A 3-POINT STEM VOLUME AND TAPER EQUATION FOR RADIATA PINE

A.D. Gordon and M. Budianto

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EXECUTIVE SUMMARY

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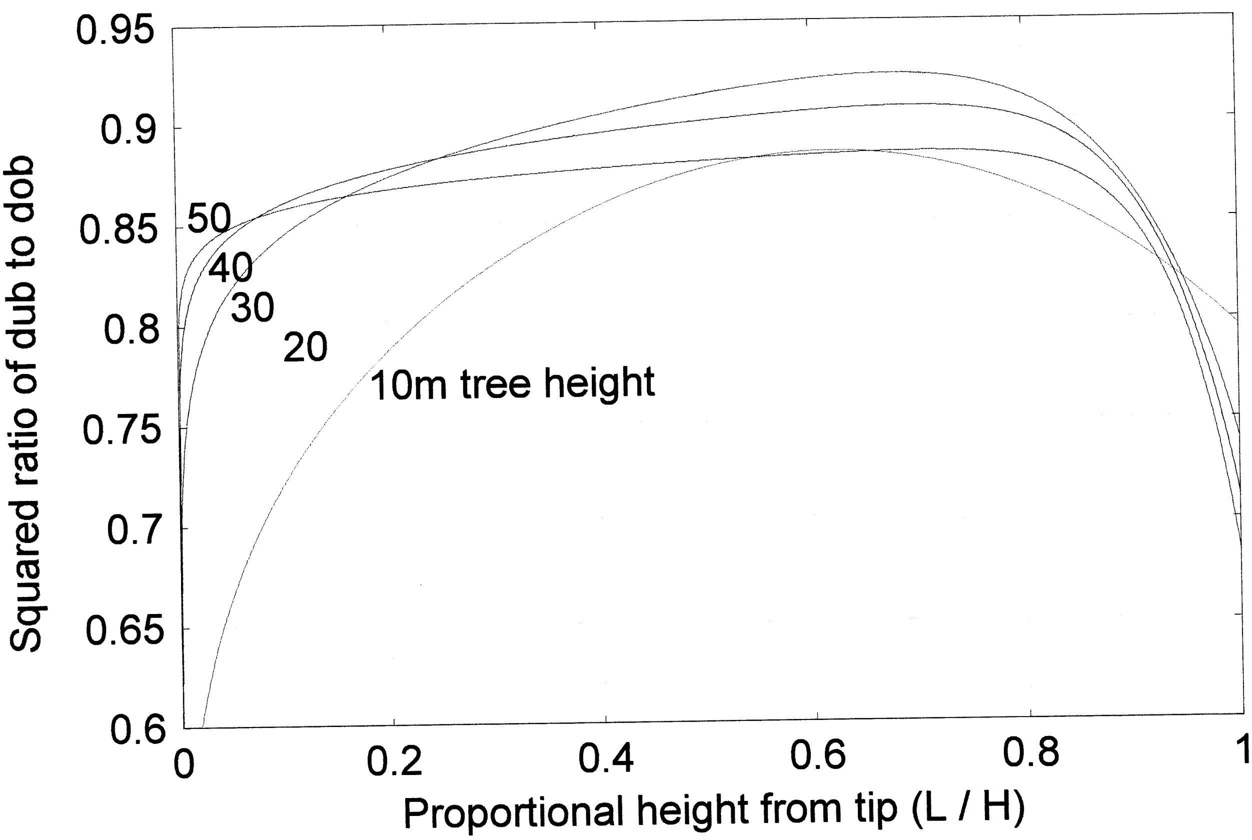
Local volume and taper equations are derived from sectionally measured trees sampled from individual stands, or from groups of stands with similar characteristics. Such equations can give precise predictions of tree and log volume, but should not be applied to trees outside their sampling frame unless carefully validated. This makes it difficult for forest managers to select an appropriate equation when predicting the volume outturn of un-sampled stands, or when examining the effect of silviculture on hypothetical stands.

This report describes the development of a general taper and volume equation for plantation-grown *Pinus radiata* in New Zealand, based on composite equations (Gordon *et al* 1995, 1999), which have been extended to incorporate an over-bark diameter at six metres (D_6). In addition, a method has been developed to determine D_6 from tree size, pruned height and the stand basal area, stocking and dominant height, so allowing the taper equation to respond to changes induced by growth and silviculture.

The three-point taper equation gives un-biased predictions of diameter across most of the locations represented in the sectional measurement data used. On a few subsets the over-bark diameter prediction residuals show some bias. The sectional measurement data used to fit the taper equation were taken from trees ranging in height from 12.4m to 49.9m. When applied below this range the taper equation predicts logical diameters and volumes for trees with heights down to about seven metres. As tree height approaches seven metres, care must be taken when applying the taper equation as it becomes sensitive to small changes in form quotient. Most precision is obtained when three measurements are made on each tree. If D_6 is predicted then the precision of the volume and diameter predictions decreases.

The bulk of tree volume is contained in the lower part of the stem. For example over 60% of volume is below the 12 metres point on an average 36m radiata pine tree. In terms of value, the gradient is more pronounced with up to half the value being in the first six metres, depending on the log products that are produced. Because the 3-point taper equation uses measurements which span the first six metres of the tree, it is likely that it will give more precise predictions of stem and log value than local two-point equations.

Figure 4. Relationship between under- to over-bark sectional area ratio and *L/H* by tree height class.

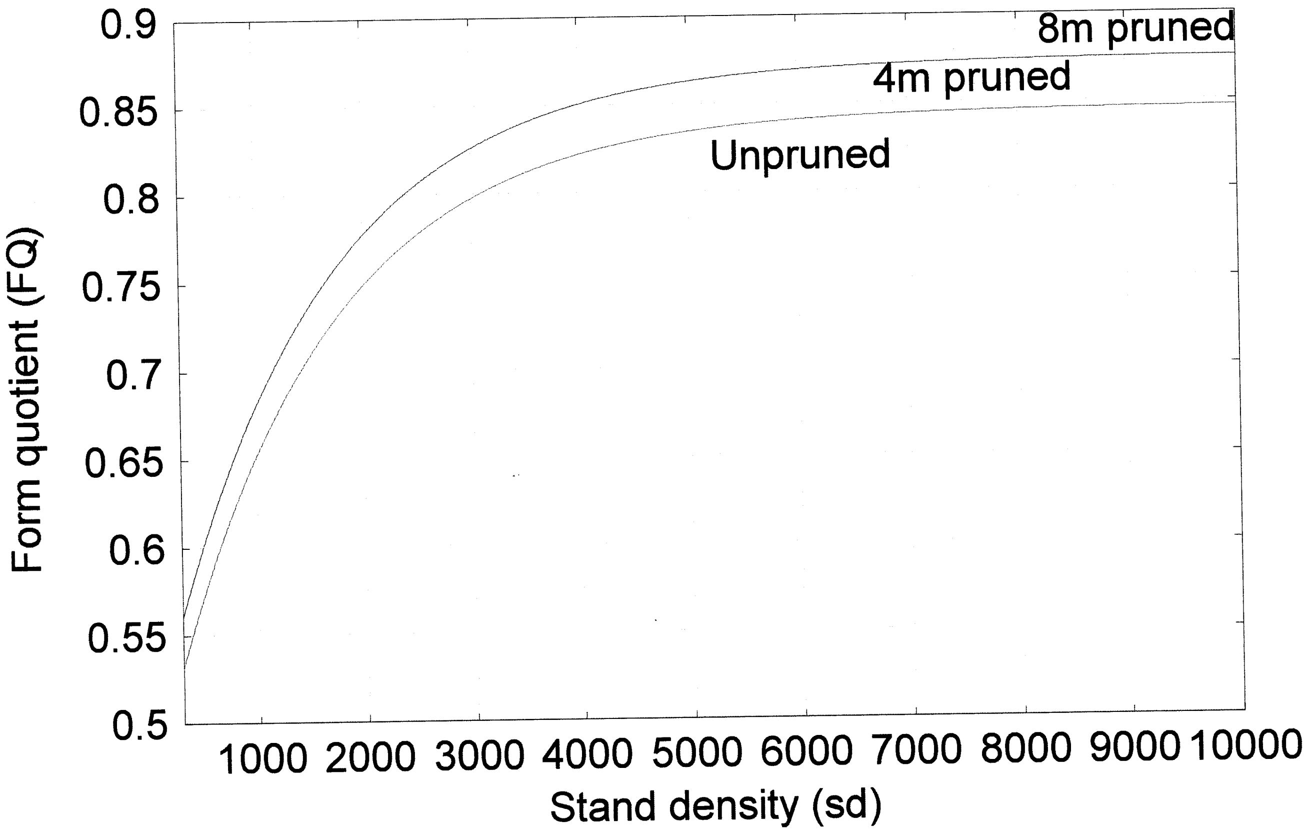


To examine the accuracy of *dub* prediction, equation 9 was fitted to one half the data selected at random and the resulting solution applied to the other half. Scatter graphs of prediction error over predicted values and proportional height (*L/H*), by tree height class, are shown in Appendix C. The means and standard deviations of the *dub* prediction errors are shown in Table 11.

Table 11. Means and standard deviations of prediction errors of *dub* from independent data.

Tree Height Class	Mean *dub* prediction error (cm)	Number of observations	Standard deviation of *dub* prediction error (cm)
$H < 20$m	0.0059051	729	0.3541580
20m $<= H <$ 35m	0.000813585	451	0.4534731
35m $<= H$	-0.1471295	205	0.7582164
Combined	-0.0184042	1385	0.4695348

Figure 3. Relationship between Form Quotient, Stand Density and Pruned height for a tree of mean basal area and mean top height.



To examine the accuracy of *FQ* prediction, equation 8 was fitted to one half of the data selected at random and the resulting solution applied to the other half. Scatter graphs of prediction error over predicted values, stand density, height dominance and pruned height are shown in Appendix B. The mean and standard deviation of the prediction errors are shown in Table 9.

Table 9. Means and standard deviations of prediction errors of *FQ* from independent data.

Number of observations	Mean *FQ* prediction error	Standard deviation of *FQ* prediction error
570	0.0022893	0.0455164

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INTRODUCTION

Tree volume and taper equations are used to determine the stem volume of trees given measurements of variables such as breast height diameter and tree height. They can also predict volume, diameters and taper of arbitrary stem sections. These equations are basic components of stand inventory, growth and yield, forest planning and product simulation systems.

In New Zealand, radiata pine stem volume has usually been predicted using functions of breast height, over-bark diameter (*dbh*) and tree height (*H*). These volume and taper equations have been derived for clearly circumscribed (local) populations, defined principally by locality, age-class and silvicultural regime. However there are many circumstances where a more general equation that could be used across different sites, stand conditions and tree sizes, would be very useful. Such a general equation may not be as precise as a local one, but could be constructed as a variable-form equation, that is, one that alters shape, and hence tree form, based on local conditions.

Previous work (Budianto and Gordon, 1998) has shown that precision can be increased substantially by incorporating a third measurement point, the over-bark stem diameter at six metres above ground (D_6), in the prediction of total stem volume under-bark. For example, the root mean square deviation of volume decreased by over 30% when D_6 was included in a commonly-used volume equation. The measurement of this upper-stem diameter on every tree could be justified in experimental work or in permanent growth plots, where the improvement in precision is worth the cost. However for routine inventory and growth and yield prediction systems, it is likely to be more cost-effective if estimates of D_6 could be made from tree dimensions, the dominance (social position) of the tree and stand parameters such as basal area and the number of stems.

Upper stem diameters have been used in taper prediction systems to calibrate two-point (*dbh* and *H*) equations (Flewelling 1993) or to introduce extra terms to an equation to make it more responsive to changes in tree shape (Czaplewski and McClure 1988, Rustagi and Loveless 1991, Kozak 1998). This paper describes the development of a general taper and volume equation for plantation-grown *Pinus radiata* in New Zealand, based on composite equations (Gordon *et al* 1995, 1999), which have been extended to use an over-bark diameter at six metres. In the new formulation, all three measurement points are incorporated into the solution and are also used to modify the predicted stem profile between the points of measurement.

DATA
Tree Sectional Measurements
Sectional measurements of 817 trees from 9 forests were used as the first data set in this study.

The locations of the forests from which trees were selected are shown in Figure 1.

Figure 1. Forest locations

Tree diameters were measured over-bark with diameter tape at 0.15, 0.7, 1.4, 3, 6,... m above ground to within 5m of the tip. All sectional measurements were subjected to a comprehensive set of computer edits to screen out possible measurement and recording errors. Plots of stem profiles were compared with sample averages to select outliers and atypical trees for more detailed checking.

Where diameters at specific heights were missing they were interpolated using a quadratic procedure involving two diameters above and two below the section of interest.

Trees were selected to cover the *dbh* range of the stands sampled in each forest. The range of the data used is summarised in Tables 1 and 2.

Table 1. Sampling details of tree sectional measurement data.

Forests (No. of stands)	No. sampled	Pruned height (m)	Age (years)	Last Record of Stocking (stems ha^{-1})
Balmoral (2)	40	5	22	325-370
Golden Downs(10)	92	4.3-6.7	25-29	217-346
Kaingaroa (19)	88	0, 4-6	26-39	190-520
Longwood (1)	96	5.5	30	370
Ngaumu (3)	30	5.5	32	150-250
Riverhead (1)	56	0	29	368
Rotoehu (3)	101	6	28-29	270-320
Te Wera (19)	103	4-6	9-29	200-700
Woodhill (3)	211	0	22-30	200-370
(61)	817	0-6.7	9-39	150-700

A total of 817 trees were sectionally measured with an average of 14 sections measured per tree.

Table 2 shows the range and distribution of breast-height diameter (*dbh*), height (*H*), total sectional stem volume under bark (*TSVub*) and tree form-factor (*FF*). Breast-height form-factor and form-quotient (*FQ*) using diameter over-bark at 6 m (D_6) were calculated as:

$$FF = \frac{40000}{\pi} \frac{TSVub}{Dbh^2 H} \quad \textbf{(1)}$$

$$FQ = \frac{D_6}{Dbh} \quad \textbf{(2)}$$

Table 2. Descriptive Statistics of sectionally measured trees by forest

Forest	Variable	n	Minimum	Mean	Maximum	Std Dev.
Balmoral	*dbh*	40	16.3000	34.1400	52.1000	8.3533
	H	40	12.4000	21.7450	27.1000	2.8351
	TSVub	40	0.0946	0.7375	1.5586	0.3852
	FF	40	0.2896	0.3404	0.3907	0.0242
	FQ	40	0.6380	0.7700	0.8510	0.0389
GoldenDowns	*dbh*	92	15.9000	41.0207	61.5000	9.4592
	H	92	19.4000	31.8652	42.5000	4.7426
	TSVub	92	0.1547	1.5852	3.5901	0.7374
	FF	92	0.3095	0.3512	0.4269	0.0235
	FQ	92	0.7430	0.8301	0.8947	0.0324
Kaingaroa	*dbh*	88	21.3000	46.7295	76.8000	10.5504
	H	88	29.4000	39.2284	49.9000	4.7750
	TSVub	88	0.4185	2.4773	5.6910	1.1602
	FF	88	0.2933	0.3491	0.4134	0.0284
	FQ	88	0.7585	0.8409	0.9050	0.0310
Longwood	*dbh*	96	30.2000	47.1594	67.3000	7.5973
	H	96	26.9000	34.3344	39.4000	2.5471
	TSVub	96	0.8813	2.2643	4.6377	0.8251
	FF	96	0.3162	0.3638	0.4287	0.0237
	FQ	96	0.7701	0.8520	0.9251	0.0329
Ngaumu	*dbh*	30	46.5000	60.7333	73.1000	7.2574
	H	30	34.5000	39.4967	42.6000	2.3898
	TSVub	30	1.9771	3.9085	5.6096	0.8989
	FF	30	0.3015	0.3385	0.3880	0.0227
	FQ	30	0.7706	0.8631	0.9180	0.0365
Riverhead	*dbh*	56	22.0000	43.6054	61.8000	9.6233
	H	56	26.5000	34.8518	40.8000	3.4572
	TSVub	56	0.3910	1.8982	3.4530	0.8017
	FF	56	0.2885	0.3463	0.4282	0.0285
	FQ	56	0.7453	0.8183	0.8974	0.0339
Rotoehu	*dbh*	101	37.1000	51.8158	76.1000	7.5005
	H	101	36.2000	41.8891	49.8000	2.4832
	TSVub	101	1.5579	2.9957	6.6317	0.9197
	FF	101	0.2716	0.3330	0.4146	0.0275
	FQ	101	0.7703	0.8543	0.9267	0.0308
Te Wera	*dbh*	103	19.9000	50.0320	70.2000	11.0057
	H	103	13.5000	35.8155	49.7000	8.5522
	TSVub	103	0.1884	2.6538	5.8045	1.3177
	FF	103	0.2818	0.3437	0.4123	0.0282
	FQ	103	0.6510	0.8398	0.9263	0.0455
Woodhill	*dbh*	211	20.1000	43.4038	66.7000	8.3385
	H	211	25.4000	31.9161	37.9000	2.1569
	TSVub	211	0.3594	1.8201	4.3847	0.7281
	FF	211	0.3104	0.3695	0.4487	0.0275
	FQ	211	0.7355	0.8342	0.9279	0.0365

The condition of the stands from which the sectionally measured trees were sampled is shown in Table 3.

