# FUNCTIONALITY CONTAINED IN THE NEW ZEALAND FARM FORESTRY ASSOCIATION RADIATA PINE CALCULATOR VERSION 2 

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FOREST AND FARM PLANTATION MANAGEMENT COOPERATIVE

# FOREST AND FARM PLANTATION MANAGEMENT COOPERATIVE EXECUTIVE SUMMARY 

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Version 2 of the radiata pine calculator, which has been jointly developed with the New Zealand Farm Forestry Association, and released for testing in July 2004, utilises some 71 algebraic functions or groups of functions that are embedded directly in it. This report describes the functions used, their sources, and provides ancillary background, including Visual Basic for Applications (VBA) code, permitting their implementation within the calculator. The functions described are as incorporated in the calculator as at July 2004. In all cases NZ-wide functions have been incorporated, with one exception. The Central North Island Pumice Plateau Weibull (diameter distribution) function no 19 has been incorporated as an interim measure.

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## INTRODUCTION

In version 1 of the NZ Farm Forestry Association calculator, some functionality was embedded in the calculator, however most of the calculations were estimated from regressions fitted to a database of STANDPAK output. In version 2, all functionality is embedded directly in the calculator.

Most of the 71 functions or groups of functions the calculator utilises are generally available to Cooperative members and researchers though the published literature or Cooperative reports, although some reside in rather obscure places, and collating them together can be time consuming. This report is designed to provide ready access to the functions to facilitate future maintenance of the radiata pine calculator version 2. It is also expected access to the functions in a single report may provide obvious benefits to Cooperative members for application in their day-to-day work.

Table 1. Summary of functions used in the Farm Forestry Calculator Version 2

| Topic | Function <br> Numbers | When <br> developed | Developed by | Reference |
| :--- | :--- | :--- | :--- | :--- |
| Stand Growth | $1-8$ | 2004 | Forest Research/FFPM | Kimberley (pending) |
| Tree-level <br> volume/taper | $9-20$ | 1999 | Forest Research/FFPM | FFPM Coop report no 66 |
| Stand level volume | 21 | 2004 | Forest Research/SFF/ <br> NZ FFA | Hansen et al. NZ J For Science <br> (submitted). |
| Height/age | $22-24$ | 2004 | Forest Research/FFPM | Budianto (pending) |
| Diameter distribution | $25-31$ | 1990 | Forest Research/SGMC | Lawrence, 1990 |
| Diameter over stubs | $32-34$ | 1986 | Forest Research/FFPM | FRI Bulletin 12 |
| Pruned Log Index | $35-38$ | 1989 | Jim Park, Interface <br> Forest \& Mill Ltd | Park, 1989. NZ J For Science <br> 19(1) 41-53 |
| Branches | $39-42$ | 1990 | Forest Research/FFPM | FFPM Coop report 1 |
| Wood density | 43 | 1997 | Forest Research | Kimberley and McConchie (1997) |
| Canopy closure | 44 | 2000 | Forest Research/FFPM | McElwee and Knowles, NZ J For <br> Science 30(3) 422-435, FFPM <br> Coop report 62, FFPM Coop <br> Proceedings, May 2000 |
| Crown height, crown <br> length | $45-46$ | 1998 | 1986 | Forest <br> Research/AgResearch |
| Understorey grazing | $47-53$ | AgResearch | FRI Bulletin 139 <br> 1998 pp9-24 proceedings, May |  |
| Livestock performance | 54 | 1986 | 1990 | Landcare Research |
| Root biomass | 55 | Watson and O'Loughlin N.Z. <br> Journal of Forestry Science 20(1): <br> 97-110. |  |  |
| Root biomass decay | $56-57$ | 1979 | O'Loughlin and Watson NZ J For <br> Science 9: 284-293. |  |
| Labour content of <br> silvicultural operations | $58-71$ | 1975 | NZ Forest Service, <br> Kaingaroa Forest, <br> Work Study Section. | Unpublished |

## Stand Growth - The 300-Index Growth Model

The 300 -index model was initially developed as an index for comparing site productivity using extensive growth and field trial data for radiata pine available in New Zealand. The 300 Index is a volume productivity index, and is defined as the mean annual volume increment, in $\mathrm{m}^{3} / \mathrm{ha} / \mathrm{yr}$, at an age of 30 years, assuming a final stocking of 300 stems $/$ ha, timely pruning to 6 m , and thinning to final crop at completion of pruning.

By placing the 300 Index and stand age together with an estimate of height (derived from site index and a ht/age curve), and basal area, (derived from the 300 index and a stand-level volume function) the 300 Index can be turned into a simple growth model. The growth model can be calibrated for any given site using the two site productivity indices of $S I$ (mean top height at age 20 years) and the 300 -index.

The initial form of the 300 Index as a growth model was installed in the Calculator version 1, however its role was to calibrate the stand, and it was not included directly in the yield calculations. In the calculator version 2, the 300 Index as a growth model is fully embedded, and is used to grow the stands directly.

The equations for this model are, in brief, as follows. To predict $D B H$ at a given age $T$ and stocking $N$ in an unthinned and unpruned stand, the following equations are used:

$$
\begin{gather*}
T_{z}=r_{1} \exp \left(r_{2} S I\right)  \tag{1}\\
a=a_{1}(1+D I)  \tag{2}\\
b=b_{1}+b_{2}(S I-28)+b_{3} D I+b_{4}(S I-28) D I  \tag{3}\\
D_{200}=a\left(\frac{1-\exp \left(b\left(T-T_{z}\right)\right)}{1-\exp \left(b\left(30-T_{z}\right)\right)}\right)^{c}  \tag{4}\\
q=q_{1}\left(1+q_{2}(S I-28)\right) \operatorname{Sign}(N-200)\left((\log (N)-\log (200) \mid)^{q_{3}}\right.  \tag{5}\\
p=p_{1}+p_{2} N+p_{3} D I  \tag{6}\\
D B H=D_{200}-q \log \left(1+\exp \left(s\left(D_{200}-p\right)\right)\right) \tag{7}
\end{gather*}
$$

