocking.

Table 5. Mean 300 Index drift up to age 30 years in post-1959 stands classified by stocking.

Stocking class	Annual drift ($\text{m}^3/\text{ha}/\text{yr}^2$)
<200sph	0.044
200-300sph	0.038
300-500sph	0.074
500-800sph	0.007
>800sph	-0.202
Overall	0.042

The performance of the model for stands thinned at different ages is shown in Fig. 7 and Table 6. These show little difference in 300 Index drift with age of final thinning indicating that the model accurately predicts the effects of thinning on yield. Similarly, Fig. 8 and Table 7 indicate that the effect of pruning on growth is also accurately represented.

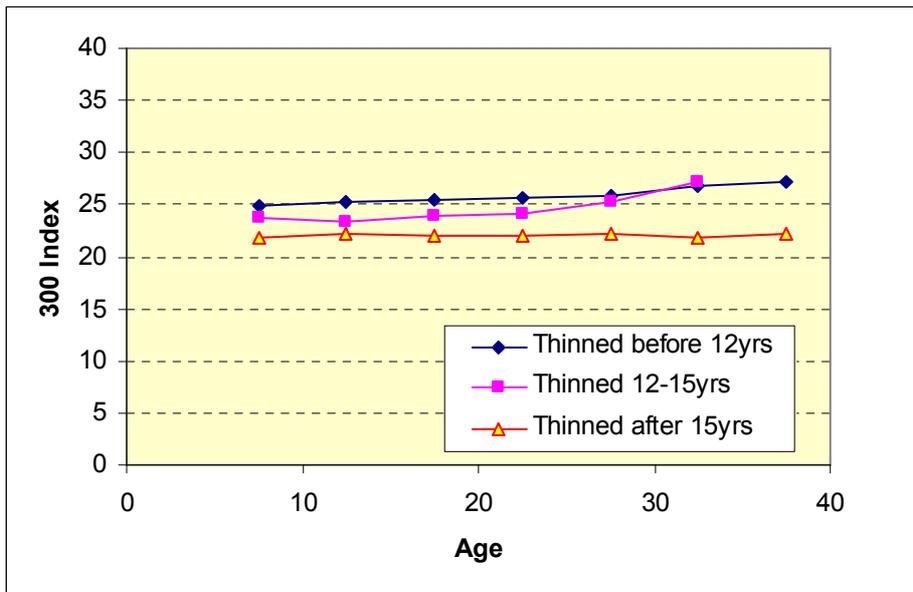


Fig. 7. Drift in the mean 300 Index over time in post-1959 stands classified by age of final thinning.

Table 6. Mean 300 Index drift up to age 30 years in post-1959 stands classified by final thinning age.

Age of final thinning	Annual drift ($\text{m}^3/\text{ha}/\text{yr}^2$)
<12yrs	0.052
12-15yrs	0.007
>15yrs	-0.003
Overall	0.042