Table 3. Range of Stand condition - sectionally measured trees.

Variable	Mean	Std Dev	Minimum	Maximum
Age (*t*)	27.8	3.8	9.0	39.0
Numbers of Stems per hectare (*N*)	316.8	101.7	150.0	700.0
Pruned height (*prht*)	3.8	2.7	0.0	6.7
Mean top height (*mth*)	36.3	5.6	14.0	49.0
Basal area (*G*)	54.2	24.5	8.5	183.7

Pruning trial data

A second data set was assembled to extend the range of information that could be used to relate the six metre diameter to tree dimensions and stand conditions. The data came from the following pruning and followers trials: FR166 Glengarry, FR133 Paengaroa, FR274 Kaingaroa, FR201 Ngaumu, FR243 Waiotahi, and FR247 Otago Coast. Measurements of upper stem diameters had been taken on many of the plots within these trials to record or control the height of the pruning lift. Measurements were selected that were within 0.3 metres of the six metre point, and where the tree dimensions and stand age, basal area, dominant height and numbers of stems were known.

Tree pruned height was included in the data. If the diameter measurement was taken immediately following a pruning treatment then the previous pruned height was used, as the expression of any changes in diameter growth rate at six metres must be related to the tree condition while that growth was occurring. A total of 345 observations were assembled in the second data set.

Table 4. Range of data in second data set from Pruning Trials

Variable	Mean	Std Dev.	Minimum	Maximum
dbh	24.7	5.4	11.6	46.3
D_6	16.3	4.9	4.0	34.8
H	13.6	3.1	8.8	25.5
Age	9.5	1.5	7.0	11.6
Numbers of Stems per hectare	350	125	150	640
Pruned height	4.2	2.1	0.0	7.5
Quadratic mean diameter (*mdbh*)	25.0	4.2	15.0	33.5
Mean top height (*mth*)	14.6	3.3	9.3	22.1
Basal area (*G*)	17.7	10.0	5.8	43.7

Bark thickness data

A third data set was used to develop the relationship between over- and under-bark diameters. These sectional diameter data, described by Gordon (1983b), were all measured with diameter tape over-bark and directly under-bark after peeling the bark. A total of 2928 measurements from 375 trees were included.

ANALYSIS AND RESULTS

Taper Equation

The taper equation was formulated to predict over-bark diameter (*dob* cm) as a function of tree height and the length (L m) between the tip of the stem and the point of measurement. To ensure the equation passed through *dbh* and the *dob* at six metres, two coefficients were expressed as functions of *dbh, H,* and D_6.

The form of the equation is

$$dob = \sqrt{dbh^2\left(\beta_1 z^{\gamma_1} + \beta_2 z^{\gamma_2} + \beta_3 z^{\gamma_3}\right)} \qquad \mathbf{(3)}$$

where

$$z = \frac{L}{H}, \qquad z_b = 1 - \frac{1.4}{H}, \qquad z_u = 1 - \frac{6}{H}$$

$$\beta_1 = \frac{1 - \frac{z_b^{\gamma_2}}{z_u^{\gamma_2}}\left(\frac{D_6^2}{dbh^2} - \beta_3\, z_u^{\gamma_3}\right) - \beta_3\, z_b^{\gamma_3}}{z_b^{\gamma_1} - \frac{z_b^{\gamma_2}\, z_u^{\gamma_1}}{z_u^{\gamma_2}}}$$

$$\beta_2 = \frac{\frac{D_6^2}{dbh^2} - \beta_1\, z_u^{\gamma_1} - \beta_3\, z_u^{\gamma_3}}{z_u^{\gamma_2}}$$

Equation 3 was fitted to each tree in the data set and the four estimated coefficients examined for relationships with *dbh, H,* and D_6 using rank correlations. These relationships were incorporated in equation 3 as subsidiary functions to alter the shape of the taper curve with tree size (Williams and Reich 1997). The subsidiary functions were:

$$\gamma_1 = \gamma_{10} + \gamma_{11}\frac{D_6}{H - 6} \qquad (4)$$

$$\beta_3 = \beta_{30} + \beta_{31}\frac{dbh - D_6}{6 - 1.4} \qquad (5)$$

$$\gamma_3 = \gamma_{31} H \frac{D_6}{dbh} \qquad (6)$$

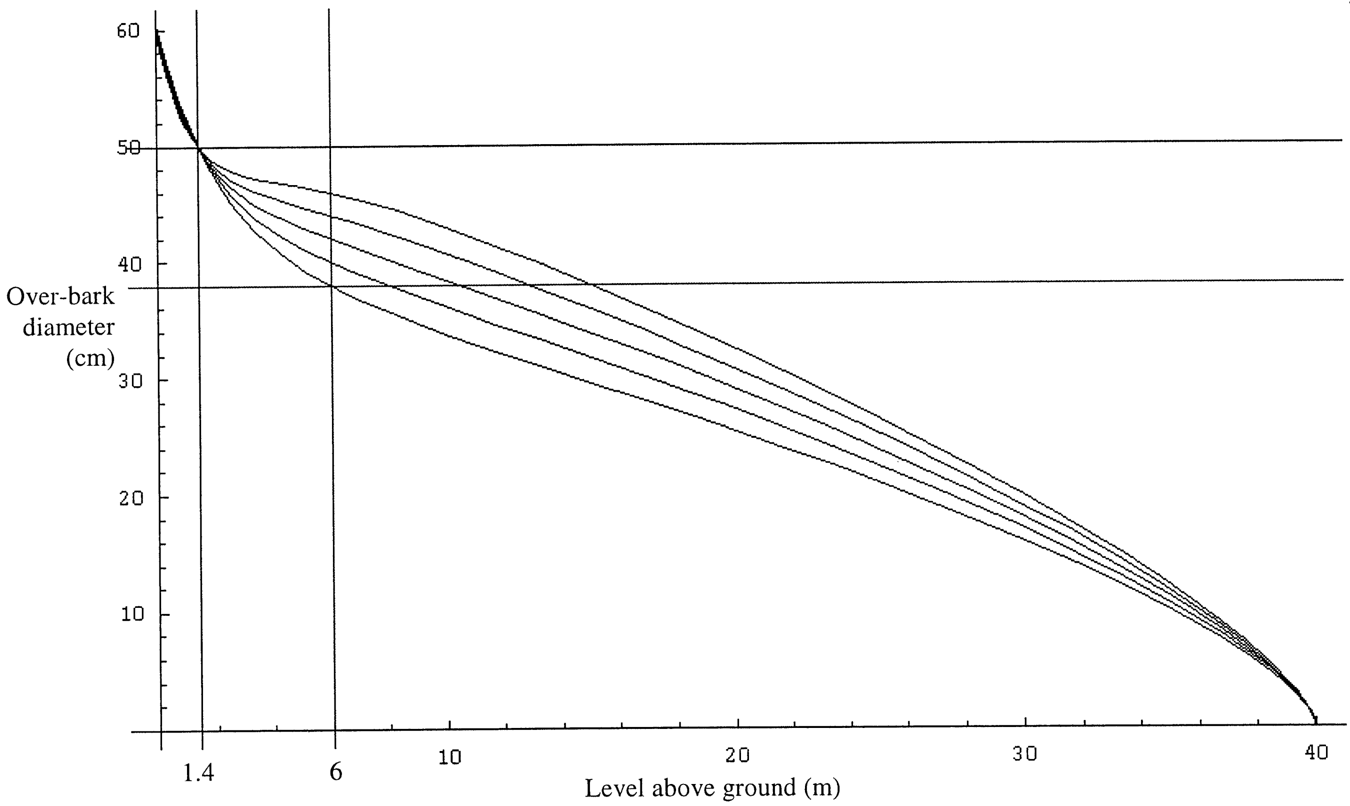
Estimates of the coefficients of equation 3 were obtained with non-linear least squares using all of the sectional measurement data set of 9580 observations. To provide more accurate estimates of the coefficient standard errors, the data set was restricted to one observation per tree selected at random, to eliminate the serial correlation between measurements (Kozak 1997). The coefficients and standard errors are shown in Table 5.

Table 5. Taper Equation Coefficient Estimates and Standard Errors

Coefficient	Estimate	Approx. Standard Error
γ_{10}	1.018	0.049
γ_{11}	0.2967	0.0344
γ_2	12.68	1.847
β_{30}	0.7768	0.1290
β_{31}	-0.1347	0.0641
γ_{31}	1.047	0.102

Taper curves derived from equation 3 for trees of the same *dbh* and *H* but with a range of form quotients from 0.76 to 0.92, are illustrated in Figure 2.

Figure 2. Over-bark taper curves for trees of dbh 50cm, H 40m and D6 of 38, 40, 42, 44 and 46 cm. Reference lines are drawn at over-bark diameters of 38 and 50 cm and at 1.4 and 6 m above ground.



To examine the prediction accuracy of the taper curve, equation 3 was fitted to one half the sectional measurement data selected at random, and the resulting solution applied to the other half. Scatter graphs by forest of prediction error over predicted values and *L/H* are shown in Appendix A. The means and standard deviations of the prediction errors are shown in Table 6.

Table 6. Means and standard deviations of prediction errors of *dob* from independent data.

Forest	Mean *dob* prediction error (cm)	Number of observations	Standard deviation of *dob* prediction error (cm)
Balmoral	-0.0950235	137	1.1405186
GldnDowns	-0.4818911	546	1.2778335
Kaingaroa	0.3119546	555	1.3740732
Longwood	0.5786554	565	1.3677371
Ngaumu	0.2459887	176	2.4539607
Riverhead	0.4005341	341	1.3422685
Rotoehu	-0.4168337	733	1.6205903
Te Wera	-0.5212553	620	1.7358806
Woodhill	0.6221574	1187	1.8032663
Combined	0.1056798	4860	1.6739940

Diameter at Six metres

Tree diameter at six metres is related to tree size and stand conditions. This relationship can be quantified so that predictions of D_6 can be used in equation 3 in situations where D_6 is not measured. In most assessments of radiata pine plantations it can be assumed that tree *dbh* and *H* will be measured, as well as details of the stand to which the tree belongs, that is stand age, basal area, mean top height and numbers of stems. In addition the pruned height of the tree, or average pruned height of each pruned element, is measurable or can be gleaned from stand records.

Tree form factor has generally been found to increase with tree size, stand density, inter-tree competition and the height of the green crown (Larsen 1963). Six metre form quotient (*FQ*) was assumed to be a measurable expression of form factor, from which estimates of D_6 could be derived. A function to predict *FQ* was developed which comprised three additive terms representing stand conditions, tree dominance and pruning. The assumption that the terms could be used in an additive manner was checked by examining the residuals against the interactions between the terms.

A combined variable was constructed which incorporated the stand conditions to give a measure of stand density (*sd*).

$$sd = \frac{mdbh^2}{\sqrt{rspace}} \tag{7}$$

where

mdbh is the *dbh* of the tree of stand mean basal area

$$rspace \text{ (relative spacing)} = \frac{100}{\sqrt{N}\, mth}$$

Relative spacing is the ratio of the average square spacing between trees (metres) to the stand mean top height (metres), and is a measure of the growing space available within the stand. Relative spacing decreases with higher stocking and higher dominant height.

Scatter graphs of the sectional measurement data (tree level) and the pruning trial data showed that tree form quotient was related to stand density in a positive but non-linear fashion.

The data were examined on a stand by stand basis for relationships between measures of dominance and the tree form quotient (*FQ*) - Table 7. All the significant correlations were positive, indicating *FQ* tends to increase with increasing dominance. This contrasts with the general, but weaker, trend for form factor to decrease with increasing tree size (Budianto and Gordon 1998, Larson 1963).

Table 7. Significant (α=0.05) correlations between form quotient and dominance measures for stands with over 20 trees sampled. Non-significant results omitted.

Number of trees sampled within stand	Rank correlation with *dbh / mdbh* (probability)	Rank correlation with *H / mth* (probability)	Rank correlation with $dbh^2H / (mdbh^2\ mth)$ (probability)
21		0.56106(0.0081)	
33		0.38034(0.0290)	
24	0.65246(0.0005)	0.78094(0.0001)	0.69652(0.0002)
47			
96	0.39537(0.0001)	0.52635(0.0001)	0.43183(0.0001)
22	0.47557(0.0253)		0.50988(0.0153)
57			
22			
56			
89		0.32094(0.0022)	
75			

Height dominance (*H/mth*) was most strongly related and was used as the dominance term in the prediction of tree form quotient. The pruned height of the tree (or stand average pruned lift) had a small but significant positive contribution to the prediction of form quotient. The resulting equation was:

$$FQ = \beta_0 + \beta_1 e^{-\beta_2 sd} + \beta_3 e^{-\left(\frac{H}{mth}\right)^2} + \beta_4 prht \qquad (8)$$

Table 8. Form quotient prediction Equation - Coefficient Estimates and Standard Errors

Coefficient	Estimate	Approx. Standard Error
β_0	0.945	0.009
β_1	-0.387	0.010
β_2	0.000686	0.000035
β_3	-0.267	0.022
β_4	0.00357	0.00055

This fit has an approximate R^2 of 79.4%. Analysis of residual graphs (Appendix B) showed no trends except for a slight over-estimation of *FQ* at high stockings. The stand density term increases with *mdbh* and *mth* and will also increase with numbers of stems if *mdbh* is held constant. Figure 3 shows the change in *FQ* with stand density for a tree of *mdbh* and *mth* at three pruned heights.

Bark Thickness

The second component of a composite taper equation is a relationship between over- and under-bark sectional area. This enables the equation to calculate under-bark diameters and volumes.

The variation in the ratio of under- to over-bark diameter in radiata pine is related mainly to the position on the stem, rather than to the locality or to individual tree characteristics (Gordon 1983b). The ratio of under- to over-bark sectional area averages about 80% at the base of the tree (Gordon 1983b, figure 3) but rises rapidly to 90% at one fifth of tree height. The ratio drops again above half height at a rate which depends on the tree size. Equation 9 was found to describe the variation in the sectional area ratio with proportional height, as well as allowing for the changes with tree height.

$$\left(\frac{dub}{dob}\right)^2 = \alpha_0 + \alpha_{01}H + \alpha_{10}\, z^{\frac{e^{-\alpha_{12}H}}{2}} + \alpha_2\, z^{\alpha_{31}H} \qquad (9)$$

Coefficient estimates are shown in Table 10. The coefficient standard errors were estimated from a data set restricted to one observation per tree selected at random to eliminate serial correlation.

Table 10. Bark thickness prediction Equation - Coefficient Estimates and Standard Errors

Coefficient	Estimate	Approx. Standard Error
α_0	0.4242	0.0509
α_{01}	-0.002822	0.000252
α_{10}	0.6067	0.0546
α_{12}	0.06129	0.00624
α_2	-0.2070	0.0082
α_{31}	0.3208	0.0254

Figure 4 shows the change in the under- to over-bark sectional area ratio with *L/H* and tree height as predicted by equation 9.

DISCUSSION

Examination of the *dob* prediction errors (Appendix A and Table 6) indicates that there is bias in the upper half of the stem in trees from some subsets, notably from Woodhill forest. The stands from which these trees were sampled may have been affected by wind exposure, which tends to reduce height growth and hence result in trees with larger diameters than normal in the upper stem. The diameters are under-predicted by equation 3 in this region of the stem. The data from Riverhead also indicates some bias in the upper stem.

The form factor of the sectionally measured trees showed weak but significant negative rank correlations with within-stand tree dominance. Three measures of dominance were used and their correlations with form factor indicate that stem volume relative to *dbh* and *H* decreases with dominance (Table 12).

Table 12. Rank Correlations between form factor and measures of tree dominance

Dominance Measure	Correlation
$\frac{dbh}{mdbh}$	-0.28
$\frac{H}{mth}$	-0.15
$\frac{dbh^2 H}{mdbh^2 mth}$	-0.28

It was expected that six-metre form quotient would also decrease with dominance, as dominant trees tend to show poorer form than trees of lower social order. This is often seen in the proportionally deeper crowns of dominant stems (Larson 1963). The form quotient does not appear to reflect this change in overall stem shape, as it was positively related to dominance (Table 7), suggesting the changes in stem shape related to dominance are occurring above six metres. However the changes in stem volume that are related to dominance are minor compared with those that are reflected in the form quotient. This can be illustrated by calculating the proportion of variation in volume and form factor accounted for, in a step-wise fashion, by linear models of *dbh, H,* measured *FQ* and *dbh/mdbh* (Table 13 and Table 14).