The coefficients as estimated are: $r_{1}=8.6877, r_{2}=-0.0539, a_{1}=56.523, b_{1}=-0.09045, q_{1}=$ 2.6416, $p_{1}=28.1224, c=1.4821, b_{2}=-0.00212, p_{3}=15.7581, p_{2}=-0.00455, b_{3}=-0.1325, s=$ $0.1702, b_{4}=-0.0084, q_{2}=0.0209, q_{3}=0.8234$. And ' $\mid{ }^{\prime}$ ' denotes absolute value and Sign is the signum function, i.e.

$$
y(x)=\left\{\begin{array}{cc}
-1, & x<0  \tag{8}\\
0, & x=0 \\
1, & x>0
\end{array}\right.
$$

These equations use $S I$, and $D I$ to calibrate the model to a particular site. $D I$ is a diameter index, which is derived from the $S I$ and 300 -index. This is accomplished by an iterative procedure in which the above equations are used to estimate volume at age 30 for a ' 300 -index' stand, i.e. a final stocking of 300 stems $/ \mathrm{ha}$, timely pruning to 6 m , and thinning to final crop at completion of pruning. The procedure finds the value of $D I$, which achieves a volume MAI equal to the 300 index.

The method of modelling thinning is based on a time-shift approach. Firstly, the $D B H$ after thinning is predicted from $D B H$ before thinning using the thinning function. An iterative procedure is then used to predict age corresponding to the $D B H$ and stocking after thinning for an unthinned stand. The thinning age shift $T_{s}$, defined as the difference between this predicted age and the actual age, is then calculated. The model gradually increases $T_{s}$ by a maximum of 0.5 years in the period following thinning, and it then remains constant until the end of the rotation or the next thinning. Predictions of $D B H$ are obtained using the above unthinned model equations, but using $T-T_{s}$ in place of $T$. Any subsequent thinning is treated in the same way with each thinning increasing the age shift, $T_{s}$.

Pruning effects are also modelled using a time-shift approach in which 'effective' age is gradually adjusted downwards from the 'actual' age by a pruning age-shift term $T_{p}$ which is a function (not described in detail here) of pruned height, crown length and stocking. $T_{p}$ continues to increase until several years after the final pruning, beyond which it remains constant until the end of the rotation.

To estimate the 300 -index from a stand measurement, an iterative procedure is used, which finds the 300 -index that achieves the measured BA (or DBH or Volume) at the given age for the specified stocking and pruning history.

## Tree-level Taper and Volume Function

The tree-level taper function works off the equations in Gordon and Budianto (1999), which uses a 3-point taper function form, which utilises DBH, ht, and an estimate of the diameter under bark at $6 \mathrm{~m}\left(D_{6}\right)$. The $D_{6}$ is predicted from tree DBH and form quotient $\left(D_{6}=D B H \cdot F Q\right)$, the latter is predicted from stand parameters using the following regression

$$
\begin{equation*}
F Q=\beta_{0}+\beta_{1} e^{-\beta_{2} s d}+\beta_{3} e^{-\left(\frac{H}{m t h}\right)^{2}}+\beta_{4} H_{p} \tag{9}
\end{equation*}
$$

Where $H$ is tree height $(\mathrm{m}), H_{p}$ is pruned height ( m ), $m t h$ is stand mean top height $(\mathrm{m})$, and $s d$ is stand density, which is calculated as

$$
\begin{equation*}
s d=\frac{m d b h^{2}}{\sqrt{\frac{100}{m t h \sqrt{n}}}} \tag{10}
\end{equation*}
$$

Where $m d b h$ is stand quadratic mean diameter at breast height ( cm ), $n$ is stocking (stems per hectare), and $m t h$ is stand mean top height ( m ). The coefficients of equation (9) are $\beta_{0}=0.945$, $\beta_{1}=-0.387, \beta_{2}=0.000686, \beta_{3}=-0.267$, and $\beta_{4}=0.00357$.

Once $D_{6}$ is estimated, the diameter over bark (dob) at height $L$ is calculated using the following equation

$$
\begin{equation*}
d o b=\sqrt{D B H^{2}\left(\beta_{1} z^{\gamma_{1}}+\beta_{2} z^{\gamma_{2}}+\beta_{3} z^{\gamma_{3}}\right)} \tag{11}
\end{equation*}
$$

Where

$$
\begin{gather*}
z=\frac{L}{H}  \tag{12}\\
z_{b}=1-\frac{1.4}{H}  \tag{13}\\
z_{u}=1-\frac{6}{H} \tag{14}
\end{gather*}
$$

$$
\begin{gather*}
\beta_{1}=\frac{1-\left(\frac{z_{b}^{\gamma_{2}}}{z_{u}^{\gamma_{2}}}\right)\left(\frac{D_{6}^{2}}{D B H^{2}}-\beta_{3} z_{u}^{\gamma_{3}}\right)-\beta_{3} z_{b}^{\gamma_{3}}}{z_{b}^{\gamma_{1}}-\frac{z_{b}^{\gamma_{2}} z_{u}^{\gamma_{1}}}{z_{u}^{\gamma_{2}}}}  \tag{15}\\
\beta_{2}=\frac{\frac{D_{6}^{2}}{D B H^{2}}-\beta_{1} z_{u}^{\gamma_{1}}-\beta_{3} z_{u}^{\gamma_{3}}}{z_{u}^{\gamma_{2}}}  \tag{16}\\
\beta_{3}=\beta_{30}+\beta_{31} \frac{D B H-D_{6}}{6-1.4}  \tag{17}\\
\gamma_{1}=\gamma_{10}+\gamma_{11} \frac{D_{6}}{H-6}  \tag{18}\\
\gamma_{3}=\gamma_{31} H \frac{D_{6}}{D B H} \tag{19}
\end{gather*}
$$

Where the coefficients are: $\beta_{30}=0.7768, \beta_{31}=-0.1347, \gamma_{10}=1.018, \gamma_{11}=0.2967, \gamma_{2}=12.68, \gamma_{31}$ $=1.047$. The diameter under bark is then estimated using the following relationship:

$$
\begin{equation*}
d u b=\sqrt{d o b\left(\alpha_{0}+\alpha_{01} H+\alpha_{10} z \exp \left(\frac{-\alpha_{12} H}{2}\right)+\alpha_{2} z \exp \left(\alpha_{31} H\right)\right)} \tag{20}
\end{equation*}
$$

Where the coefficients are: $\alpha 0=0.4242, \alpha_{01}=-0.002822, \alpha_{10}=0.6067, \alpha_{12}=0.06129, \alpha_{2}=-$ 0.207 , and $\alpha_{31}=0.3208$.