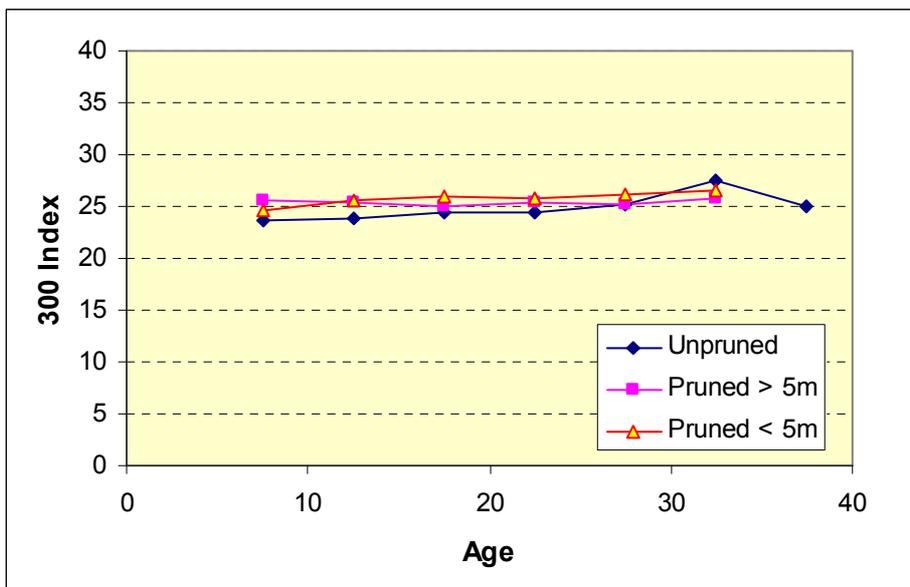


Fig. 8. Drift in the mean 300 Index over time in post-1959 stands classified by pruning treatment.

Table 7. Mean 300 Index drift up to age 30 years in post-1959 stands classified by pruning treatment.

Pruning treatment	Annual drift (m ³ /ha/yr ²)
Unpruned	0.055
Pruned<5m	-0.034
Pruned>5m	0.085
Overall	0.042

In Fig. 9, the performance of the model in stands planted on the three site types defined on the basis of previous vegetation cover is shown. Coastal sand sites show a clear negative index drift as noted in the previous validation (ref), although in the current analysis this is less pronounced. Fig. 9 indicates that there is almost no drift in stands established on ex-pasture ‘Farm’ sites, but that some positive drift on ‘Forest’ sites (previously in forest or scrub). The model can therefore be expected to over-predict on Sand sites, under-predict on Forest sites, and be unbiased on Farm sites.

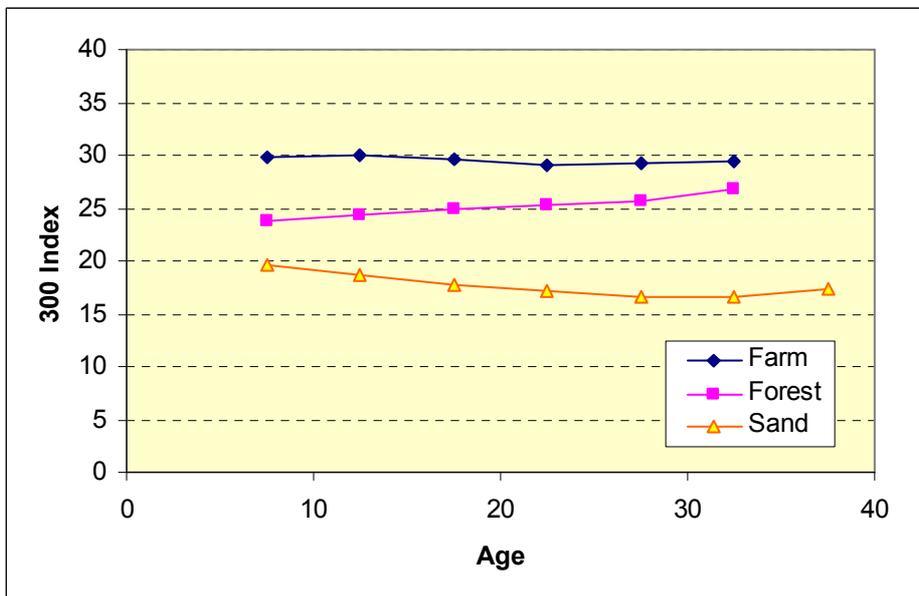


Fig. 9. Drift in the mean 300 Index over time in post-1959 stands classified according to previous cover.

When stands are classified on the basis of productivity (300 Index), there is significant negative drift on sites of low overall productivity (300 Index less than 19) and slight positive drift for stands of average to high productivity (Fig. 10).