Table 13. Proportion of variation in ln(*TSVub*) - sectional measurement data set (817 observations)

Model	Proportion of Variation Accounted for
ln(*dbh*)	93.1%
ln(*dbh*), ln(*H*)	98.4%
ln(*dbh*), ln(*H*), ln(*dbh/mdbh*)	98.4%
ln(*dbh*), ln(*H*), ln(*FQ*)	99.3%
ln(*dbh*), ln(*H*), ln(*FQ*), ln(*dbh/mdbh*)	99.3%

Table 14. Proportion of variation in form factor - sectional measurement data set (817 observations)

Model	Proportion of Variation Accounted for
dbh	16.0%
dbh, H	16.4%
dbh, H, dbh/mdbh	16.6%
dbh, H, FQ	62.1%
dbh, H, FQ, dbh/mdbh	62.2%

It is unlikely that a 3-point taper equation based around a six metre diameter will be able to account for small changes in the upper stem shape but it will clearly give better estimates of volume than an equation bases solely on *dbh* and *H*.

An indication of the drop in precision expected by using predictions of D_6, rather than measuring at three points on every tree, is shown in Table 15. This table was derived in the same way as Table 6, by using an equation fitted to independent data, but shows results using both measured and predicted D_6 values. The standard deviation of the error in predicting *dob* increases from 1.67cm to 1.96cm when D_6 is predicted, a decrease in precision of 17%.

Table 15. Means and standard deviations of prediction errors of *dob* from independent data, comparing measured with predicted six metre *dob*.

Source of D_6 value	Mean dob prediction error (cm)	Number of observations	Standard deviation of *dob* prediction error (cm)
Measured	0.1056798	4860	1.6739940
Predicted by equation 8	0.0600411	4860	1.9584423

The under-bark volume of any stem section can be calculated by combining equations 3 and 9 and integrating (Gordon *et al* 1995). Appendix D gives a function (as Pascal code) for calculating the volume between the tip of a tree and a specified level above ground.

This taper equation is limited in use to trees greater than six metres in height. When implemented it was found that the taper curve for trees less than approximately seven metres became sensitive to the size of D_6 and could fail if extreme values were used. Radiata pine in New Zealand will usually reach seven metres in height within five or six years of establishment so it is only a minor limitation to be unable to calculate the volume of trees less than this height.

The data used in this study were collated in an opportunistic fashion rather than as a designed experimental set. Any further attempts to quantify the relationship between form quotient and tree and stand conditions would benefit from a designed, experimental data set in order to properly test hypotheses. One refinement could be the inclusion of the time (or height growth) since pruning into the prediction of D_6. This would avoid step change in the taper curve that will currently occur when D_6 is predicted and the tree is further pruned. Obviously the changes in tree shape due to pruning as expressed in the form quotient will appear over a period of time after the event, but the size of this lag is unknown at present.

When implementing the taper equation, several checks have been found necessary to ensure that the taper curve resulting from any (dbh, D_6, H) triplet predicts monotonically decreasing *dob* from ground level to the tip of the tree. Simply requiring $0 < D_6 < dbh$ is insufficient as the taper curve will accommodate extreme form quotients by assuming a non-monotonic shape. However when restricted to the range of the data to which equation 3 was fitted all solutions appear logical.

SUMMARY

The three-point taper equation produces un-biased predictions of diameter across most of the locations represented in the sectional measurement data used. On a few subsets the dob prediction residuals show some bias. Taper equations based on only two measurement points may give better estimates if fitted and applied to "local" populations such as represented by these subsets. However the three-point equation will provide useful predictions throughout New Zealand, especially if no local equation is available. It will also tend to give more realistic predictions than two-point equations when used in growth and yield simulation systems across a range of ages and stand conditions, as the predicted stem shape responds to changes in stand density and pruned height

The sectional measurement data used to fit the taper equation was taken from trees ranging in height from 12.4m to 49.9m. When applied below this range the taper equation predicts logical diameters and volumes for trees with heights down to about seven metres. Care must be taken when applying the taper equation as it becomes sensitive to small changes in *FQ* as tree height approaches seven metres.

Most precision is obtained when three measurements are made on a tree. If D_6 is predicted via *FQ* (equation 8) then the precision of the volume and diameter predictions decreases.

The bulk of tree volume is contained in the lower part of the stem. For example over 60% of volume is below the 12 metres point on an average 36m radiata pine tree. In terms of value the gradient is more pronounced, with up to half the value being in the first six metres, depending on the log products that are produced. Because the 3-point taper equation uses measurements which span the first six metres of the tree, it is likely that it will give more precise predictions of stem and log value than local two-point equations.

ACKNOWLEDGMENTS

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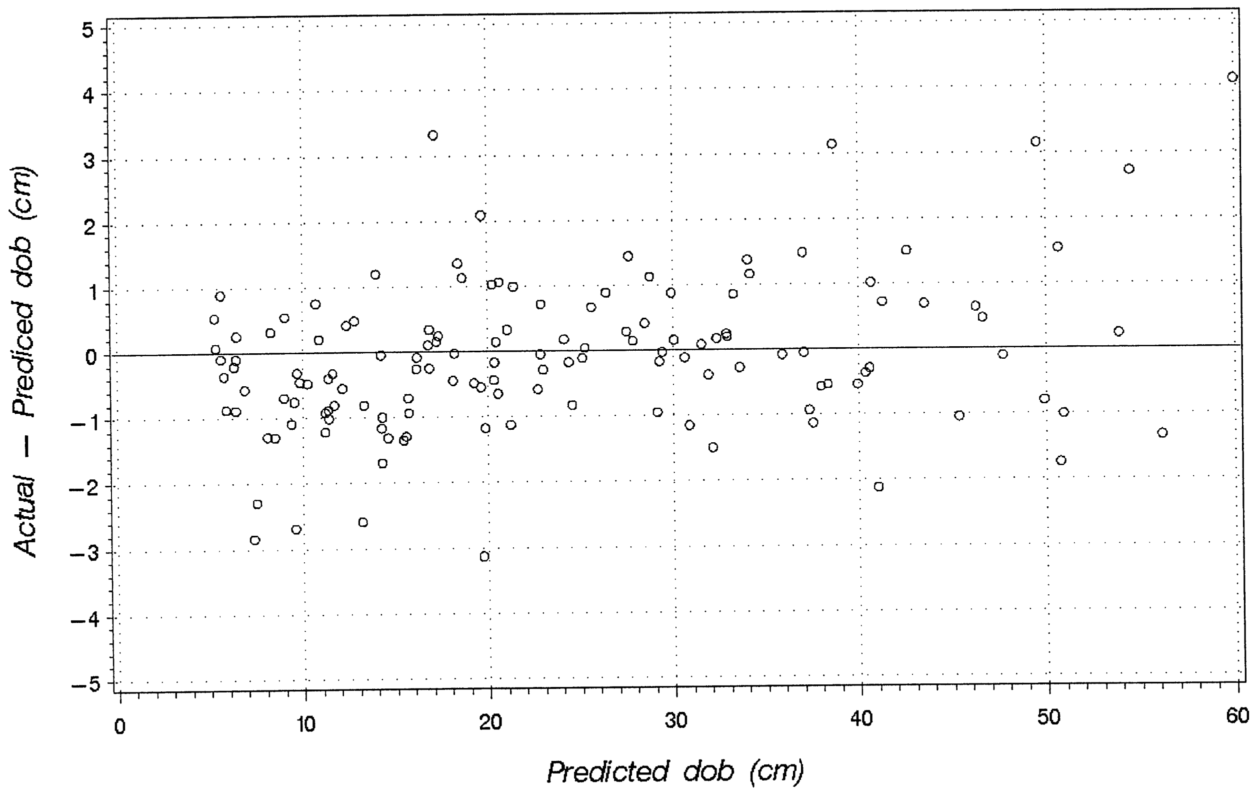
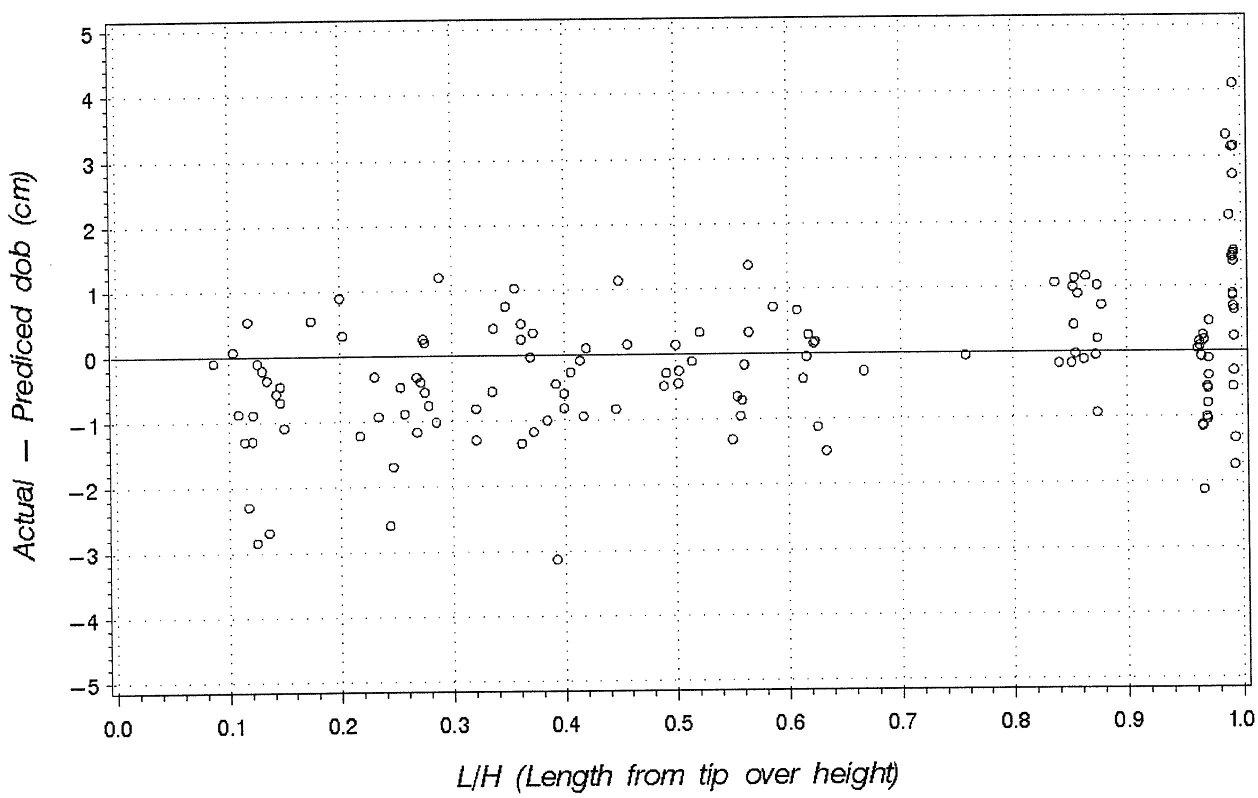
APPENDIX A

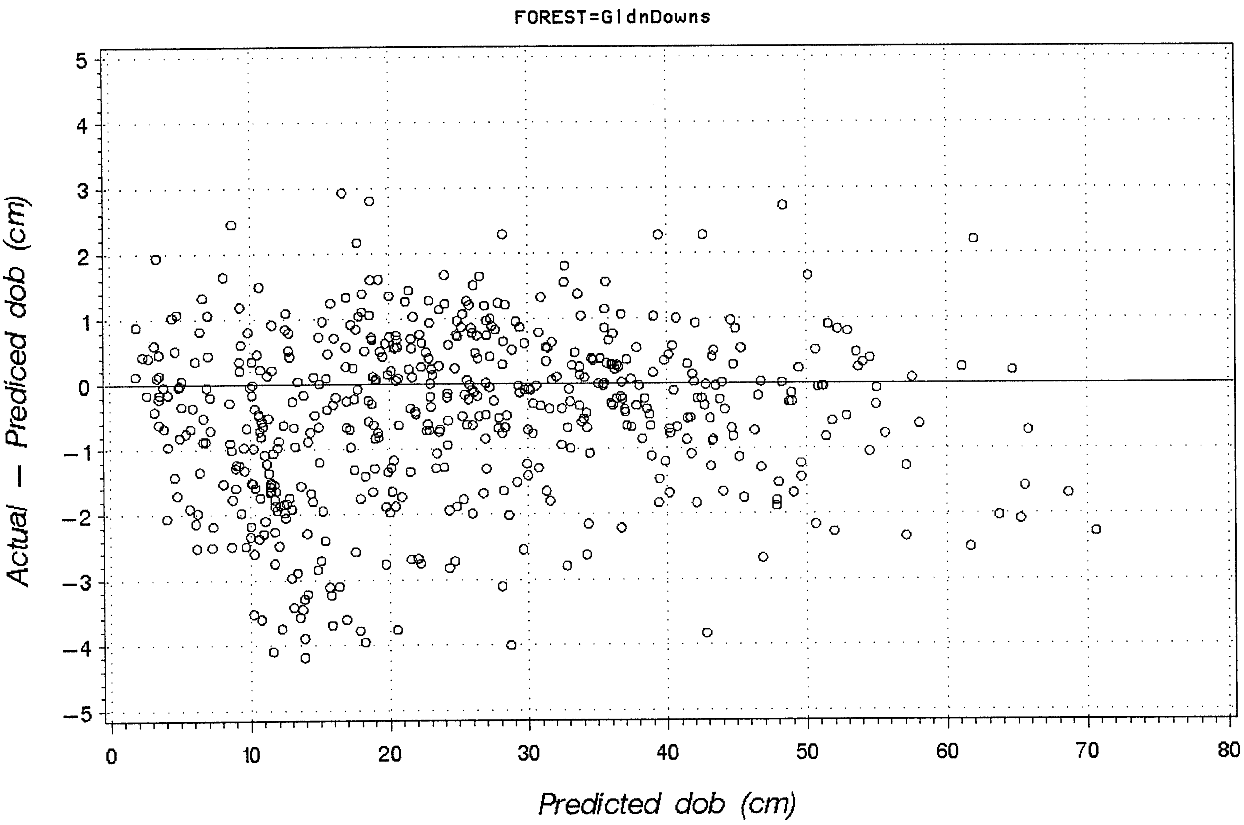
Dob Prediction errors with predicted values and proportional height by forest using independent fitting and testing data.