The volume function essentially integrates the above equation, and works with the algorithm given in Appendix D in Gordon and Budianto (1999), which is not repeated here.

## Stand-level Volume Function

A New-Zealand-wide dataset available to Forest Research was collated (Carolyn Andersen pers.comm.) and the individual tree volumes were estimated for a range of stands using the above three-point single-tree taper/volume function. Using time-specific and plot-specific $\mathrm{dbh} /$ height regressions the height was estimated for all trees in the plots, including those measured for height. Based on the measured DBH and the estimated heights the individual tree volumes were estimated for all trees in the plots using the single-tree taper/volume function of Gordon and Budianto (1999). The stem diameter at 6 m for individual trees, which is a requirement of the individual tree volume function, was all estimated from diameter at breast height and the plot-level parameters following the approach outlined in Gordon and Budianto (1999), I.e. An actual diameter at 6 m was not used. Finally, total plot volume was calculated as the sum of the individual tree volumes. The estimated stand-level volume was then regressed against stand mean parameters using:
$\mathrm{V}=\exp \left(\left(\log (B A)-a-b \log (M T H)-d \log (N)-e \log (N)^{2}-f \log (M T H)^{2}-g \log (M T H) \log (N)\right) /(\right.$ $h \log (N)+c))$

Where $V$ is stand volume ( $\mathrm{m}^{3} / \mathrm{ha}$ ), $B A$ is basal area ( $\mathrm{m}^{2} / \mathrm{ha}$ ), MTH is mean top height ( m ), and $N$ is stocking (stems/ha). The coefficients were estimated as $a=2.5880, b=-2.1434, c=1.3972, d$ $=-0.4376, e=0.0509, f=0.1070, g=0.1238$, and $h=-0.0814$.

A paper describing this work has been submitted to the NZ Journal of Forestry Science for publication (Hansen et al. 2004).

## Height/Age Function

The height/age function is based on a model made by Budianto (Cooperative report and paper in prep). The model is of the form:

$$
\begin{equation*}
M T H=0.25+\left(M T H_{20}-0.25\right)\left(\frac{1-\exp (-T \exp (a))}{1-\exp (-20 \exp (a))}\right)^{b} \tag{22}
\end{equation*}
$$

Where MTH is mean top height, $M T H_{40}$, and the coefficients $a$ and $b$ are given as

$$
\begin{equation*}
a=\exp \left(e_{0}+e_{1} L+e_{2} A\right) \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
b=\frac{1}{e_{3}+e_{4} S I} \tag{24}
\end{equation*}
$$

Where $L$ is latitude (degrees south), $A$ is altitude (m), and $S I$ is site index (MTH at age 20), and the coefficients are $e_{0}=-1.335, e_{1}=-0.03581, e_{2}=-0.0006306, e_{3}=0.499$, and $e_{4}=0.005059$.

## Log Cutting Overview

The total standing volume is distributed to $\log$ grades through simulated log cutting of model trees. To simulate the distribution of tree and branch sizes the cutting simulation is iterated for model trees in each diameter class ( 2 cm steps) across the diameter distribution, and for each BIX class (steps of $\sigma / 2$ ) within each diameter class ( $\sigma$ is the standard deviation). The result from each simulation is multiplied by the probability of finding a tree in that particular diameter and BIX class. Once the volume is distributed to log grades across both distributions, a user-input percentage of each grade is downgraded to the poorest grade (usually pulp), and the total merchantable volume across grades is automatically adjusted to fit the user-input conversion percentage.

## Modelling Diameter Distributions

The probability of each diameter class is modelled using the probability density function of the three-parameter Weibull distribution:

$$
\begin{equation*}
W_{p d f}(x)=1-e^{-\alpha(x-\gamma)^{\beta}} \tag{25}
\end{equation*}
$$

The coefficients ( $\alpha, \beta$, and $\gamma$ ) are estimated iteratively from stand minimum $\operatorname{DBH}\left(D B H_{\text {min }}\right)$, maximum DBH ( $D B H_{\max }$ ) and the DBH variation (Var ${ }_{\mathrm{DBH}}$ ) following the approach of Goulding and Shirley (1979). The VBA implementation is as follows:

```
Function Wparms(minDBH, maxDBH, DBH, SPH)
    Dim Out(0, 2)
    Pi}=3.1415926535987
If DBH > 0 Then
    DBHvar = Wvar(minDBH, maxDBH) 'Estimate DBHvar
    BA1 = DBH ^ 2 * Pi / 40000
    BA2 = minDBH ^ 2 * Pi / 40000
    hat = DBHvar / ((BA1 - BA2)^ 2)
    counter = 0
    betad = 0.5
    betau = 15
    While Abs(betau - betad) > 0.00001 And counter < 200
        beta = 0.5 * (betad + betau) 'guess halfway between up and down, i.e. binary search
        g1 = Exp(Excel.WorksheetFunction.GammaLn(1 + 2 / beta))
        g2 = Exp(Excel.WorksheetFunction.GammaLn(1 + 1/ beta))
        diff = (1/ (SPH^^ (-1 / beta) - 1) ^ 2) * (g1 / g2^ 2-1) - hat
        If diff < 0 Then
            betau = beta
        Else
            betad = beta
        End If
        counter = counter + 1
    Wend
```

```
    If Abs(betau - betad) < 0.00001 Then 'make sure the search has converged
        beta = 0.5 * (betau + betad)
        k = Abs((BA1 - BA2) / (1-1 / SPH ^ (1 / beta))) / g2
        Out(0,0) = 1 / k^ beta 'Estimate alpha parameter
        Out(0,1) = beta 'Estimate beta parameter
        Out(0, 2) = BA1 - k * g2 'Estimate gamma parameter
    End If
End If
Wparms = Out 'Return parameters
End Function
```