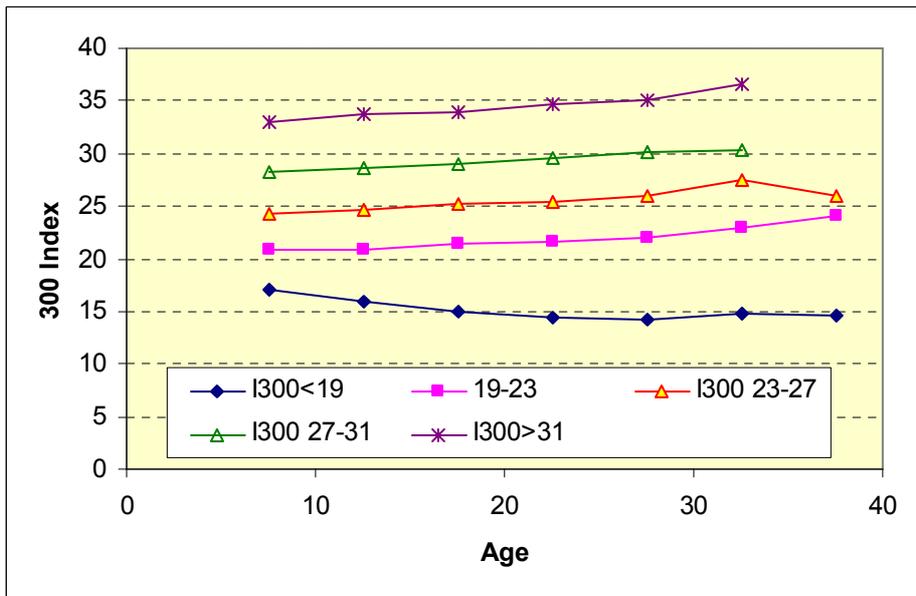


Fig. 10. Drift in the mean 300 Index over time in post-1959 stands classified by 300 Index.

The combined effects of previous vegetation and site productivity are summarized in Table 8. This shows that mean index drift is consistently about 0.1-0.15 less on Farm sites than Forest sites when both are of similar overall productivity. This provides evidence of a decline in site fertility on ex-pasture sites compared with Forest sites. It is generally hypothesised that the ‘Farm effect’ is largely due to a build up of nitrogen in the soil from a history of fertilization, and some decline in fertility would therefore be expected to happen at some point although no evidence of this has been noted previously. Given that the 300 Index on Farm sites typically averages about 5 units more than Forest sites (Fig. 9), the rate of decline shown in Table 8 suggests that typically the additional fertility of ex-pasture sites may be exhausted after about 2 rotations.

Table 8. Mean 300 Index drift ($\text{m}^3/\text{ha}/\text{yr}^2$) up to age 30 years in post-1959 stands classified by 300 Index and previous cover.

	Overall	Farm	Forest	Sand
I300<19	-0.185	-0.445	-0.071	-0.243
I300 19-23	0.054	-0.044	0.096	-0.088
I300 23-27	0.088	-0.041	0.104	0.045
I300 27-31	0.104	0.003	0.134	0.246
I300>31	0.131	0.033	0.226	
Overall	0.042	-0.026	0.107	-0.185

When stands are classified on the basis of Site Index, a slight tendency toward negative drift at low Site Indices and positive drift at high Site Indices was evident (Fig. 11). However, Table 9 indicates that this is due to the correlation between Site Index and 300 Index. For a given Site Index, the drift becomes more negative with decreasing 300 Index. However, for a given 300 Index, Site Index has no systematic effect on drift.