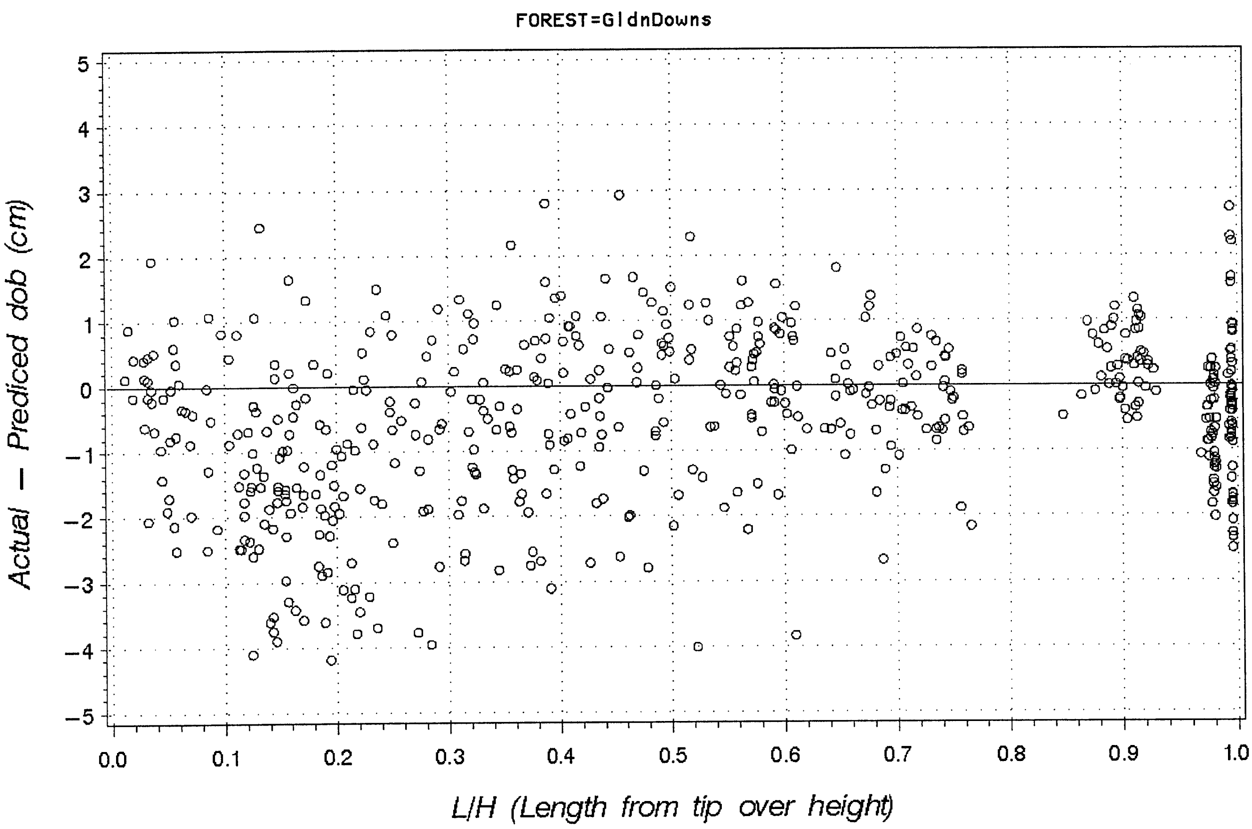
FOREST=Balmoral

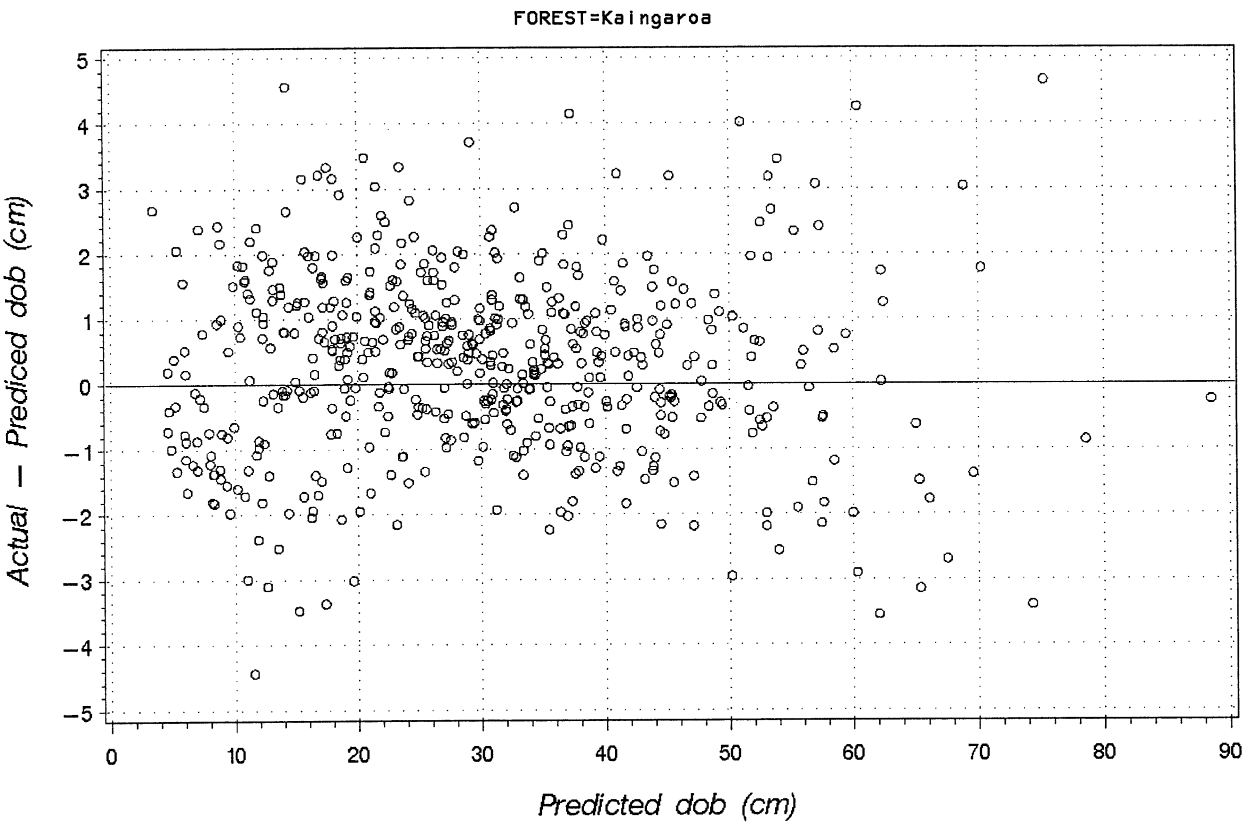


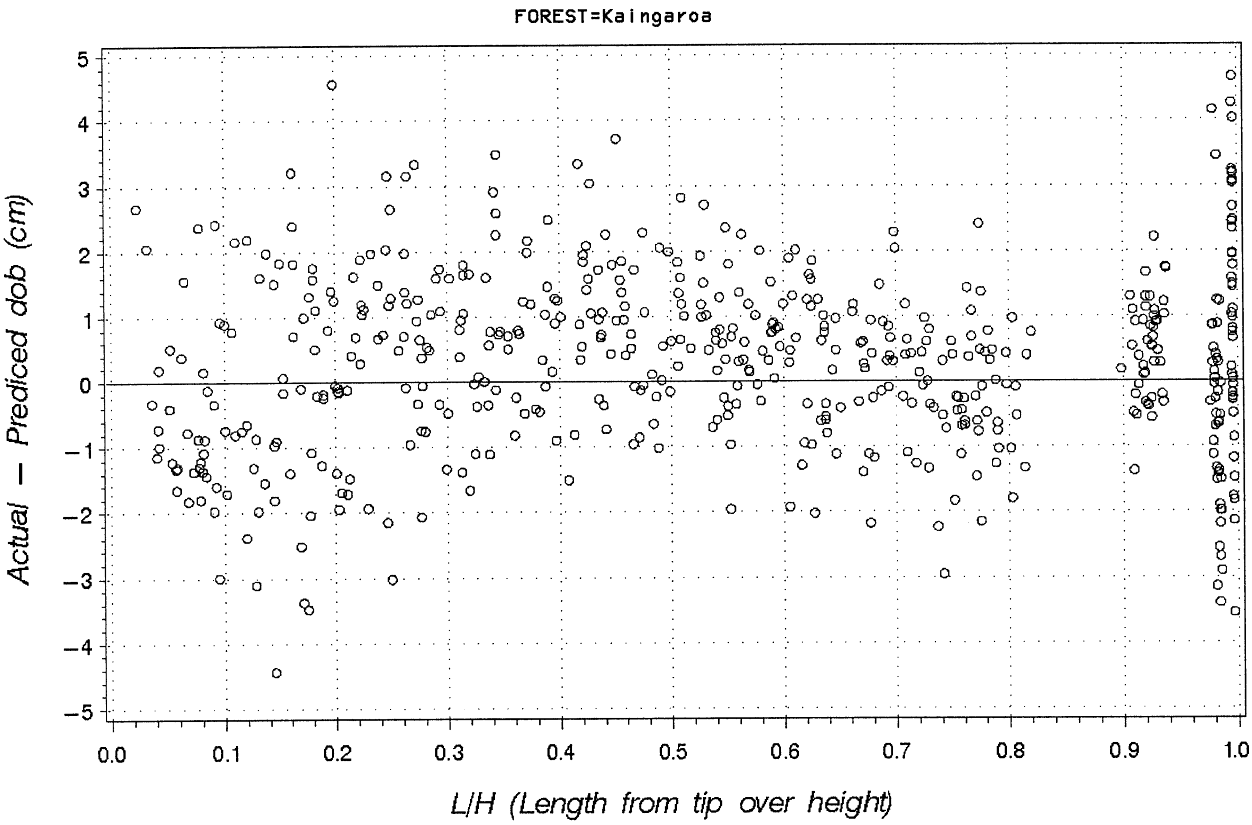
FOREST=Balmoral



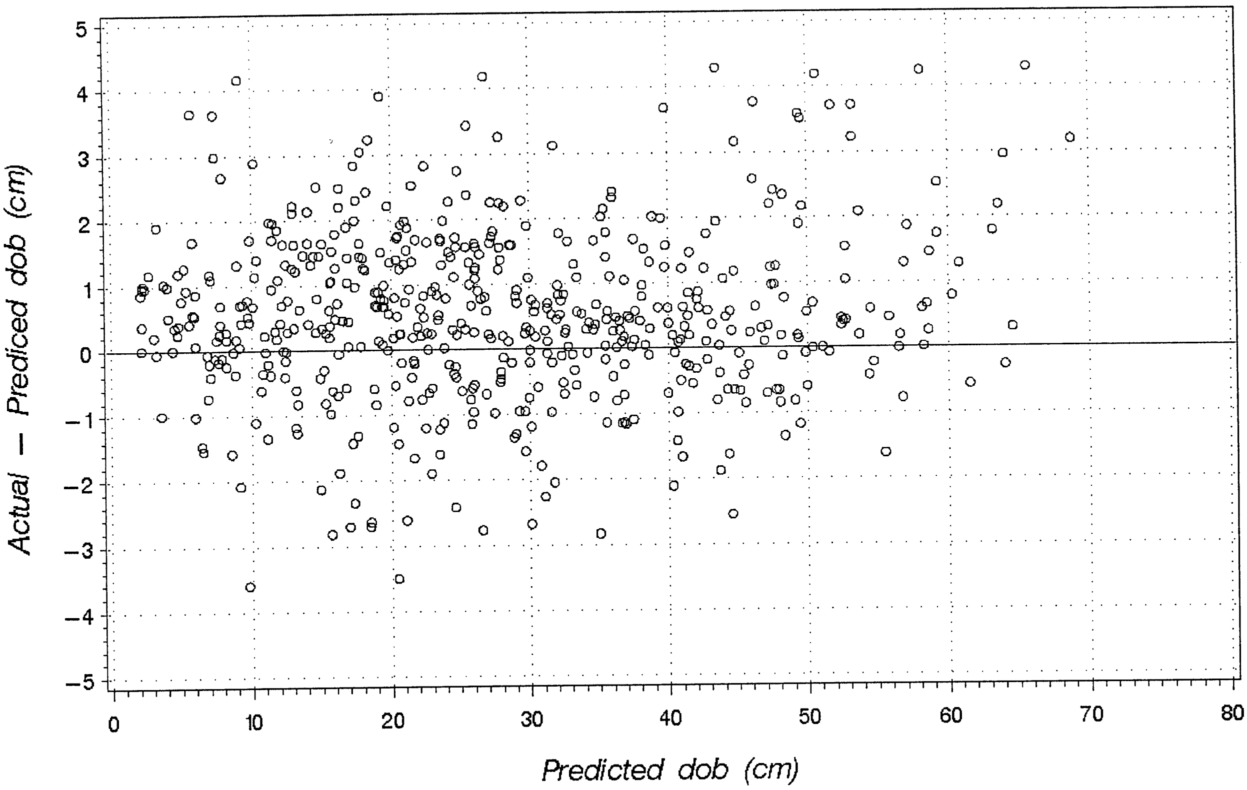

FOREST=GldnDowns
Actual − Prediced dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
80
Predicted dob (cm)


FOREST=GldnDowns
Actual − Prediced dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
L/H (Length from tip over height)

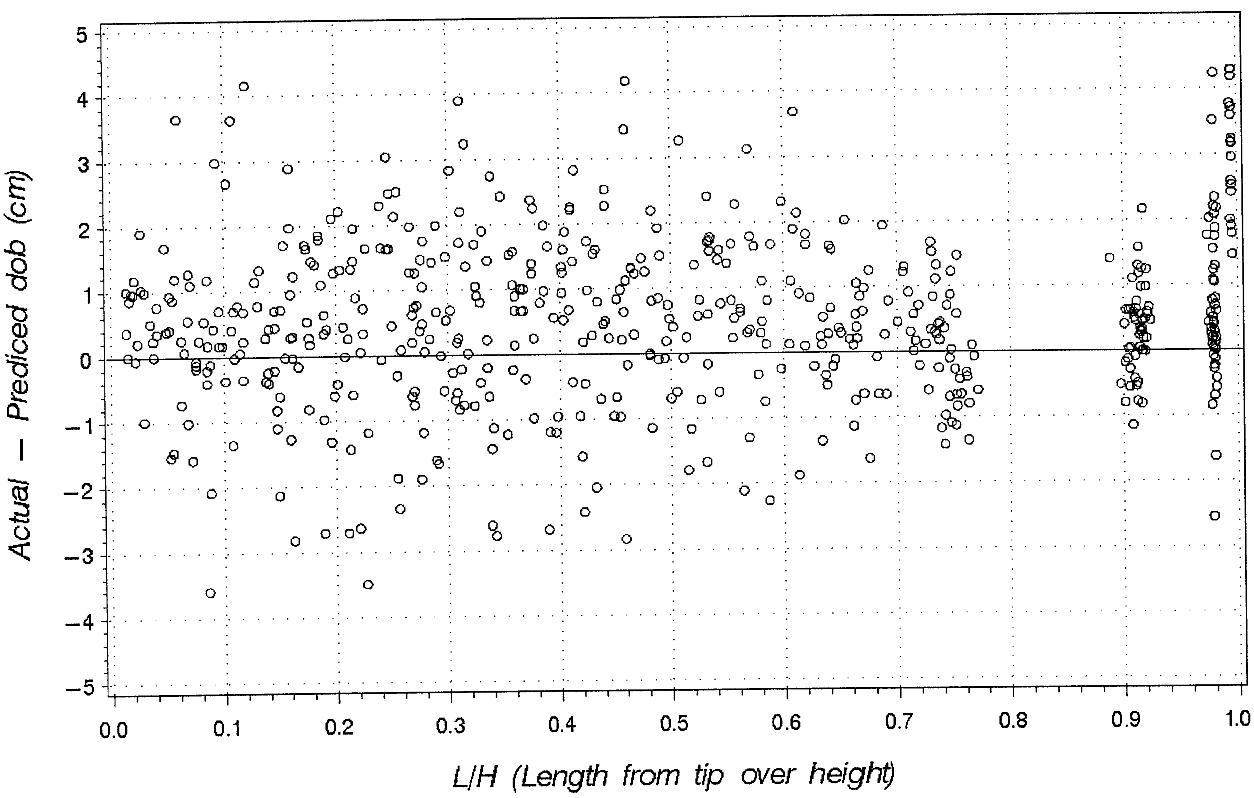

FOREST=Kaingaroa
Actual – Prediced dob (cm)
Predicted dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
80
90

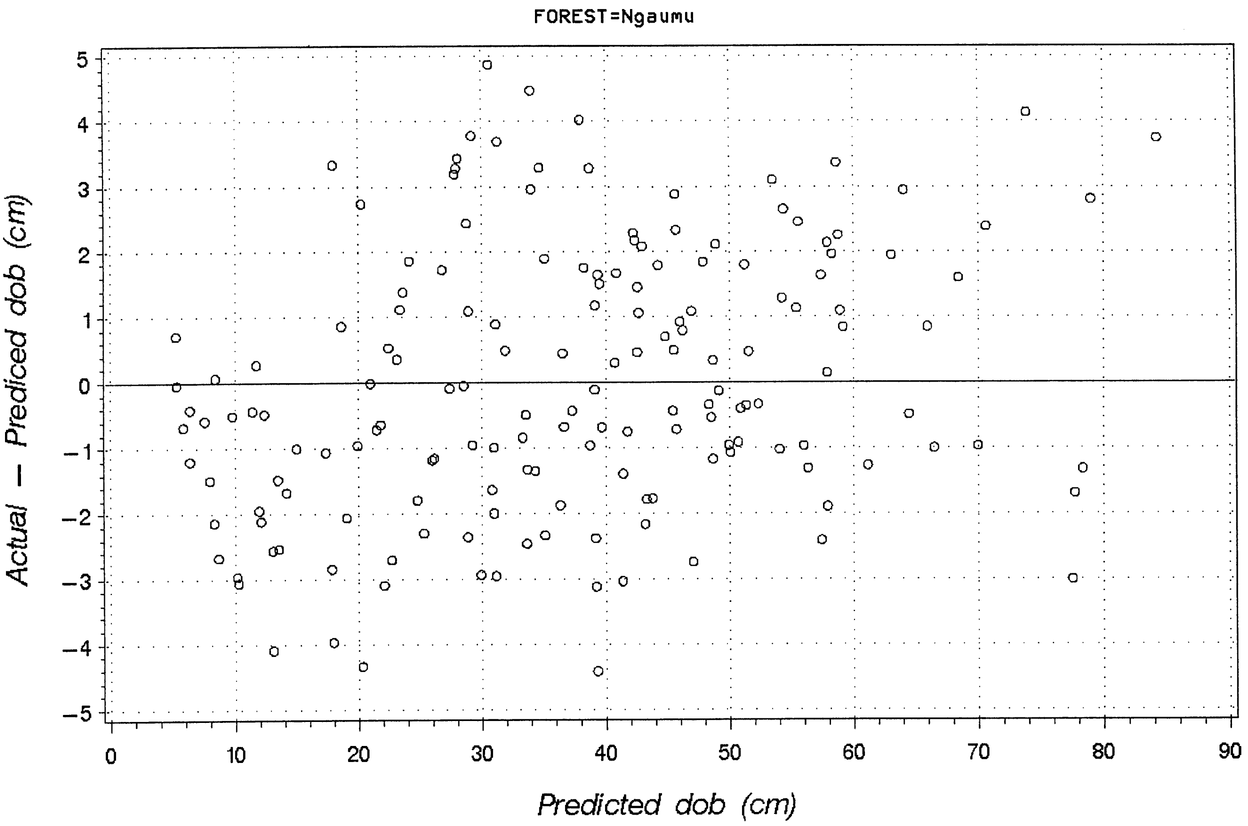

FOREST=Kaingaroa
Actual – Prediced dob (cm)
L/H (Length from tip over height)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