The minimum DBH is estimated from Table 19 empiric regression CNI Pumice Plateau (Lawrence, 1990) using

$$
\begin{equation*}
D B H_{\min }=a+b D B H+c T+d B A \tag{26}
\end{equation*}
$$

Where $D B H$ is the quadratic mean diameter at breast height (cm), $T$ is stand age (years), $B A$ is stand basal area ( $\mathrm{m}^{2} / \mathrm{ha}$ ), and the coefficients are: $a=-0.545, b=1.03064, c=-0.4172$, and $d=-$ 0.13705 .

The maximum DBH is estimated from Table 19 empiric regression in Lawrence (1990) using

$$
\begin{equation*}
D B H_{\max }=a+b D B H+c T+d B A \tag{27}
\end{equation*}
$$

Where $D B H$ is the quadratic mean diameter (cm) at breast height, $T$ is stand age (years), $B A$ is stand basal area ( $\mathrm{m}^{2} / \mathrm{ha}$ ), and the coefficients are: $a=1.135, b=0.8301, c=0.649$, and $d$ $=0.13085$.

The DBH variation ( Var $_{\mathrm{DBH}}$ ) is estimated from minimum DBH $\left(D B H_{\text {min }}\right)$ and maximum DBH $\left(D B H_{\max }\right)$ as:

$$
\begin{equation*}
V a r_{D B H}=a+b\left(\frac{c}{4}\left(D B H_{\max }^{2}-D B H_{\min }^{2}\right)\right)^{2} \tag{28}
\end{equation*}
$$

The height $\left(H_{i}\right)(\mathrm{m})$ of a tree in $i^{\prime}$ th diameter class is estimated from its diameter $\left(D B H_{i}\right)(\mathrm{cm})$ using a general Peterson height/DBH curve of the form:

$$
\begin{equation*}
H_{i}=1.4+\left(a+\frac{b}{D B H_{i}}\right)^{-2.5} \tag{29}
\end{equation*}
$$

Where the coefficients $a$ and $b$ are determined from empiric (undocumented) regressions as:

$$
\begin{gather*}
a=\frac{\alpha D B H+\beta B A+\gamma \ln (T)}{M T H}  \tag{30}\\
b=D B H\left(\exp \left(\frac{\ln (M T H-1.4)}{-2.5}\right)-a\right) \tag{31}
\end{gather*}
$$

Where $D B H$ is stand mean DBH (cm), $B A$ is stand basal area ( $\mathrm{m} 2 / \mathrm{ha}$ ), $T$ is stand age (years), MTH is stand mean top height ( m ), and the coefficients are: $\alpha=0.05003, \beta=0.039241$, and $\gamma=$ 1.2555.

## Model for Diameter-Over-Stubs (DOS)

The diameter-over-stubs $(D O S)$ is estimated using the original approach of Knowles et al. (1987), which was later verified and redeveloped by Knowles and McElwee (1999).

$$
\begin{equation*}
D O S=a+b D A_{D O S}+c B r_{\text {max }}+d B r_{\text {max }}^{2}+e H_{D O S}+f H_{D O S}^{2} \tag{32}
\end{equation*}
$$

Where $D A_{D O S}$ is the diameter at the DOS height $\left(H_{D O S}\right)(\mathrm{cm}), B r_{\text {max }}$ is the maximum branch in the DOS whorl (mm), and the coefficients are $a=0.6787, b=0.8597, c=0.1439, d=-$ $0.0007354, e=0.4777$, and $f=-0.03793$.

The size of the maximum branch $\left(B r_{\max }\right)$ in the DOS whorl is estimated as

$$
\begin{equation*}
B r_{\max }=a\left(H-H_{D O S}\right)\left(\frac{D B H}{H-1.4}\right)^{2}+b \sqrt{H_{D O S}} \tag{33}
\end{equation*}
$$

Where $H$ is tree height (m), $H_{D O S}$ is the DOS height(m) (the height from the ground to the largest whorl), $D B H$ is diameter at breast height (cm), and the coefficients are $a=0.7011$ and $b=$ 12.122.

The diameter at the DOS whorl $\left(D A_{D O S}\right)(\mathrm{cm})$ is calculated from tree height $(H)(\mathrm{m})$ and DOS height $\left(H_{D O S}\right)(\mathrm{m})$ using

$$
\begin{equation*}
D A_{D O S}=D B H \frac{H-H_{D O S}}{H-1.4} \tag{34}
\end{equation*}
$$

## Model for Pruned Log Index (PLI)

Pruned $\log$ index is estimated using the approach of Park (1989), i.e.

$$
\begin{equation*}
P L I=\left(\sqrt{\frac{D_{13}-D_{c}}{10}}\right)\left(\frac{D_{13}}{D_{c}}\right) V_{R}^{1.6} \tag{35}
\end{equation*}
$$