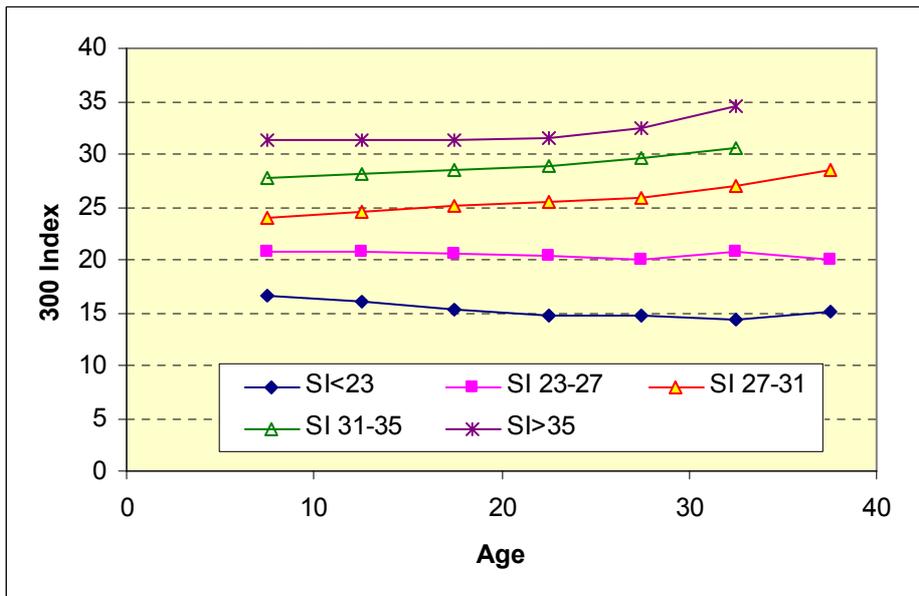


Fig. 11. Drift in the mean 300 Index over time in post-1959 stands classified by Site Index.

Table 9. Mean 300 Index drift ($\text{m}^3/\text{ha}/\text{yr}^2$) up to age 30 years in post-1959 stands classified by Site Index and 300 Index.

	Overall	I300<19	I300 19-23	I300 23-27	I300 27-31	I300>31
SI<23	-0.140	-0.214	0.043	0.089	0.155	-0.302
SI 23-27	-0.038	-0.168	0.032	0.051	0.175	0.213
SI 27-31	0.093	-0.173	0.066	0.099	0.187	0.204
SI 31-35	0.097	-0.188	0.079	0.105	0.065	0.128
SI>35	0.043	-	-0.097	-0.007	0.060	0.069
Overall	0.042	-0.185	0.054	0.088	0.104	0.131

Regional trends in the 300 Index against age are shown in Figs. 12 to 14. Some regional differences are apparent. Among North Island regions, there is a strong positive index drift in Wellington (although this is based mainly on limited data was from Ngaumu Forest) and Gisborne (although based on only a few plots). Coastal sand forests show a clear negative drift and there is a slight negative drift in Auckland. Other regions show either a slight positive drift (Waikato and Bay of Plenty) or no drift. In the South Island, there is moderate positive drift in Nelson, Marlborough, Otago and Southland while the West Coast and Canterbury show a slight negative drift. These results are generally confirmed by Table 10, although the negative drift in Northland shown in this Table is not clearly apparent in Fig. 12. Table 10 shows that the trend of a more negative or less positive drift on Farm compared with Forest sites is apparent in all regions except Bay of Plenty, Canterbury and Otago.

These results suggest that there will be some regional differences in model performance. On coastal sand sites, the model will tend to over-predict yield. There may also be a tendency to over-predict yield in Northland, Auckland, Canterbury and the West Coast. In the remainder of the South Island, and in Wellington and Gisborne, the model will tend to under-predict, and there may also be slight under-prediction in Waikato and Bay of Plenty.

The reasons for these regional differences in growth trends are not obvious, except for the negative drift in sand forests which is probably due to a declining nitrogen status, and is therefore of a similar nature to the more gradual index decline on Farm compared with Forest sites. When individual growth plots are mapped, it appears there may be some concentrations of plots showing either positive or negative drift (Figs. 15 and 16). This suggests that there may be scope for identifying environmental drivers for these differences in growth trajectories.