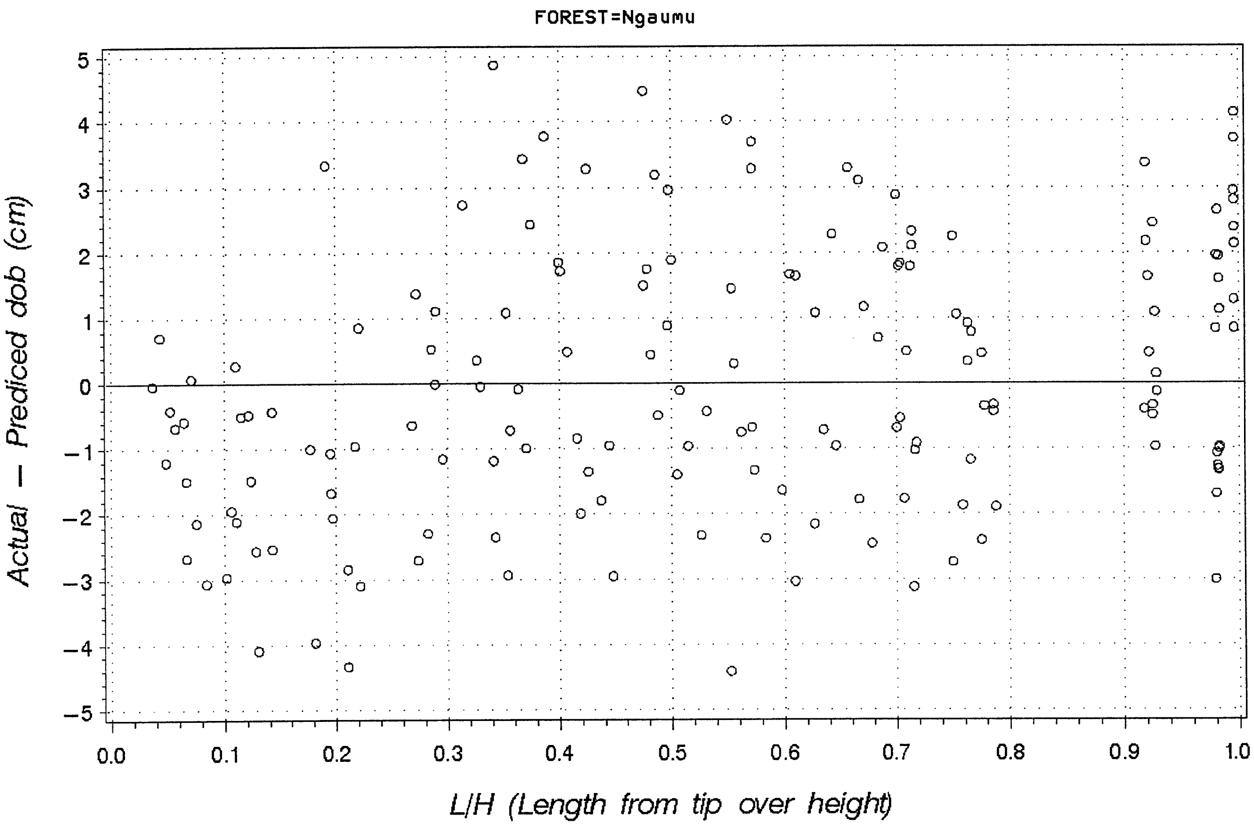
FOREST=Longwood

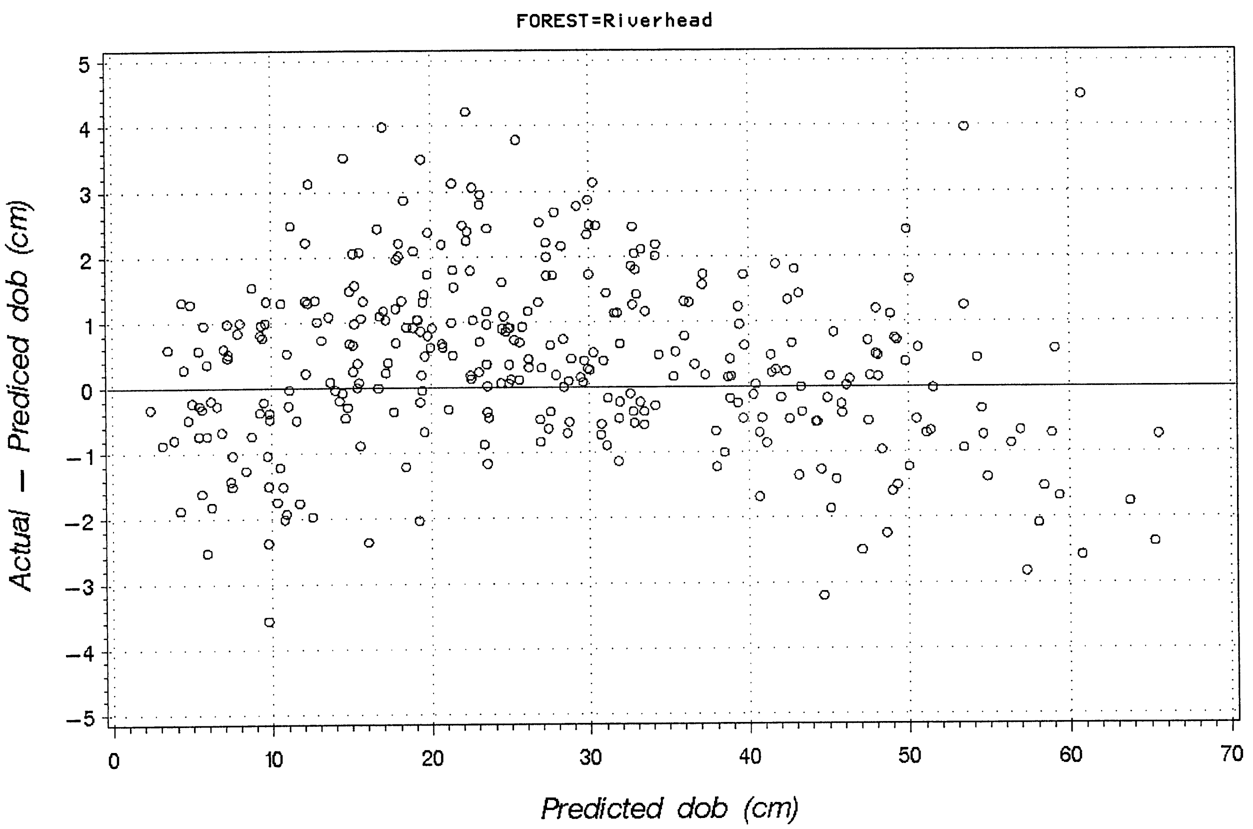


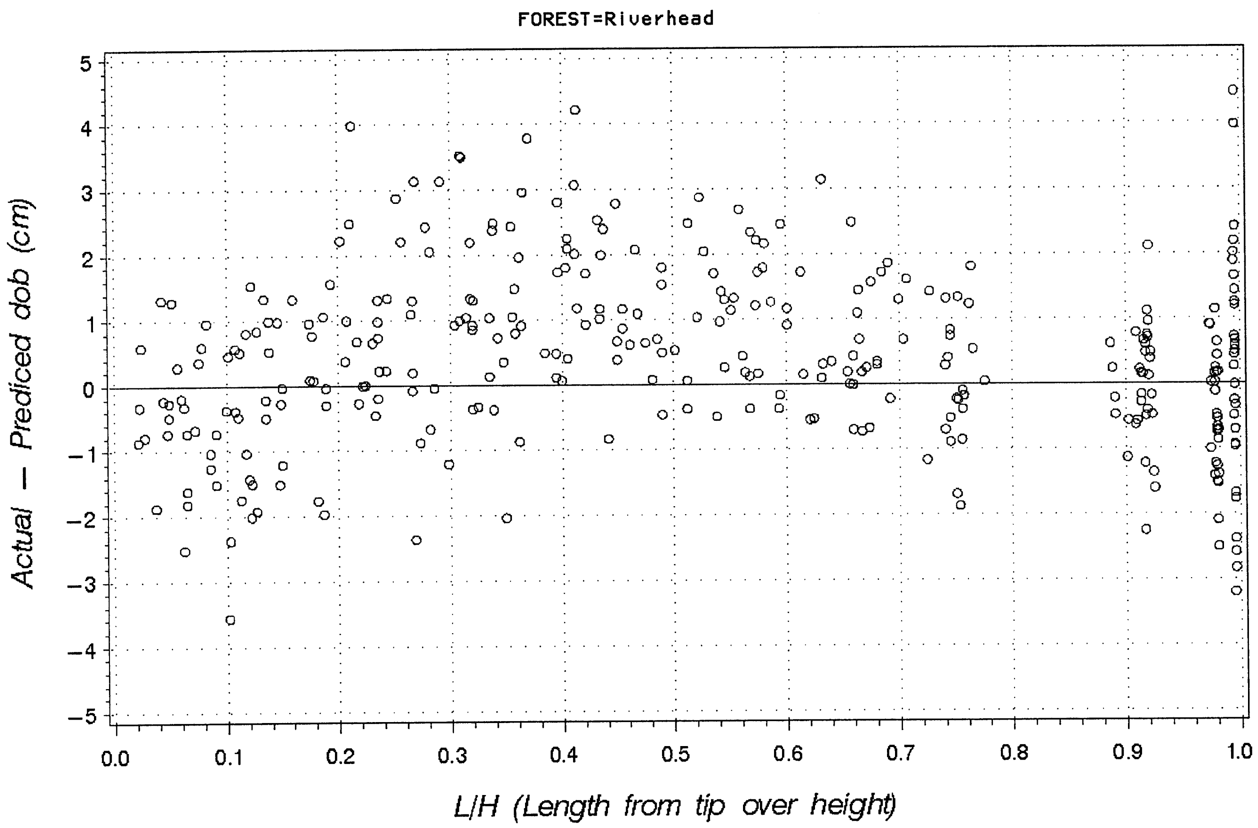
FOREST=Longwood

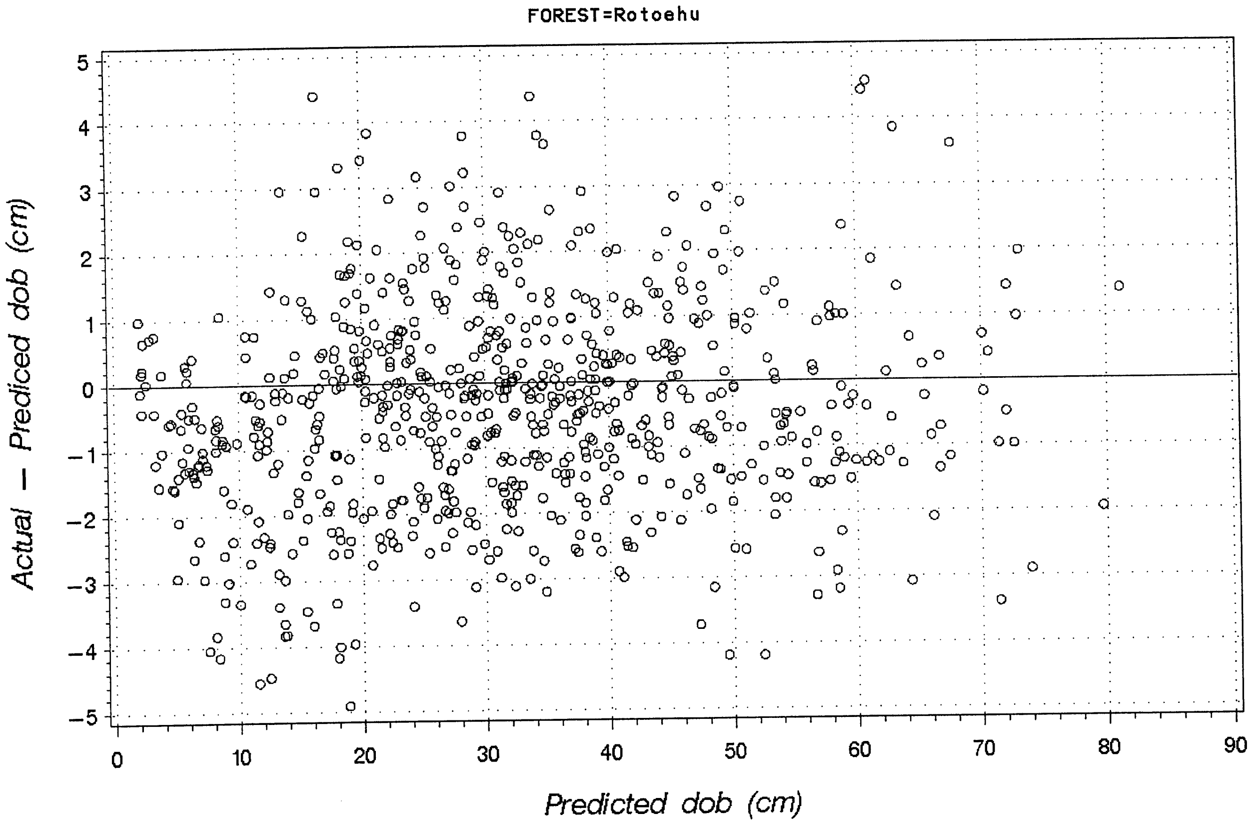


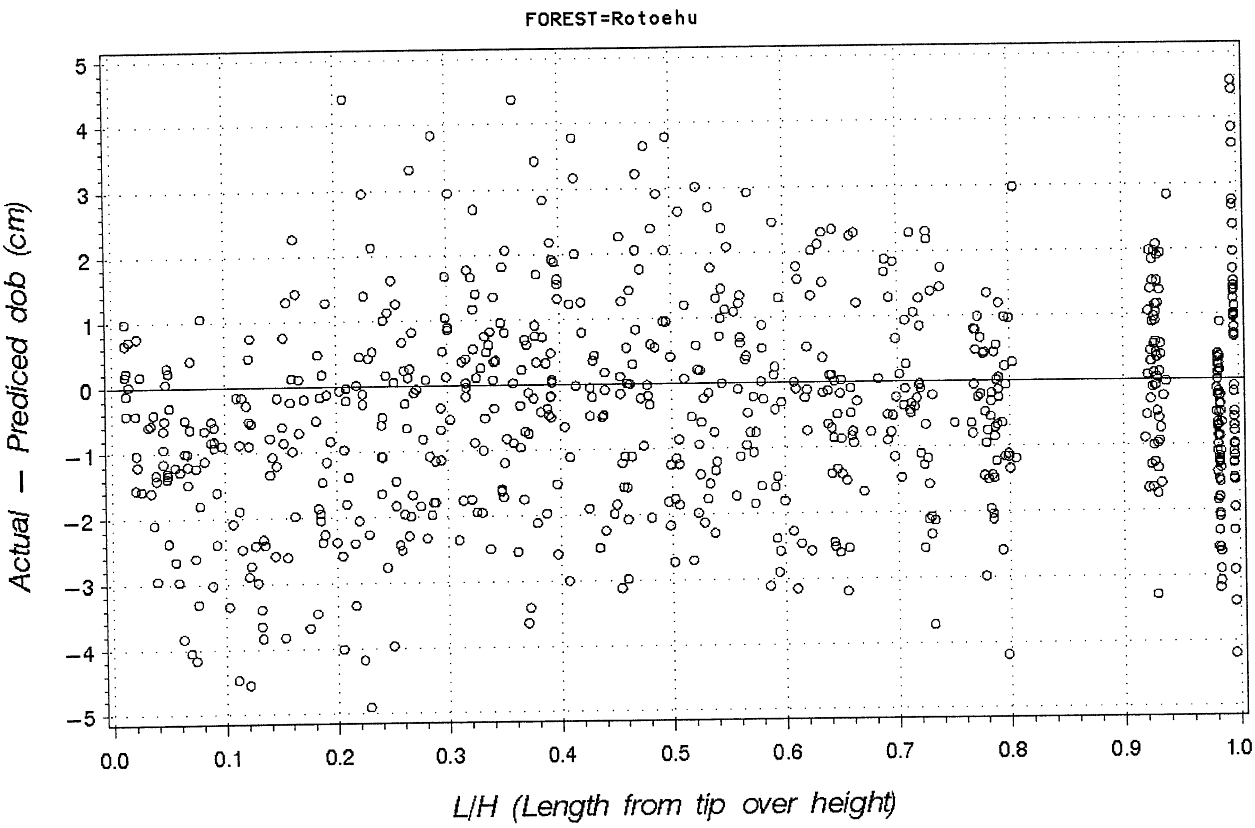

FOREST=Ngaumu
Actual – Prediced dob (cm)
Predicted dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
80
90