Where $D_{13}$ is the diameter (mm) of the log 1.3 m from the large end, $D_{c}$ is the diameter of the defect core ( mm ), and $V_{R}$ is the ratio between the common volume and the log volume, where common volume is the volume of the log not intercepted by the defect core. The coefficients were estimated from a matrix of output generated from an Excel implementation of the PLI calculator (Park 2004)
$D_{13}$ is estimated from the small-end diameter (SED) (mm), the length of the $\log (L)(\mathrm{m})$ and the taper $(\Delta t)(\mathrm{mm} / \mathrm{m})$ as

$$
\begin{equation*}
D_{13}=S E D+(L-1.3) \Delta t \tag{36}
\end{equation*}
$$

$D_{c}$ is estimated from sweep $(S W)(\mathrm{mm} / \mathrm{m})$ and diameter-over-stubs (DOS) in mm, as

$$
\begin{equation*}
D_{c}=a+b S W+D O S \tag{37}
\end{equation*}
$$

Where the coefficients are $a=46.375$, and $b=1.841$.
$V_{R}$ is calculated as a multiple linear regression using log length $(L)(\mathrm{mm})$, small-end-diameter $(S E D)(\mathrm{mm})$, sweep $(S W)(\mathrm{mm} / \mathrm{m})$ and taper $(\Delta t)(\mathrm{mm} / \mathrm{m})$ using the following equation:

$$
\begin{equation*}
V_{R}=a L+b S E D+c S W+d \Delta t+e \tag{38}
\end{equation*}
$$

With coefficients $a=-0.008, b=0.00019, c=-0.0093, d=-0.00354$, and $\underline{e}=0.8694$.

## Model for Branch Index (BIX)

The branch index models (Kimberley and Knowles 1993) are centred on BIX for the second log (BIX ${ }_{2}$ )

$$
\begin{equation*}
B I X_{2}=a+b \ln \left(1+\exp \left(\frac{c}{b}+\frac{d}{b} D B H_{20}+\frac{e}{b} H_{\text {thin }}+\frac{f}{b} S I+\frac{g}{b} G F\right)\right) \tag{39}
\end{equation*}
$$

Where $D B H_{20}$ is the DBH at age $20(\mathrm{~cm}), H_{\text {thin }}$ is the mean top height $(\mathrm{m})$ at first thinning, $S I$ is site index (mean top height $(\mathrm{m})$ at age 20), and $G F$ is the GF rating, nominally set at 14 . The coefficients are $a=3, b=3.52, c=0.985, d=0.356, e=-0.321, f=-0.354$, and $g=-0.212$

The BIX of the first $\log \left(B I X_{1}\right)(\mathrm{cm})$ is then calculated from $B I X_{2}$ as

$$
\begin{equation*}
B I X_{1}=B I X_{2}\left(a-b^{c S P H}\right) \tag{40}
\end{equation*}
$$

Where SPH is the stocking (stems $/$ ha) and the coefficients are $a=1.61, b=0.947$, and $c=0.01$.

The BIX for the $n$ 'th log is calculated from height at thinning $\left(H_{\text {thin }}\right)(\mathrm{m})$ and final crop stocking (SPH) (stems/ha) as

$$
\begin{equation*}
B I X_{n}=B I X_{2}\left(a+b H_{t h i n}+c S P H+d H_{\text {thin }} S P H\right) \tag{41}
\end{equation*}
$$

Where the coefficients are $\mathrm{a}=0.622, \mathrm{~b}=0.0361, c=0.0000953$, and $d=-0.0000769$.

## BIX Class Distribution

The branch index for the $n$ 'th log of a model tree in the $i$ 'th diameter class is assumed normal distributed with a mean of $B I X_{i, n}$ as given in the equations above, and a standard deviation of $\sigma=$ 0.6. The effects of this distribution on the log cutting are simulated through cutting model trees in 10 different BIX classes with boundaries from $-2 \sigma$ to $2 \sigma$, in steps of $\sigma / 2$. The probability and mean value for each class is determined from the normal distribution, and the mean for each diameter class.

## Model for Maximum Branch ( $\mathbf{B r}_{\text {max }}$ )

Maximum branch diameter in the $\log$ (as against BIX) is used to determine the log grade. The size of the maximum branch for the $n$ 'th log of the $i$ 'th model tree $\left(B r_{i, n, \max }\right)$ is modelled from the BIX $\left(B I X_{n}\right)$ of the mean tree, and the difference between the $\mathrm{DBH}(\mathrm{cm})$ of the model tree $\left(D B H_{i}\right)$ and the mean tree $(\overline{D B H})$, i.e.

$$
\begin{equation*}
B r_{i, n, \text { max }}=a+b B I X_{n}+c\left(D B H_{i}-\overline{D B H}\right) \tag{42}
\end{equation*}
$$

Where the coefficients are $\mathrm{a}=0.133, \mathrm{~b}=1.111$, and $\mathrm{c}=0.05$.

## Model for Wood Density

The mean density of the wood $(\rho)\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ at harvest time $(T)$ (years) is estimated from outerwood measurements at an earlier time ( $T_{\text {meas }}$ ) (years) The function is from Kimberley and McConchie (1997) and is of the form:

$$
\begin{equation*}
\rho=a+b\left(\rho_{B H, O W}+c\left(d^{T_{\text {meas }}}-d^{T}\right)\right)+e T+f T\left(\rho_{B H, O W}+c\left(d^{T_{\text {meas }}}-d^{T}\right)\right) \tag{43}
\end{equation*}
$$

Where $\rho_{B H, O W}$ is the breast height outerwood density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ measured at time $T_{\text {meas }}$. The coefficients are $a=202.3, b=0.415, c=298, d=0.923, e=3.12$, and $f=0.0081$.

## Understorey Grazing

The pasture production within the plantation is calculated by reducing the livestock carrying capacity prior to afforestation by an amount proportional to the crown closure of the stand. This amount is then reduced again by the amount of slash produced from silvicultural operations. No grazing is assumed in the first three years.