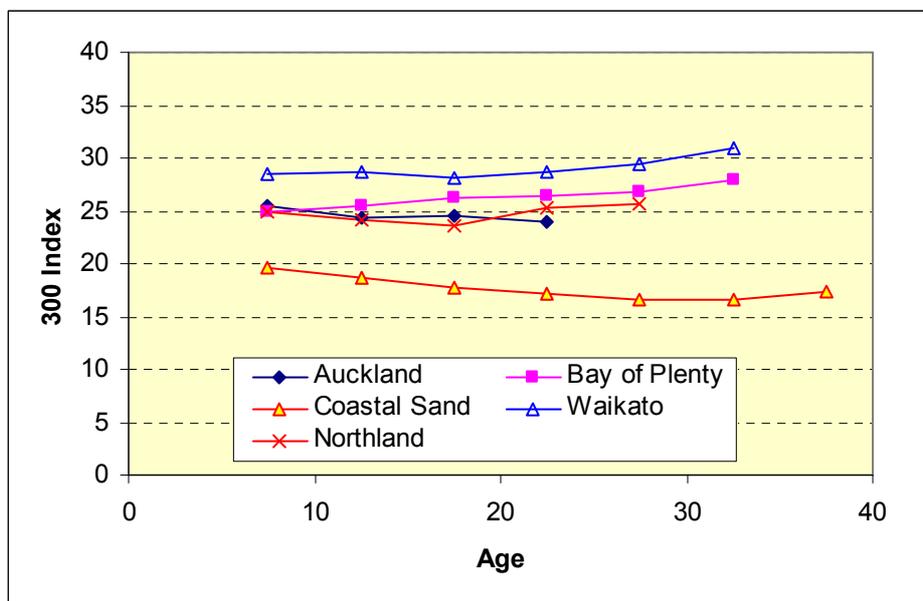


Fig. 12. Drift in the mean 300 Index over time in post-1959 stands for northern North Island regions.

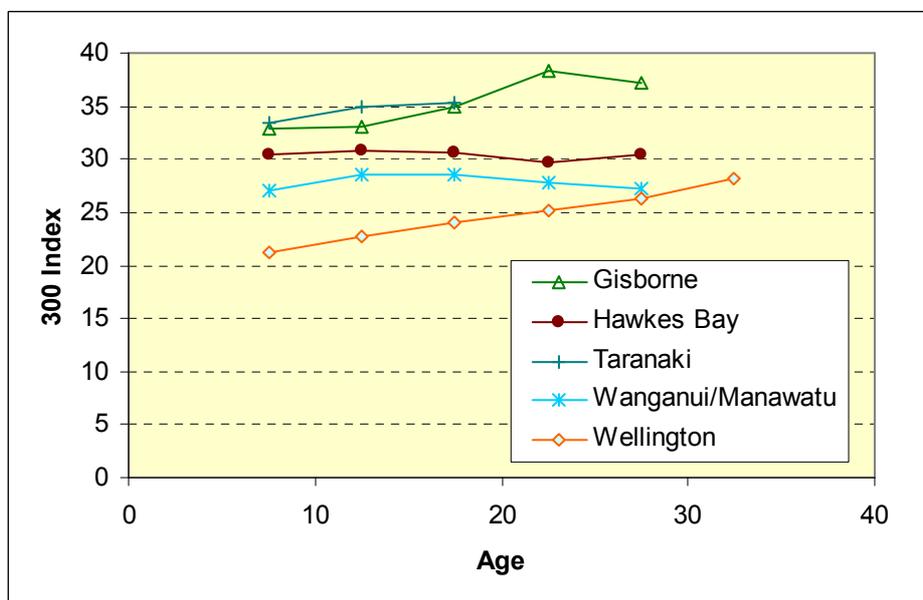


Fig. 13. Drift in the mean 300 Index over time in post-1959 stands for central/southern North Island regions.