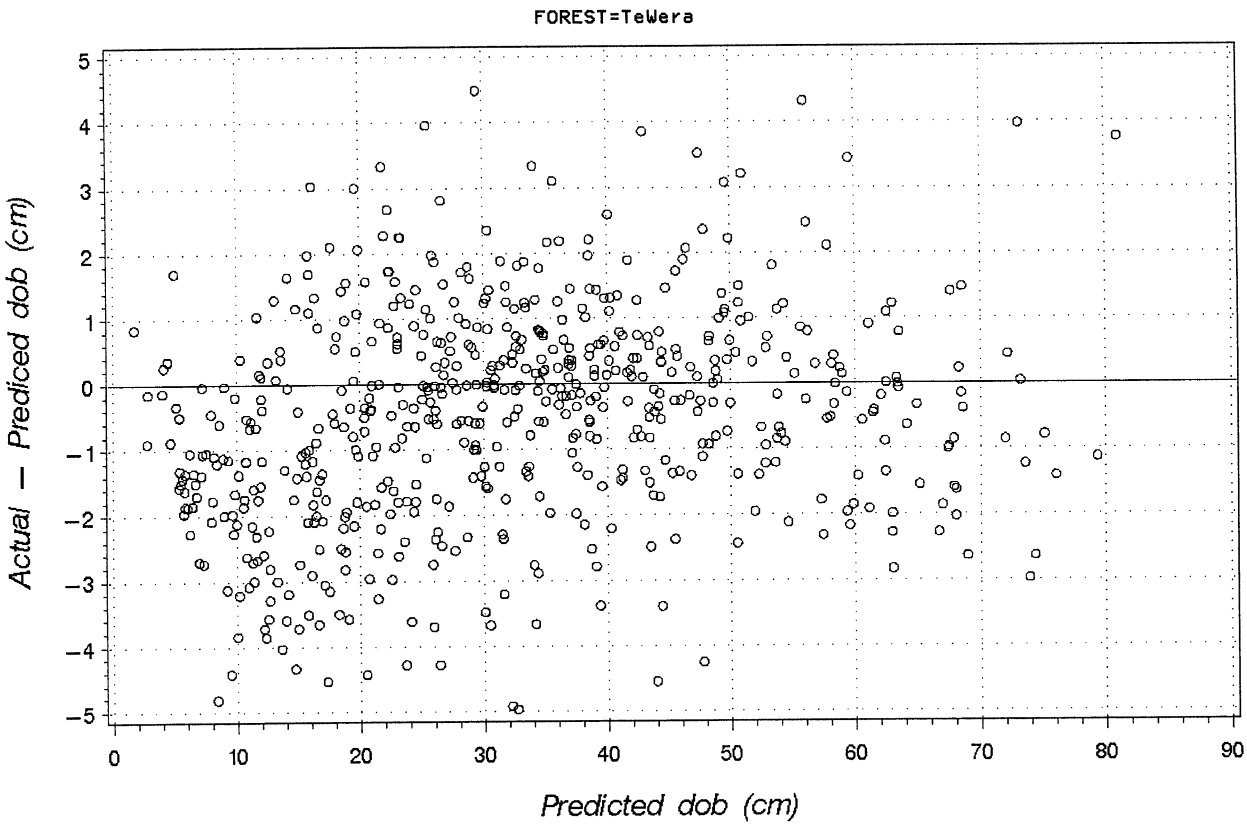

FOREST=Ngaumu
Actual – Prediced dob (cm)
L/H (Length from tip over height)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

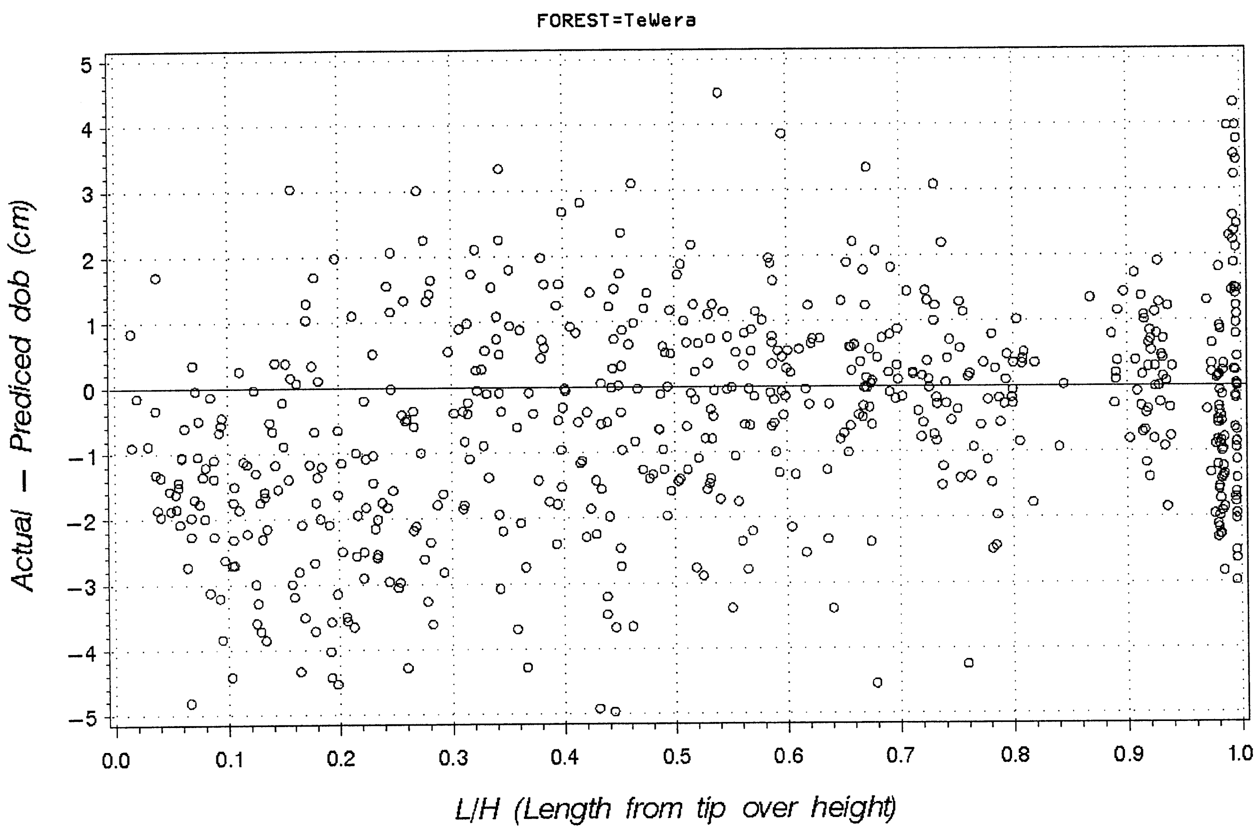

FOREST=Riverhead
Actual – Prediced dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
Predicted dob (cm)

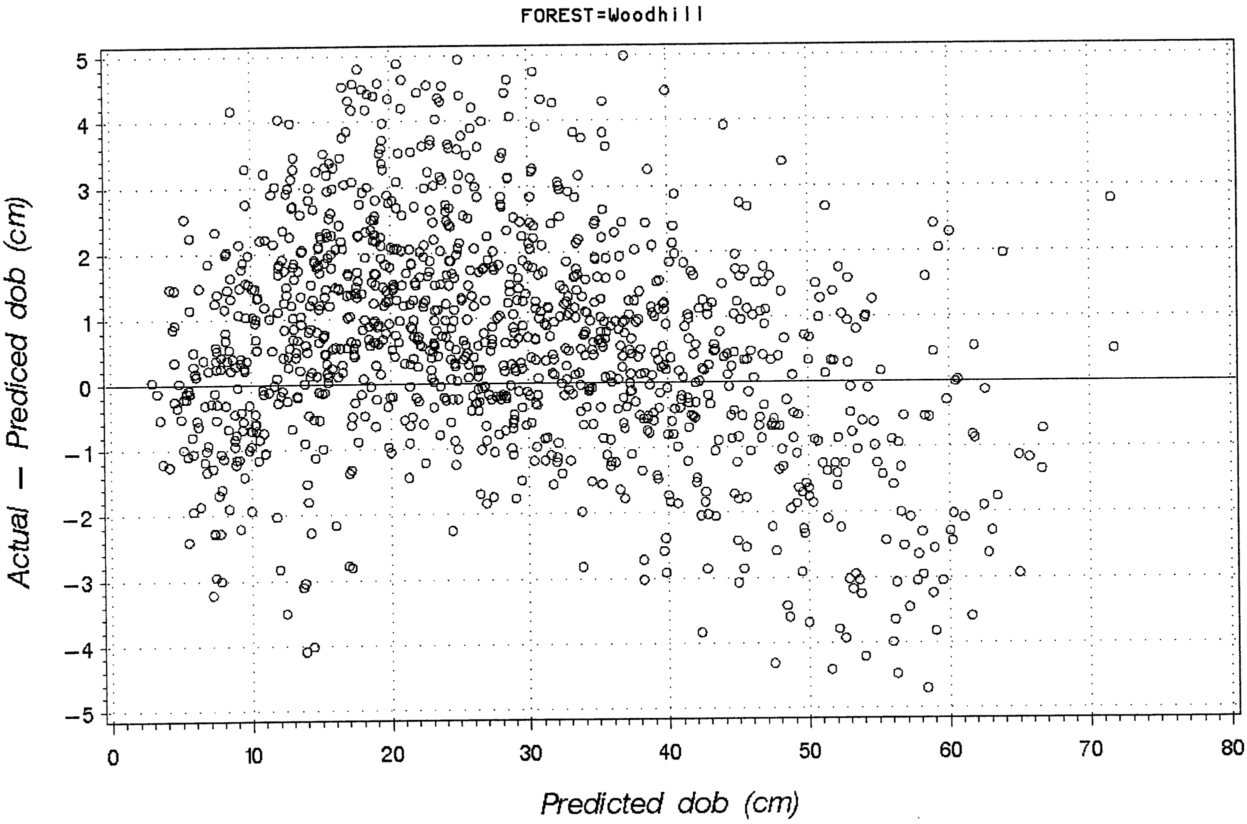

FOREST=Riverhead
Actual – Prediced dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
L/H (Length from tip over height)

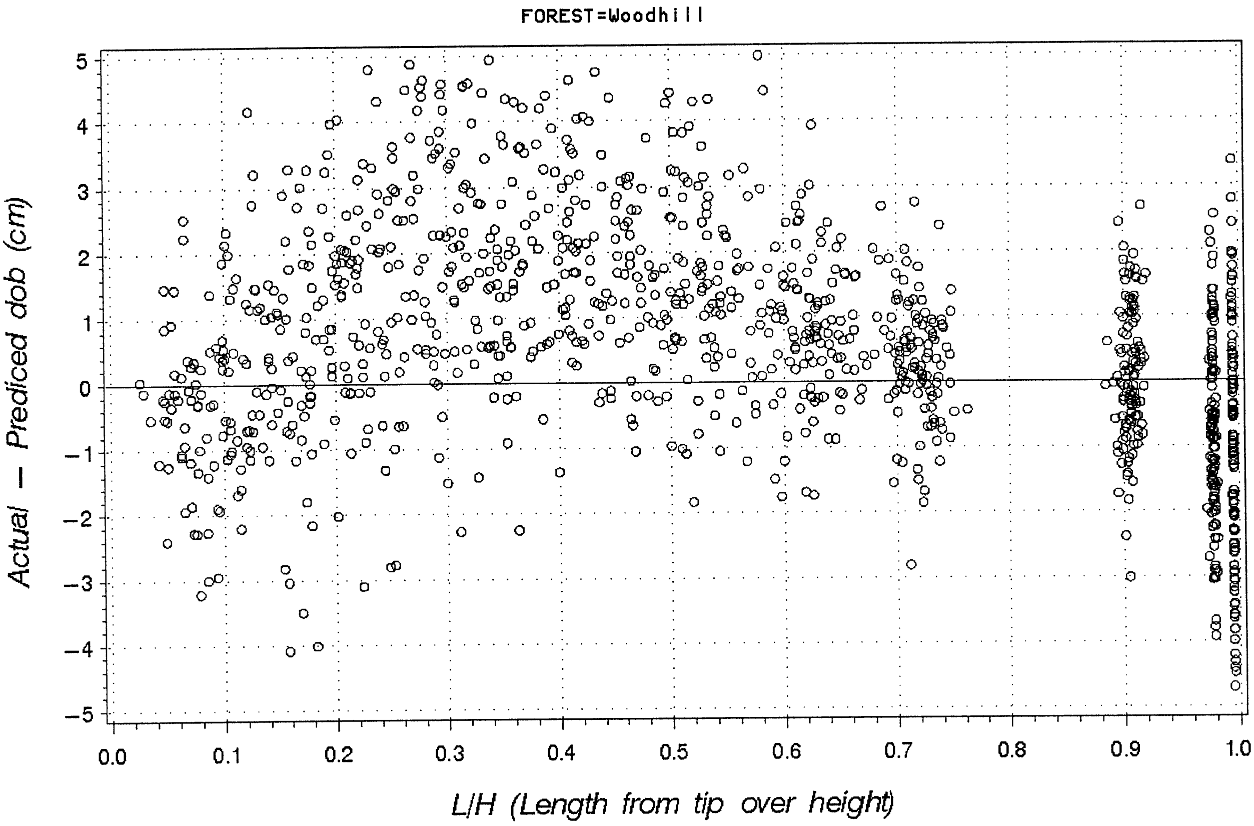

FOREST=Rotoehu
Actual — Prediced dob (cm)
Predicted dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
80
90


FOREST=Rotoehu
Actual — Prediced dob (cm)
L/H (Length from tip over height)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


FOREST=TeWera
Actual – Prediced dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
80
90
Predicted dob (cm)

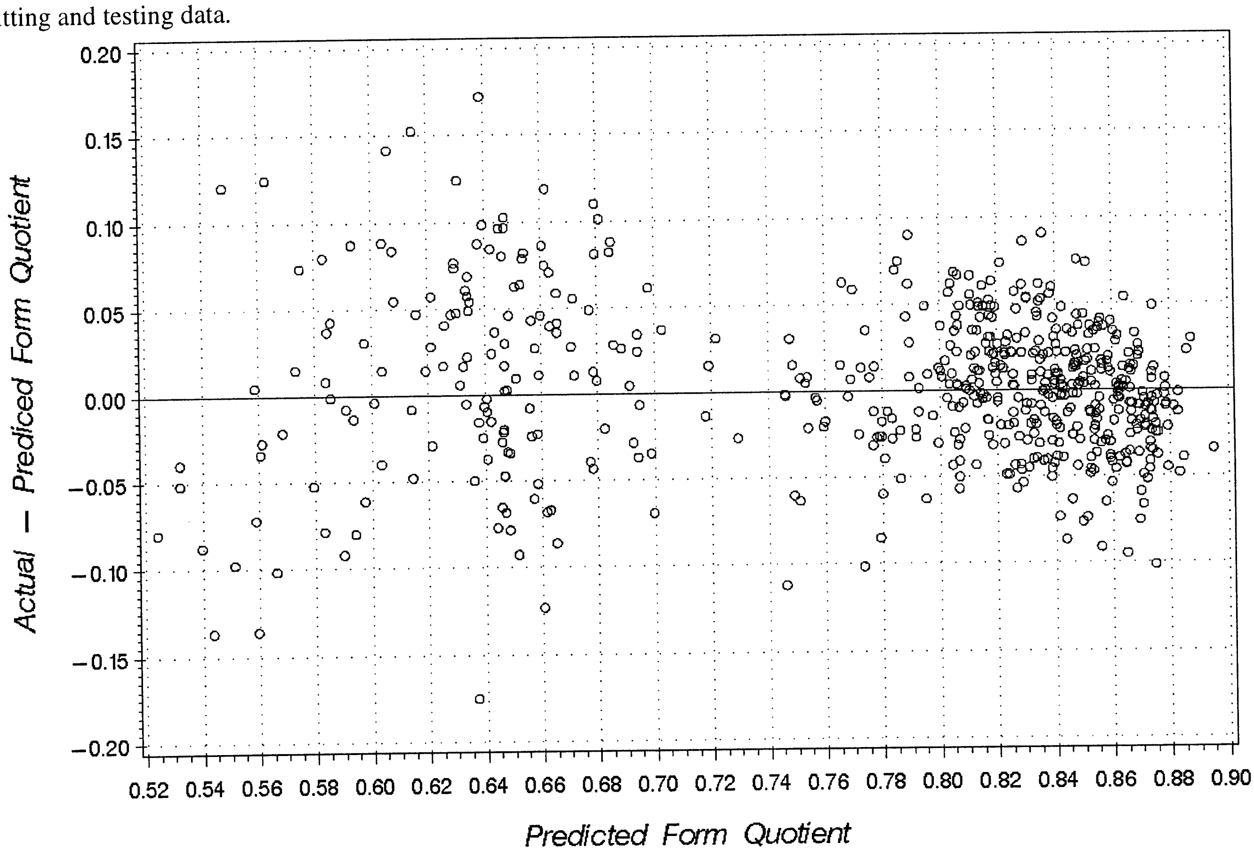

FOREST=TeWera
Actual – Prediced dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
L/H (Length from tip over height)