The canopy closure (CC) (\%) model uses the model form described by Knowles et al (1997), McElwee (1999) and McElwee and Knowles (2000), but refitted (specifically for the calculator) using the original data plus an extra set from a more recent validation study by Dean (2000). The model is of the form

$$
\begin{equation*}
C C=a\left(1-\exp \left(-b B A\left(1-c\left(\frac{H_{G C}}{M T H}-0.4\right)\right)\right)\right)^{\frac{1}{d}} \tag{44}
\end{equation*}
$$

Where $B A$ is stand basal area ( $\mathrm{m}^{2} / \mathrm{ha}$ ), $H_{G C}$ is the height to the green crown ( m ), MTH is stand mean top height (m), and the coefficients are $a=85.8279, b=0.05967, c=1.5027$, and $d=$ 0.6989 .

The height of the green crown $\left(H_{G C}\right)$ is modelled from mean height $(H)(\mathrm{m})$ and crown length (CL) (m) as

$$
\begin{equation*}
H_{G C}=H-C L \tag{45}
\end{equation*}
$$

Crown length $(C L)(\mathrm{m})$ of unpruned stems is modelled using the approach of Turner (1998), i.e.

$$
\begin{equation*}
C L=\min \left(k(M T H-0.1), a+\frac{b}{S P H}\right) \tag{46}
\end{equation*}
$$

Where SPH is the stocking (stems/ha) and MTH is the mean top height ( m ), with coefficients $k=$ $0.71, a=13.48$, and $b=598.63$. The crown length of a pruned tree is then given by whichever is the shortest, the unpruned crown length or the total height minus pruned height

The reduction in pasture production due to slash has two sources, pruning and thinning, based on Paton (1986). The proportionate reduction is calculated as the proportion of ground area covered by slash at any one time. The area $\left(\mathrm{m}^{2}\right)$ of slash from pruning one tree $\left(S A_{P}\right)$ is estimated as

$$
\begin{equation*}
S A_{P}=a+b D O S \ln (L+1) \tag{47}
\end{equation*}
$$

Where DOS is diameter over stubs (cm) and $L$ is the length (m) of stem pruned (originally referred to as effective length), with coefficients $a=-0.252$ and $b=0.465$. This value is converted to ground area coverage ( $G C_{P}$ ) (\%) by

$$
\begin{equation*}
G C_{P}=O v l \frac{S P H_{P} S A_{P}}{10000} 100 \tag{48}
\end{equation*}
$$

Where $S P H_{P}$ is the number of stems pruned (stems/ha), and $O v l=0.80$ is the amount of slash not overlapping.

The ground coverage from thinning slash involves two elements: crown and stem. The contribution from one stem $\left(S A_{T S}\right)\left(\mathrm{m}^{2}\right)$ is calculated as

$$
\begin{equation*}
S A_{T S}=\frac{D_{1}+D_{2}}{2} H_{G C} \tag{49}
\end{equation*}
$$

Where $D_{1}$ is the diameter $(\mathrm{cm})$ at the base of the stem $(\mathrm{cm}), D_{2}$ is the diameter $(\mathrm{cm})$ at the height of the green crown $\left(H_{G C}\right)$. The slash contribution from one crown $\left(S A_{T C}\right)\left(\mathrm{m}^{2}\right)$ is calculated as

$$
\begin{equation*}
S A_{T C}=C W \frac{C L}{2} \tag{50}
\end{equation*}
$$

Where $C W$ is the width of the crown ( m ) and $C L$ is the crown length ( m ). The crown width is estimated using an empiric regression for all New Zealand (the mean for the values given for North Island and South Island in Paton (1986)).

$$
\begin{equation*}
C W=a+b D B H \tag{51}
\end{equation*}
$$

Where DBH is diameter at breast height (cm), and the coefficients are $a=0.567$ and $b=0.1445$.
The ground coverage by thinning slash is then multiplied by the number of stems thinned per hectare $\left(S P H_{T}\right)$, reduced to $80 \%$ due to overlap, and expressed as percentage of land area covered;

$$
\begin{equation*}
G C_{T}=\frac{\left(S A_{T S}+S A_{T C}\right) S P H_{T}}{10000} 100 \tag{52}
\end{equation*}
$$

The accumulation and decay of the slash (and its ground coverage) is modelled on a yearly basis as

$$
\begin{equation*}
G C_{S, T}=G C_{S, T-1}(1+r)^{-1}+\Delta G C_{T}+\Delta G C_{P} \tag{53}
\end{equation*}
$$

Where $T$ is time (years), $G C_{T-1}$ is ground coverage (\%) at time $T-1$ (years), $\mathrm{r}=0.5$ is the decay rate, $\Delta G C_{T}$ and $\Delta G C_{P}$ is the increment in ground coverage (\%) caused by thinning and pruning operations in the period from time $T-1$ to time $T$.

Livestock performance under trees is poorer for a given intake of dry matter than on open pasture, so carrying capacities are adjusted downward according to the equation given in Percival et. al. 1988.

Adjusted carrying capacity $(\mathrm{ACC} \%)=(0.75 * \mathrm{CC} \%)+\left(0.0025 * \mathrm{cc} \%^{2}\right)$

## Root Biomass

Root biomass is a useful surrogate for estimating the effect of a stand of trees on slope stability, and predicted from stand stocking and mean DBH using the following function provided by Watson and O'Loughlin, (1990):

$$
\begin{equation*}
\text { RootBiomass }=\operatorname{SPH}\left(\left(10 \wedge\left(2.24 \log _{10}(\mathrm{DBH})-1.16\right)\right) / 1000\right) \tag{55}
\end{equation*}
$$

Where RootBiomass is the root biomass in tons/hectare, SPH is the stocking in stems/ha, and DBH is the stand quadratic mean DBH.