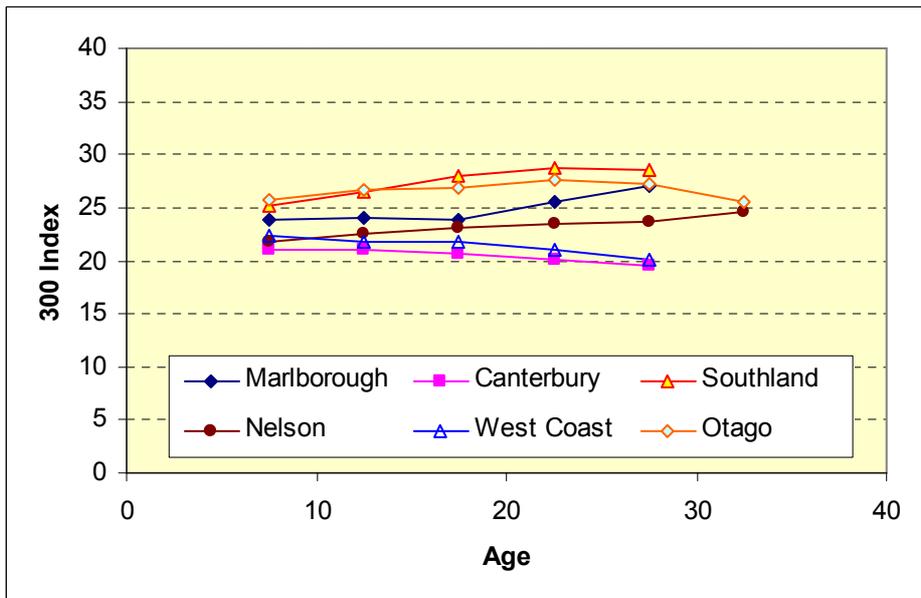


Fig. 14. Drift in the mean 300 Index over time in post-1959 stands for South Island regions.

Table 10. Mean 300 Index drift ($\text{m}^3/\text{ha}/\text{yr}^2$) up to age 30 years in post-1959 stands classified by region and previous cover.

	Overall	Farm	Forest
Northland	-0.141	-0.188	-0.056
Auckland	-0.113	-0.313*	-0.008
Waikato	0.034	-0.026	0.089
Bay of Plenty	0.097	0.113	0.097
Gisborne	0.164	0.031	0.534*
Hawkes Bay	-0.008	-0.099	0.183
Taranaki	0.222*	0.222*	
Wanganui/Manawatu	-0.021	-0.063	0.025
Wellington	0.226	0.177*	0.229
Nelson	0.110	-0.037	0.112
Marlborough	0.110	-0.235	0.285
West Coast	-0.103	-0.299*	-0.102
Canterbury	-0.063	0.052	-0.080
Otago	0.168	0.186	0.146
Southland	0.262	0.195	0.293