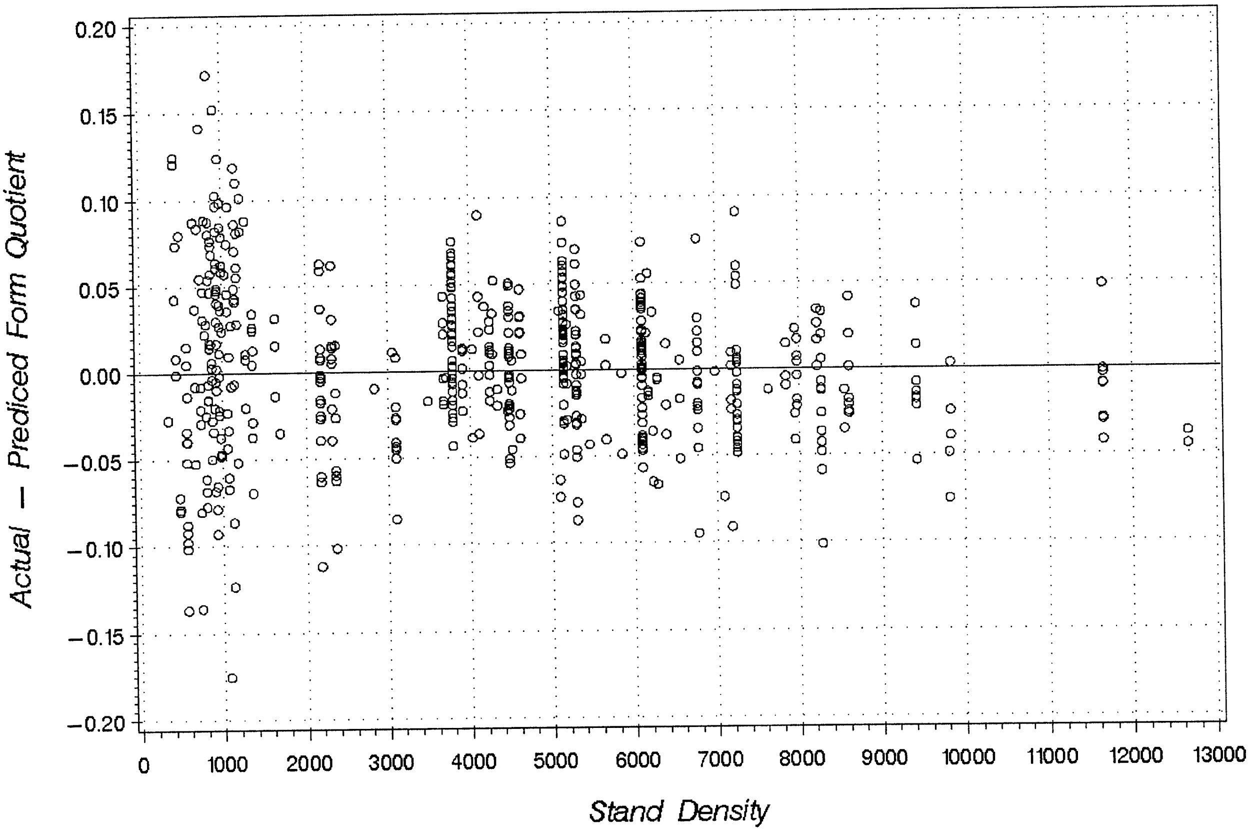

FOREST=Woodhill
Actual – Prediced dob (cm)
Predicted dob (cm)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
10
20
30
40
50
60
70
80

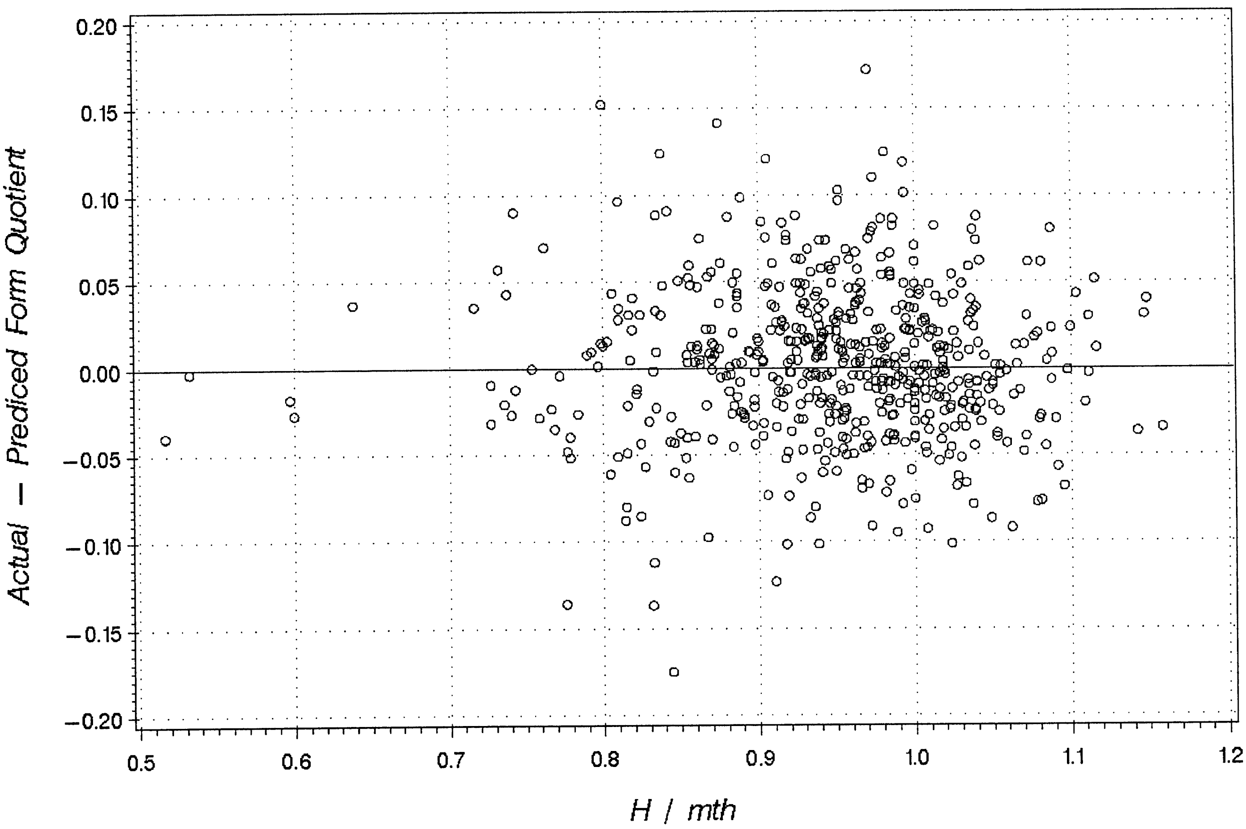

FOREST=Woodhill
Actual – Prediced dob (cm)
L/H (Length from tip over height)
5
4
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

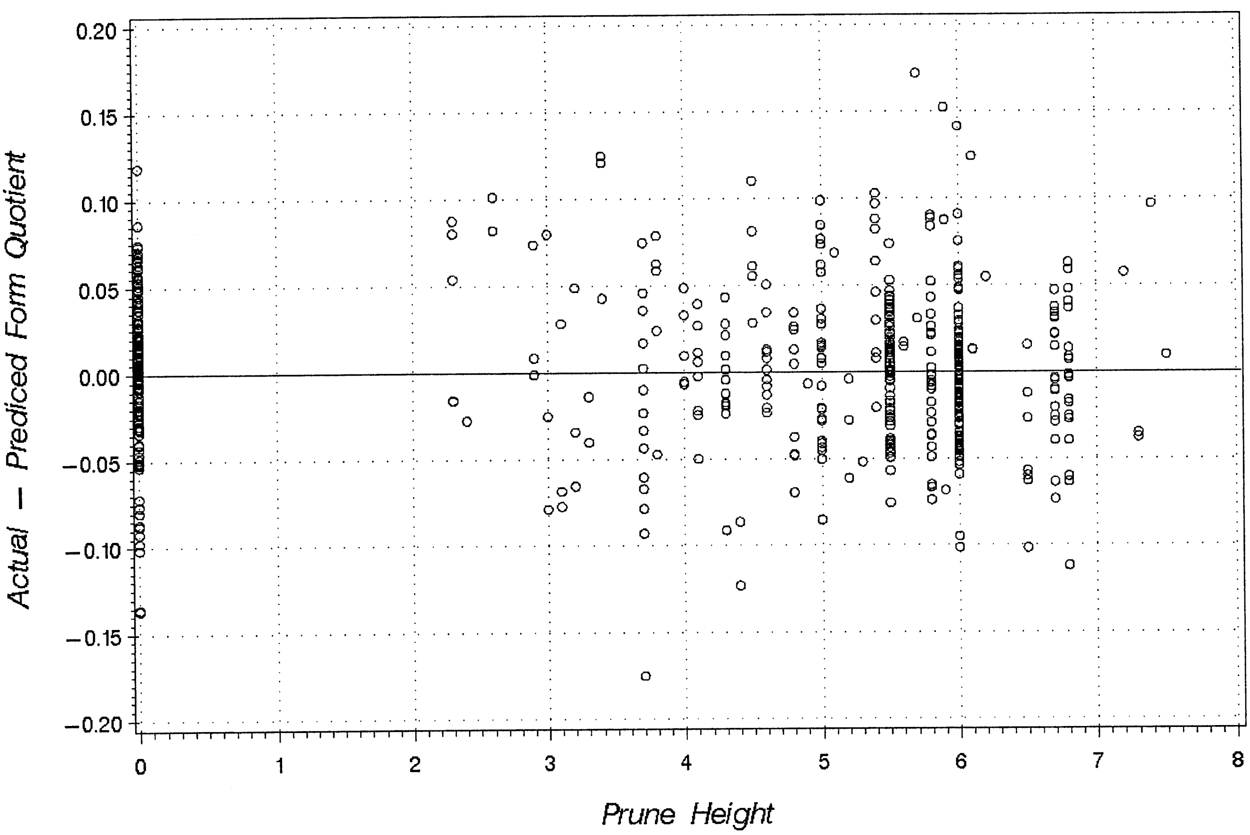
APPENDIX B

FQ Prediction errors against predicted values, stand density, height dominance and prune height using independent fitting and testing data.





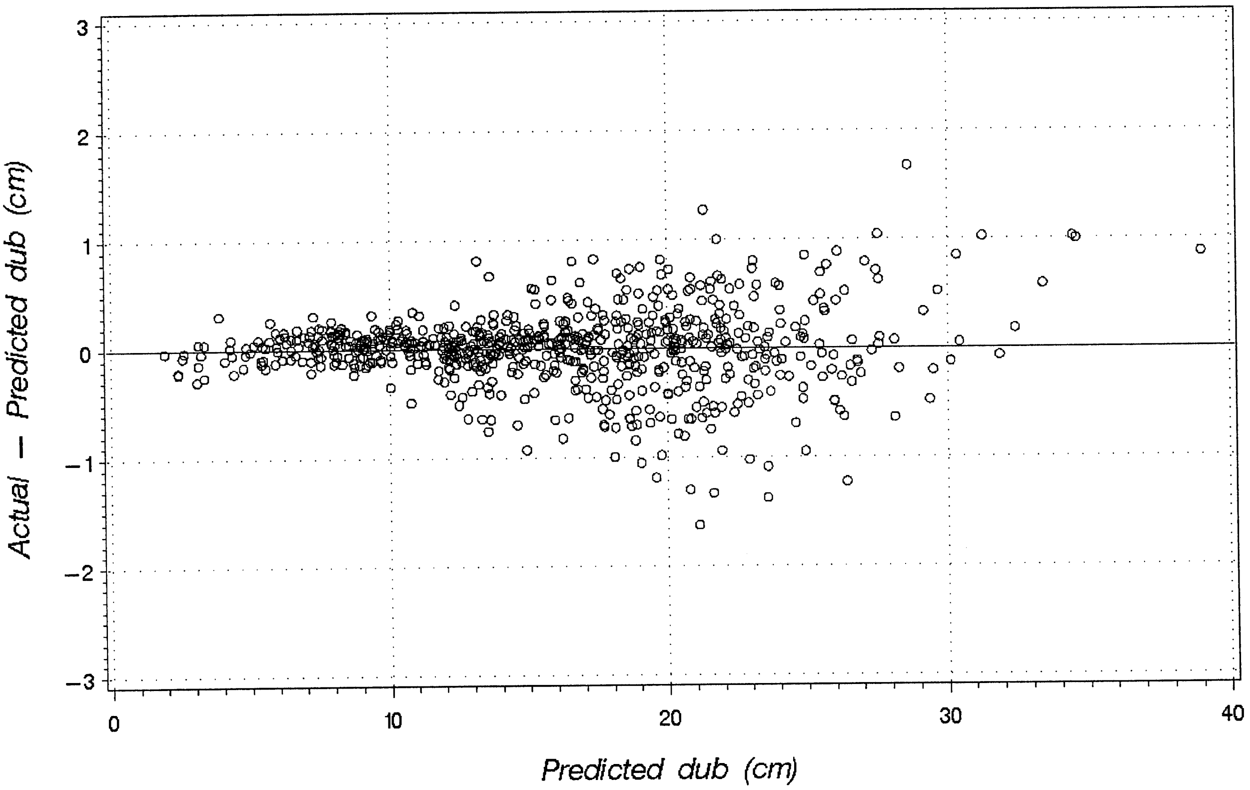

0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
Actual - Prediced Form Quotient
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
H / mth


0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
Actual - Prediced Form Quotient
0
1
2
3
4
5
6
7
8
Prune Height