## Root Decay

Root decay following logging is based on O'Loughlin and Watson (1979):

$$
\begin{equation*}
\mathrm{S}_{t 2}=\mathrm{S}_{t 1} e^{-b t 2} \tag{56}
\end{equation*}
$$

Where $\mathrm{S}_{\mathrm{t} 1}$ is root tensile strength (MPa) at clear-fellingt 2 is the time since clear-felling (months), and the coefficient $b$ is 0.056
Compared to other tree species, Pinus radiata has a relatively high root weight for a given tree DBH, but the roots have relatively low tensile strength and a rapid decay rate. If it is assumed that over any given period, loss in root strength and loss in root weight will have an equivalent effect on soil stability, the magnitude of the root effect after clearfelling can be estimated from the following equation :

$$
\begin{equation*}
\mathrm{D}_{t 2}=\mathrm{M}_{t l}\left(\mathrm{~S}_{t 2} / \mathrm{S}_{t l}\right) \tag{57}
\end{equation*}
$$

where: $\mathrm{D}=$ weight of decaying tree roots ( $\mathrm{t} / \mathrm{ha}$ ), a value equivalent to root tensile strength in terms of its effect on soil erosion, $\mathrm{M}=$ weight of all tree roots prior to the onset of root decay $(\mathrm{t} / \mathrm{ha}), \mathrm{S}=$ tensile strength of roots $(\mathrm{MPa}), t 1=$ time at beginning of period $(\mathrm{yr}), t 2=$ time at end of period ( yr ).

These equations can be used to estimate changes in total tree root weight over the course of one or more rotations, including the effects of pruning and thinning.

## Work Study Standards

## Waste Thinning

These estimates are based on regressions fitted to the work study standards as per Kaingaroa Forest Work Study Section (1975).

Waste thinning fell time ( $F T$ ) (minutes per tree)

$$
\begin{equation*}
F T=0.0587+0.0011 D B H+0.0005 D B H^{2} \tag{58}
\end{equation*}
$$

Where $D B H$ is the stand mean diameter at breast height (cm)

Waste thinning hang-up time (HAT) (minutes per tree)

$$
\begin{equation*}
H A T=0.4 \tag{59}
\end{equation*}
$$

Waste thinning clear-away time (CAT) (minutes per tree)

$$
\begin{equation*}
C A T=0.006(H-1) \tag{60}
\end{equation*}
$$

Where $H$ is hindrance (on a scale from 1 to 4 ).
Walk-and-select time ( $W S T$ ) (minutes per tree)

$$
\begin{equation*}
W S T=0.0303 H+0.0332-\frac{8.112}{S P H}+\frac{34.26 H}{S P H} \tag{61}
\end{equation*}
$$

Where $S P H$ is the stand stocking before thinning (stems per ha) and $H$ is hindrance (on a scale from 1 to 4).

Slope allowance ( $S A$ ) (multiplier)

$$
\begin{equation*}
S A=1+0.0139 S-0.001 S^{2}=0.00004 S^{3} \tag{62}
\end{equation*}
$$

Where $S$ is the slope in degrees.
Total allowance ( $T A$ ) (multiplier)

$$
\begin{align*}
& S \leq 20, H \leq 2: T A=1.643 \\
& S>20, H \leq 2: T A=1.653 \\
& S \leq 20, H>2: T A=1.653  \tag{63}\\
& S>20, H>2: T A=1.663
\end{align*}
$$

Where $S$ is slope in degrees, and $H$ is hindrance (on a scale from 1 to 4 ).
Total time per tree (TTPT)

$$
\begin{equation*}
T T P T=S A \cdot T A \cdot(F T+H A T+C A T+W S T) \tag{64}
\end{equation*}
$$

## Pruning Time

Based on regressions fitted to the work study standards as per Kaingaroa Forest Work Study Section (1975).

Prune time ( $P T$ )

$$
\begin{array}{ccc}
H_{p} \leq 2.2 & : & P T=0.383+0.0466 D B H+0.0859 H \\
2.2<H_{p} \leq 4.0 & : & P T=0.1820+0.0559 D B H-138.3 / S P H+14.50 D B H / S P H  \tag{65}\\
H_{p}>4.0 & : & P T=0.69+0.05(D B H-14)
\end{array}
$$

Where $H_{p}$ is pruning height $(\mathrm{m}), D B H$ is stand mean diameter at breast height ( cm ) , SPH is stocking (stems per ha).

Ladder handling (LH)

$$
\begin{array}{ccc}
H_{p} \leq 2.2 & : & L H=0 \\
2.2<H_{p} \leq 4.0 & : & L H=0.135+0.071 H  \tag{66}\\
H_{p}>4.0 & : & L H=0.221+0.00675 S+0.053 H
\end{array}
$$

Where $H_{p}$ is pruning height ( m ), $H$ is hindrance (on a scale from 1 to 4 ), and $S$ is slope in degrees.

Walk-and-select time (WST)

$$
\begin{array}{cl}
H_{p} \leq 2.2 & : \quad W S T=0.243-0.00017 S P H+0.1118 H \\
2.2<H_{p} \leq 4.0 & :  \tag{67}\\
H_{p}>4.0 & : \quad W S T=0.2346+\frac{35.02}{S P H}-0.1988 H+0.0692 H^{2} \\
& \quad W S T=0.1448+\frac{11,89}{S P H}+0.0292 H+8.56 \frac{H}{S P H}
\end{array}
$$

Where $H_{p}$ is pruning height $(\mathrm{m}), D B H$ is stand mean diameter at breast height ( cm ) , $S P H$ is stocking (stems per ha), and $H$ is hindrance (on a scale from 1 to 4 ).