*Based on less than 10 plots

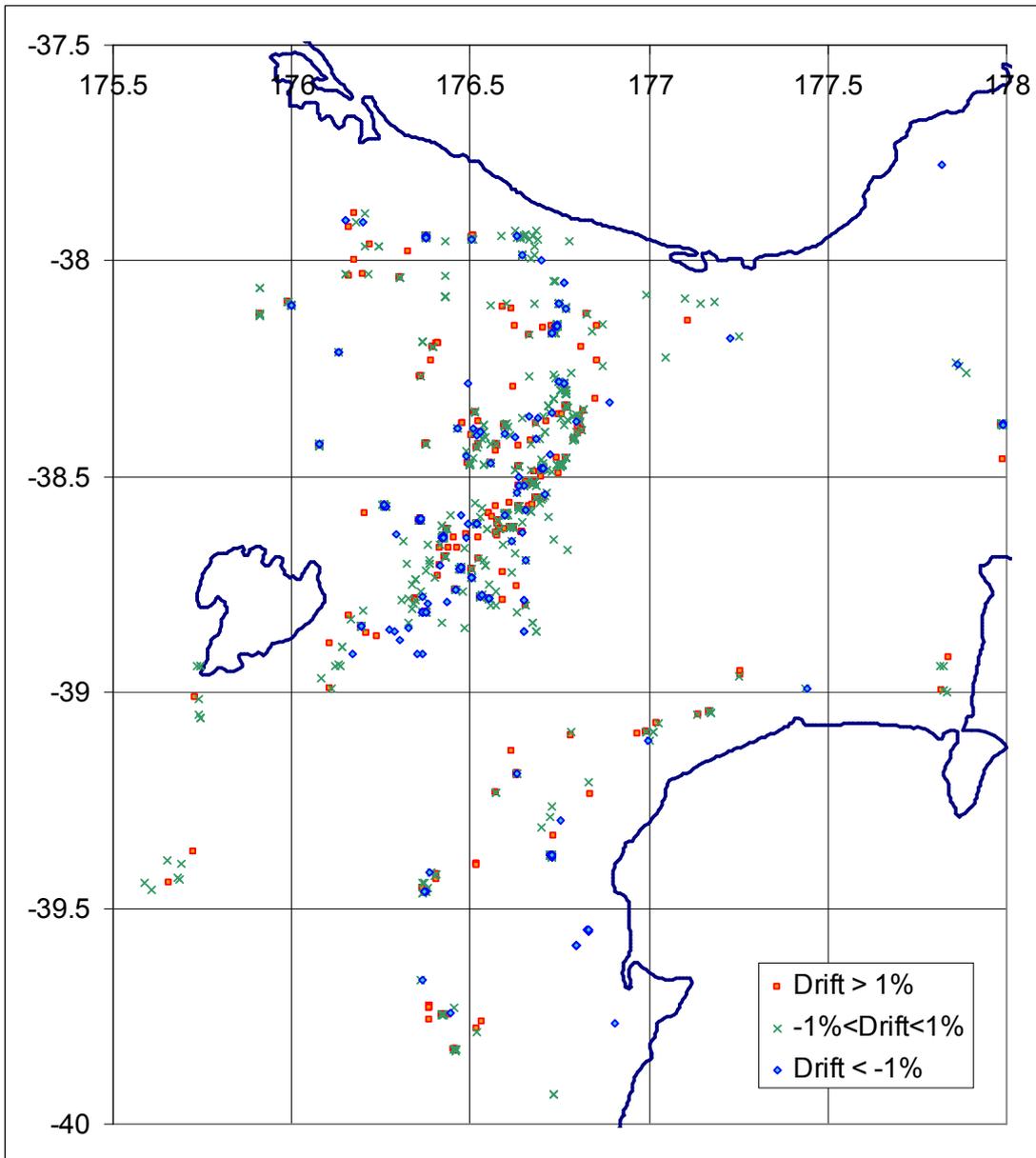


Fig. 15. Index drift in central and eastern North Island. Points are validation PSPs with known grid coordinates.