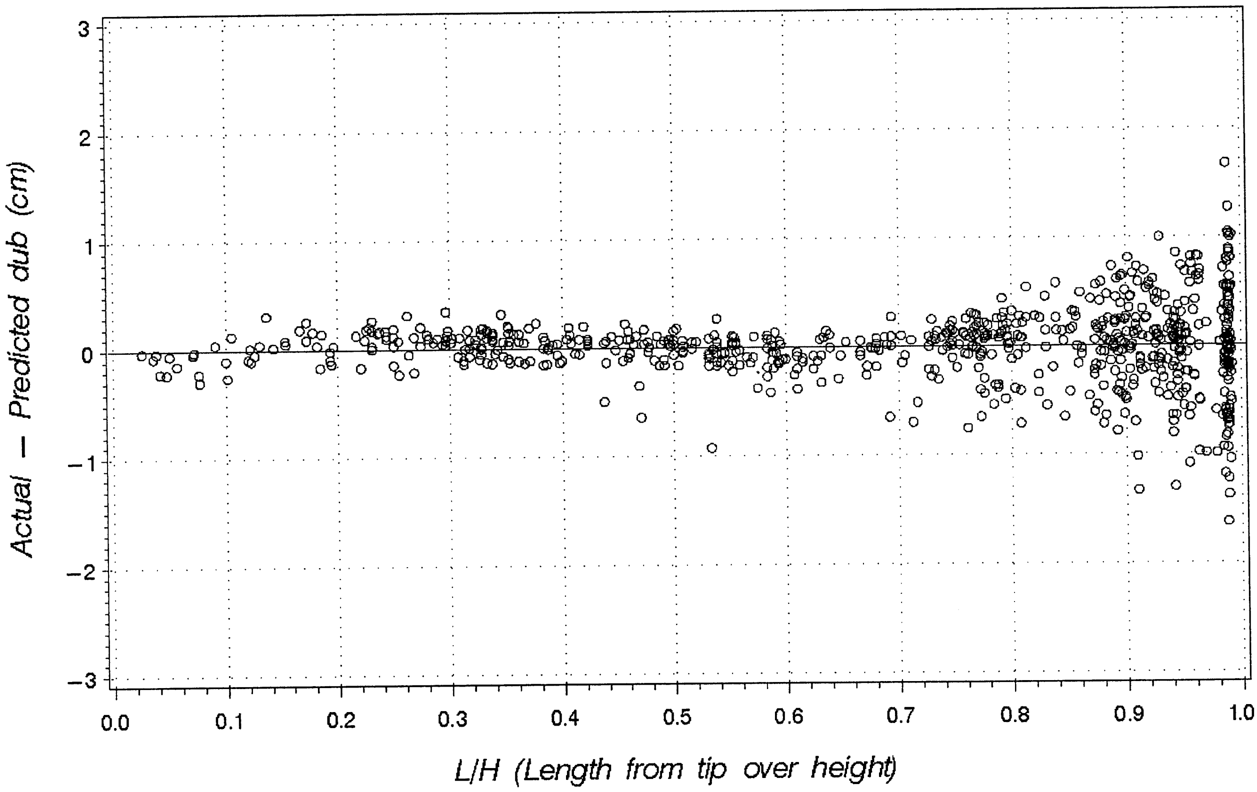
APPENDIX C

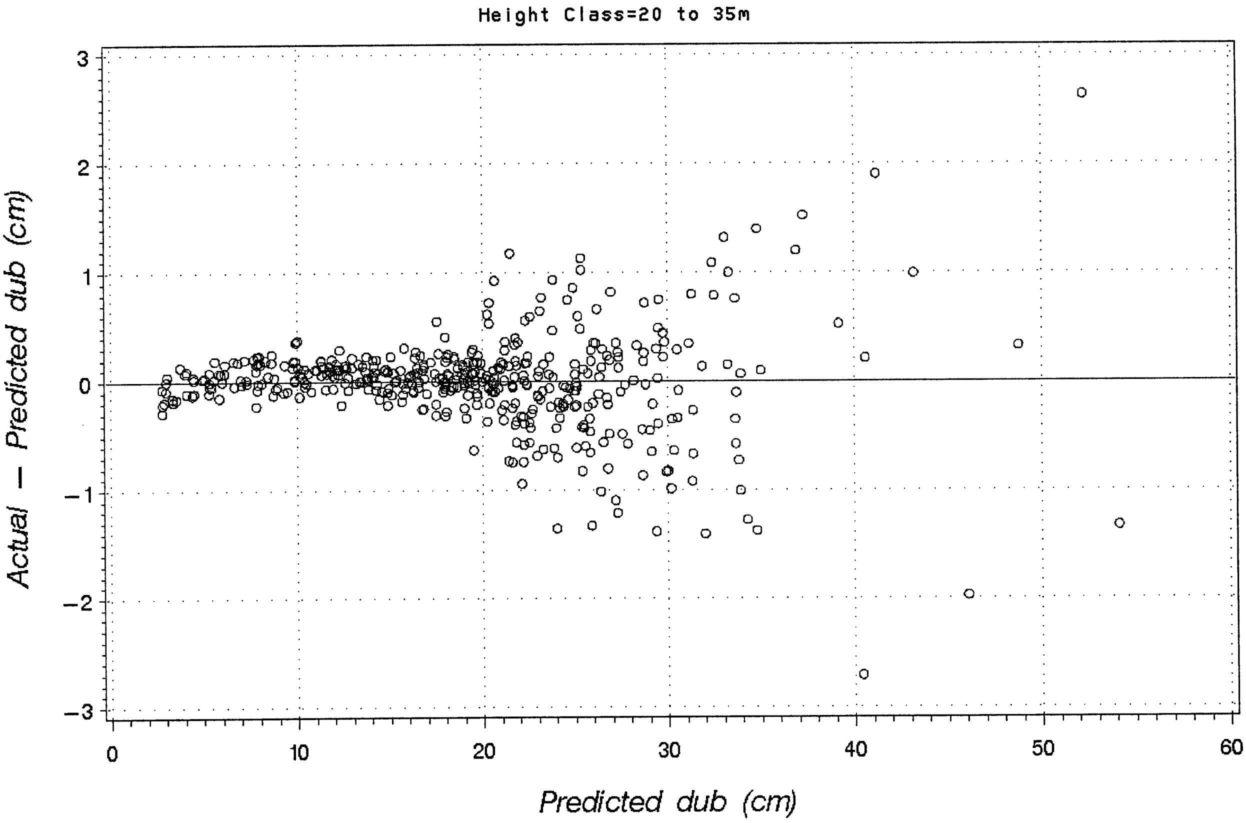
dub Prediction errors against predicted values and proportional height (*L/H*) by tree height class using independent fitting and testing data.

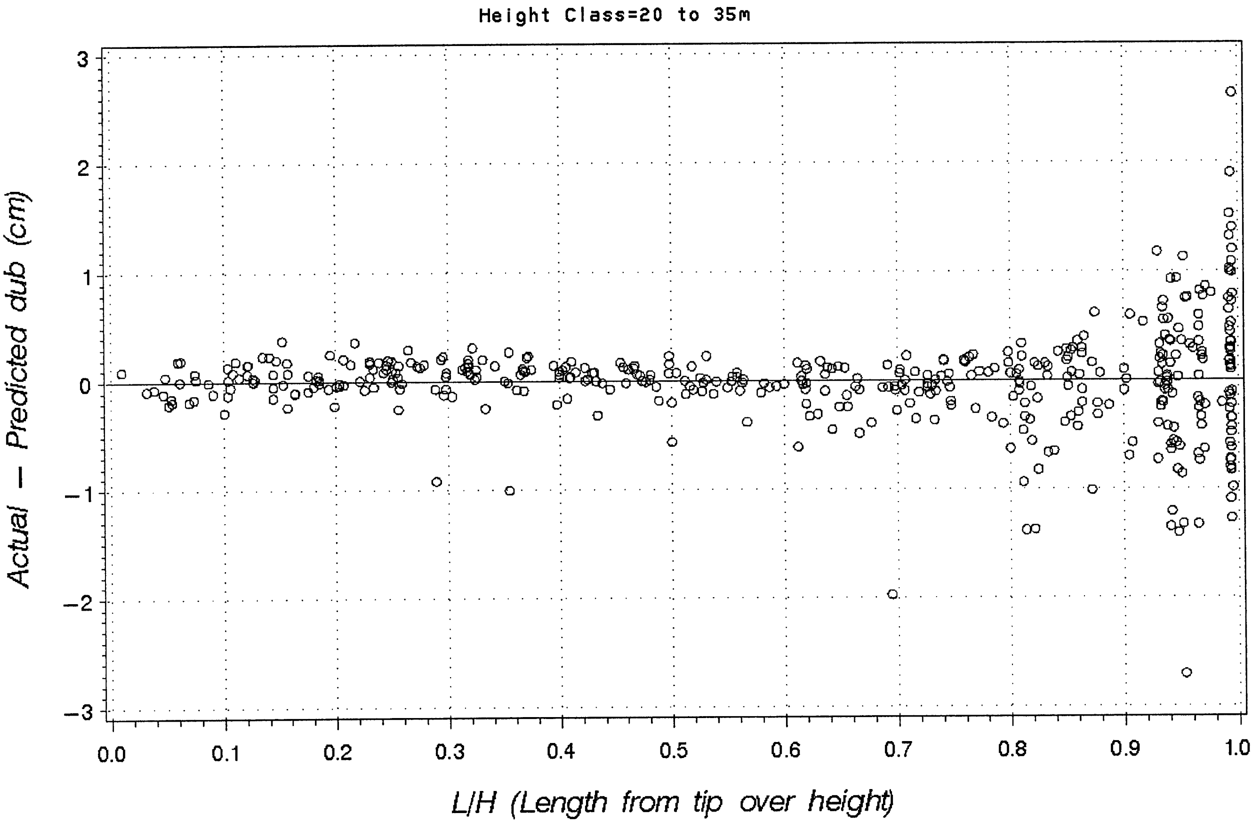
Height Class=less than 20m

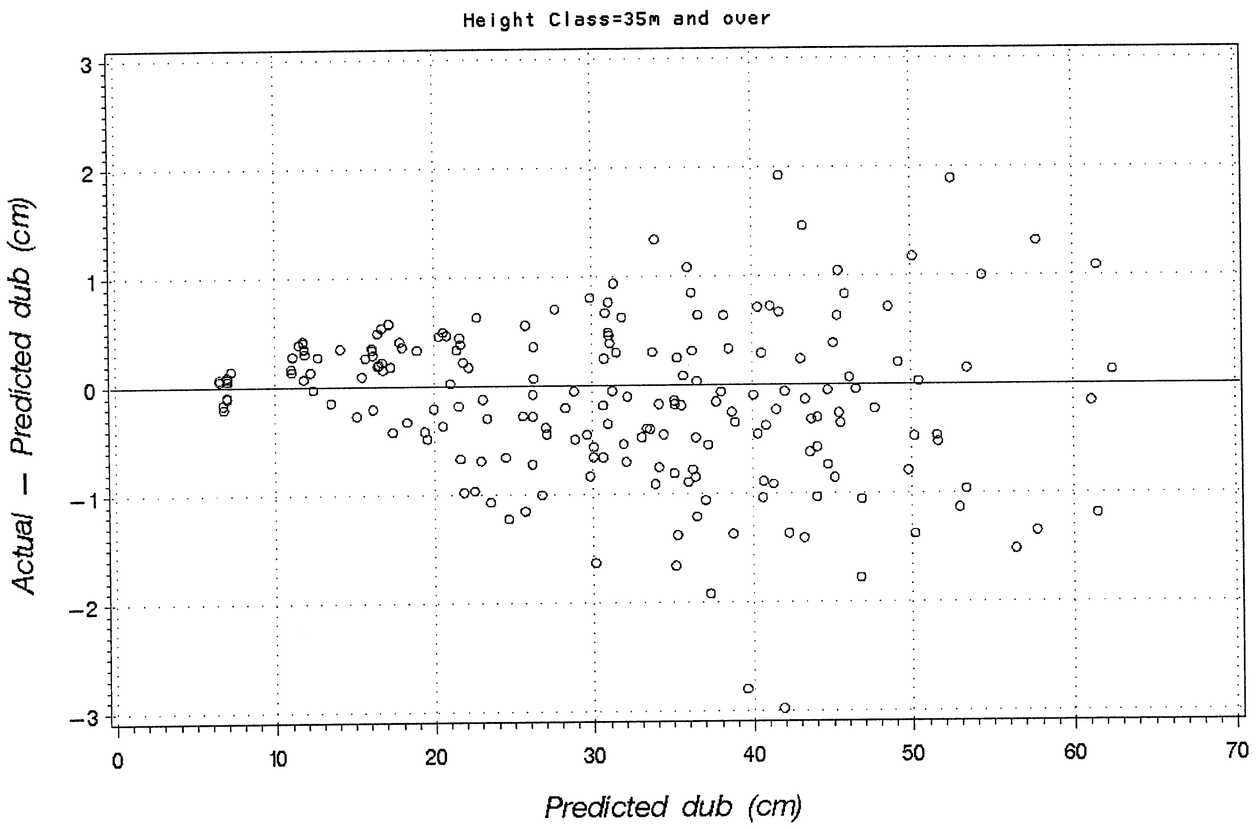


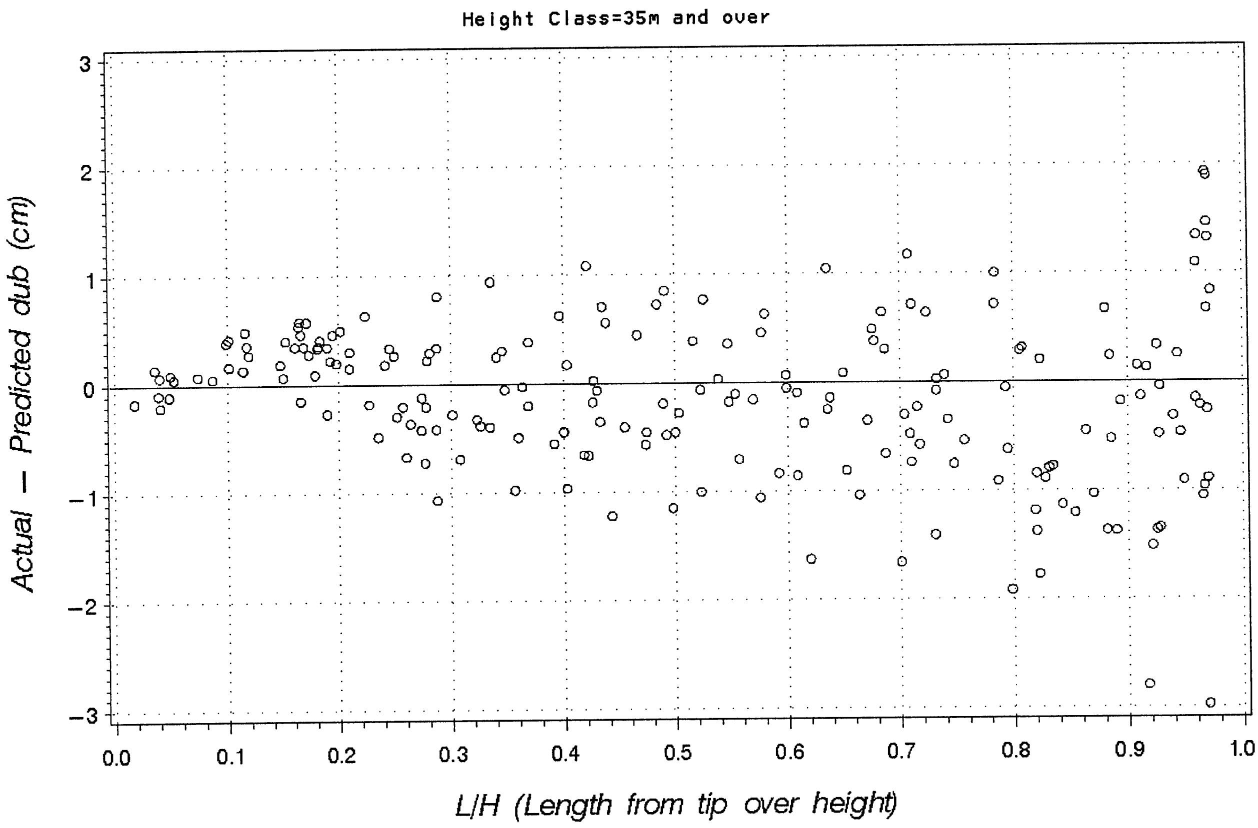
Height Class=less than 20m




Height Class=20 to 35m
Actual − Predicted dub (cm)
3
2
1
0
−1
−2
−3
0
10
20
30
40
50
60
Predicted dub (cm)


Height Class=20 to 35m
Actual − Predicted dub (cm)
3
2
1
0
−1
−2
−3
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
L/H (Length from tip over height)


Height Class=35m and over
Actual − Predicted dub (cm)
3
2
1
0
−1
−2
−3
0
10
20
30
40
50
60
70
Predicted dub (cm)


Height Class=35m and over
Actual − Predicted dub (cm)
3
2
1
0
−1
−2
−3
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
L/H (Length from tip over height)

APPENDIX D

An algorithm to calculate the under-bark volume in cubic metres from the tip of a tree down a specified level above ground.

```
function calc_volume( dbh: double; height: double; level: double): double;
// volume ub from tip down to level
var
  l: double;
  h: double;
begin
  l := height -level;
  h := height;
  result := (Pi *dbh *dbh *height /40000.0) *(
  Power(l/h,g1) *((b1*(a0 + a01*h)*l)/((1 + g1)*h) +
      (a10*b1*Power(l/h,1 + 0.5/exp(a12*h)))/
       (1 + 0.5/exp(a12*h) + g1))
  +Power(l/h,g2) *((b2*(a0 + a01*h)*l)/((1 + g2)*h) +
      (a10*b2*Power(l/h,1 + 0.5/exp(a12*h)))/
       (1 + 0.5/exp(a12*h) + g2))
  +Power(l/h,g3) *((b3*(a0 + a01*h)*l)/((1 + g3)*h) +
      (a10*b3*Power(l/h,1 + 0.5/exp(a12*h)))/
       (1 + 0.5/exp(a12*h) + g3))
  +Power(l/h,a31*h) *((a2*b1*Power(l/h,1 + g1)) /(1 + g1 + a31*h)
      +(a2*b2*Power(l/h,1 + g2)) /(1 + g2 + a31*h)
      +(a2*b3*Power(l/h,1 + g3)) /(1 + g3 + a31*h))  );
end;
```