Total allowance ( $T A$ ) (multiplier) for low pruning $\left(H_{p}<=2.2 \mathrm{~m}\right.$ ) is $T A=1.335$.
Total allowance (TA) (multiplier) for medium pruning is:

$$
\begin{align*}
& S \leq 20, H \leq 2: T A=1.353 \\
& S>20, H \leq 2: T A=1.363 \\
& S \leq 20, H>2: T A=1.363  \tag{68}\\
& S>20, H>2: T A=1.373
\end{align*}
$$

Where $S$ is slope in degrees, and $H$ is hindrance (on a scale from 1 to 4 ).
Total allowance ( $T A$ ) (multiplier) for high pruning ( $H_{p}>4.0 \mathrm{~m}$ )

$$
\begin{align*}
& S \leq 20, H \leq 2: T A=1.359 \\
& S>20, H \leq 2: T A=1.369 \\
& S \leq 20, H>2: T A=1.369  \tag{69}\\
& S>20, H>2: T A=1.379
\end{align*}
$$

Where $S$ is slope in degrees, and $H$ is hindrance (on a scale from 1 to 4 ).
Slope allowance ( $S A$ ) (multiplier) for all pruning types are:

$$
\begin{equation*}
S A=1+0.0139 S-0.001 S^{2}=0.00004 S^{3} \tag{70}
\end{equation*}
$$

Where $S$ is the slope in degrees.
Total time per tree (TTPT)

$$
\begin{equation*}
T T P T=S A \cdot T A \cdot(P T+L H+W S T) \tag{71}
\end{equation*}
$$

Where all parameters are defined in the above.

## CONCLUSIONS

From the above, it is clear that considerable functionality exists to readily enable modelling of stand level growth and quality of radiata pine at the national level in New Zealand. Where obvious gaps have presented themselves, for example to predict stand level volume at the national level, we have been able to fit a new function from existing NZ-wide data (Hansen et al. 2004). Several NZ-wide functions have been incorporated which have not previously been employed in decision support systems, or have been updated so they can be utilised at the national level. These include the three-point taper/volume function (Gordon and Budianto, 1999), canopy closure (McElwee, 1999; Dean, 2000), understorey pasture production (Knowles et al. 1997), height/age curves (Van der Colff, in prep), and 300 Index (Kimberley, in prep).

One obvious gap is that no NZ-wide diameter distribution function is available. As an interim measure, we have incorporated the Weibull model 19 Central North Island Pumice Plateau (Lawrence, 1990). It is recommended that a new national diameter distribution function be derived and incorporated in the calculator.

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## REFERENCES

DEAN, M. 2000. Validation of the function to predict canopy closure using independent regional data. Proceedings of the Forest and Farm Plantation Management Cooperative, Rotorua. June 2000 pp 9-15

GORDON, A.D. AND BUDIANTO, M. 1999. A 3-point stem volume and taper equation for radiata pine. Report 66. Forest and Farm Plantation Management Cooperative.

GOULDING, C.J. AND SHIRLEY, J.W. 1979. A method to predict the yield of log assortments for long term planning. In: Elliot, D.A. (Compiler) " Mensuration for management planning of exotic forest plantations". New Zealand Forest Service, FRI Symposium No. 20.

HANSEN, L.M., KIMBERLEY, M. AND KNOWLES, R.L., 2004. A New Zealand-wide standlevel volume function for radiata pine. New Zealand Journal of Forestry Science (submitted).

## KAINGAROA FOREST WORK STUDY SECTION 1975. Silviculture time standards Rotorua Conservancy.

KIMBERLEY, M. AND KNOWLES, R.L. 1993. A model to predict branch index in radiata pine direct sawlog regimes. Report 1. Forest and Farm Plantation Management Cooperative.

KIMBERLEY, M. AND MCCONCHIE, D. 1997. Procedures for sampling P. radiata stands for outerwood density. Forest Research. (unpublished "Value Recovery' report)

KNOWLES, R.L.; HORVATH, G.C., CARTER, M.A., AND HAWKE, M.F. 1997. Using a canopy closure model to predict understorey pasture production in Pinus radiata silvopastoral systems. In Proceedings, L'agroforesterie pour un developpement rural durable. Atelier International. CIRAD-Foret. Montpellier, France, 23-29 June, 1997 pp 409-413.

KNOWLES, R.L.; WEST, G.G. AND KOEHLER, A.R. 1987. Predicting diameter-over-stubs in pruned stands of radiata pine. FRI Bulletin 12. Ministry of Forestry, Forest Research Institute, Private Bag, Rotorua, New Zealand.

KNOWLES, R.L. AND MCELWEE, H. 1999.Validation and improvement of models to predict 'diameter-over-stubs'. Report 61. Forest and Farm Plantation Management Cooperative.

LAWRENCE, M.E. 1990. Diameter distributions for the regional growth models. FRI Project record no 2369 (unpublished).

MCELWEE, H. 1999. Validation of a model to predict canopy closure for radiata pine. Report 62. Farm and Forestry Plantation Management Cooperative.

MCELWEE, H. AND KNOWLES, R.L. 2000. Estimating canopy closure and understorey pasture production in New Zealand-grown poplar plantations. New Zealand Journal of Forestry Science 30(3):422-435.

O'LOUGHLIN C.L. AND WATSON, A.J. 1979: Root-wood strength deterioration in radiata pine after clearfelling. N.Z. Journal of Forestry Science 9: 284-293.

PARK, J.C. 1989. Pruned log index. New Zealand Journal of Forestry Science 19(1):41-53.
PARK, J.C. 2004. Interface PLI estimator - Version 2. Interface Forest \& Mill Limited, PO Box 7078, Rotorua, New Zealand.

PATON, V.J. 1988. Predicting the maximum area of pasture covered by radiata pine thinning and pruning slash. Pp 130-144 in: Agroforestry Symposium Proceedings 24-27 Nov 1986. FRI Bulletin 139. Forest Research Institute.

PERCIVAL, N.S., HAWKE, M.F., JAGUSH, K.T., KORTE, C.J., AND GILLINGHAM, A.G. 1988. Review of factors affecting animal performance in pine agroforestry. Pp 165-174 in: Agroforestry Symposium Proceedings 24-27 Nov 1986. FRI Bulletin 139. Forest Research Institute.

TURNER, J. 1998. Predicting green crown length in radiata pine. Pp 9-24 in: Proceedings of Forest and Farm Plantation Management Cooperative, Rotorua, May, 1998.

WATSON, A.J. AND C.L. O'LOUGHLIN 1990: Structural root morphology and biomass of three age classes of Pinus radiata. N.Z. Journal of Forestry Science 20(1): 97-110.