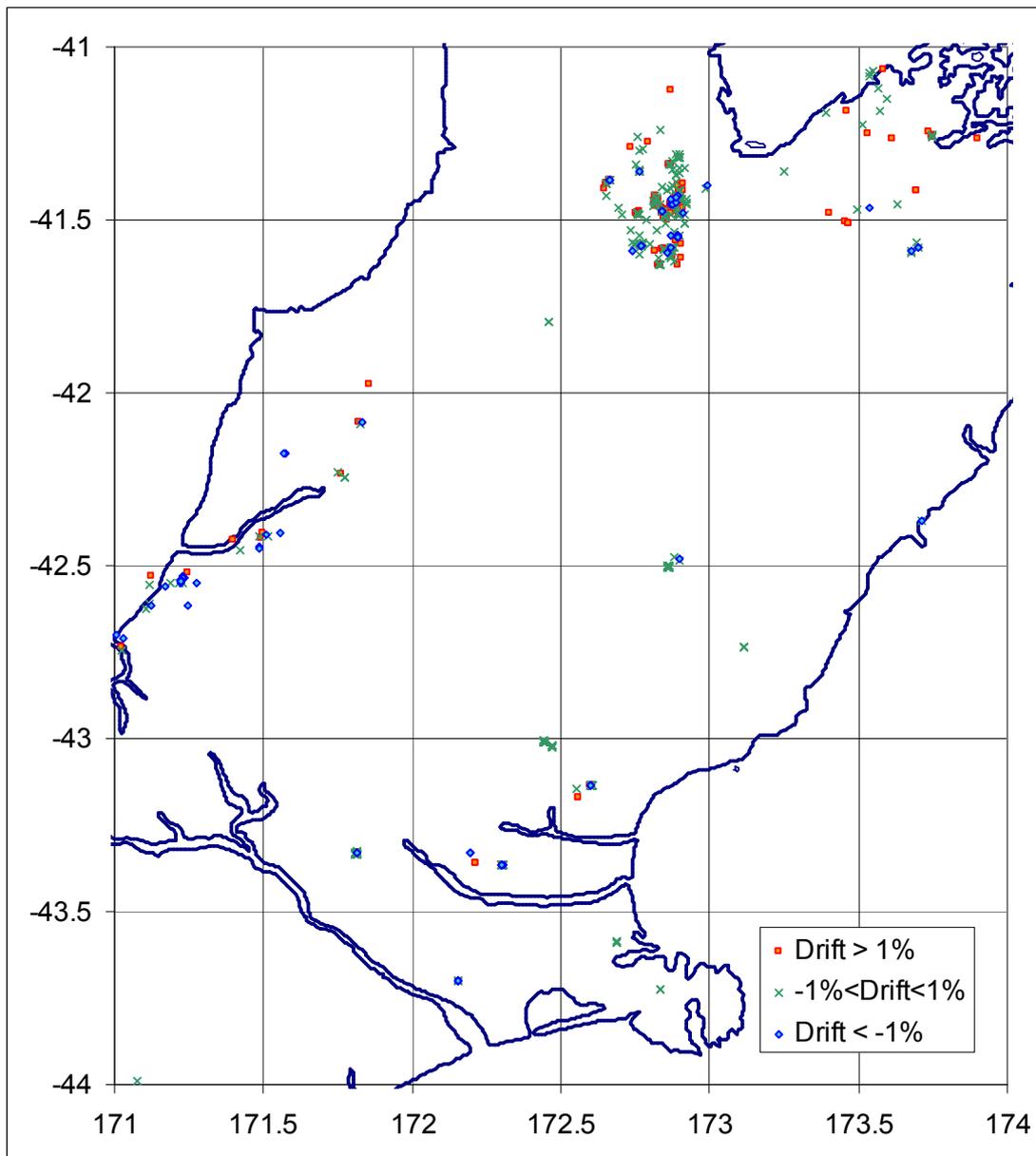


Fig. 16. Index drift in northern South Island. Points are validation PSPs with known grid coordinates.

CONCLUSIONS

- On average, the model performs well up to age 30 years, and for final crop stockings below 800 stems/ha.
- The effects of pruning and thinning are accurately modelled.
- The model under-predicts yield above age 30 years; total yield is under-predicted by 4% at age 35 years and 9% at age 40 years.
- The model over-predicts yield at stockings above 800 stems/ha; at a stocking of 1250 stems/ha, yield may be over-predicted by as much as 20%.
- There is a tendency to over-predict yield for very low productivity sites where the 300 Index is less than 19 m³/ha.yr; at a 300 Index of 15, over-prediction may be about 15%.
- Overall there is a tendency to under-predict yield on traditional Forest sites by an average of about 8%, but to be unbiased on Farm sites.

- The model over-predicts yield on Coastal Sand sites.
- There may also be a slight tendency to over-predict yield in Northland, Auckland, the South Island West Coast and Canterbury.
- The model under-predicts yield in Otago and Southland and also in Gisborne and Wellington (based on limited data).
- There is also a slight tendency to under-predict yield in Bay of Plenty, Nelson and Marlborough.

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