TOWARDS A GENERAL STEM VOLUME AND TAPER EQUATION

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EXECUTIVE SUMMARY

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This project studies the possibility of improving the volume estimation of *Pinus radiata* tree stems by including an additional diameter measurement in the volume equation. The data selected for this project covered a wide range of tree sizes sampled from forests all over New Zealand. The result shows that incorporating the stem diameter at 6 m above the ground level reduced the Root Mean Squared Deviation of Volume Prediction (*RMDVp*) for under-bark stem data by over 30%. Further work is needed to ensure that a flexible taper equation which incorporates an additional diameter can be constructed.

Introduction

Tree volume and taper equations are used to determine the stem volume of trees given measurements of variables such as breast height diameter and tree height. They can also predict volume, diameters and taper of arbitrary stem sections. These equations are basic components of stand inventory, growth and yield, forest planning and product simulation systems.

The most visible uses of volume equations are in inventory processing and product simulation systems but forest planning problems, such as regime evaluation or long-term harvest planning, often apply volume equations to representative trees from diameter distributions simulated at different ages. It is unreasonable to expect volume predictions based only on diameter at breast height (*Dbh*) and tree height (*H*) to be very precise when applied to trees from a range of dominance classes, sites, ages and silvicultural regimes. Tree form is influenced by stand attributes and site differences (Larson 1963). For example, stand treatments like pruning, thinning and fertilization influence the length of the crown and hence the shape of the stem. Site characteristics like altitude and site index influence the stem form through crown development (Muhairwe *et al.* 1993).

Many studies have tried to include site and silvicultural effects in the prediction of volume by incorporating variables like crown ratio, site index, age (Candy 1989), form factor or form quotient. Crown ratio has often been preferred to other variables as the stem tapers more rapidly in the crown. However the work of Muhairwe showed that incorporating age, a dummy value for site and crown ratio only resulted in a small improvement for taper prediction.

Rustagi and Loveless (1990) used the ratio of height at two-third *Dbh* to total height to improve the volume prediction for Douglas Fir. They showed that the prediction was improved by 65% compared to the 5% improvement using crown ratio in the equation by Hann (1986) on the same statistics. Rustagi and Loveless identified the height of two-thirds *Dbh* at an average of 50% of total tree height for their 98 trees. The tree height for the sample ranged from 12.37 m to 52.73 m. However identifying this upper stem diameter could be tedious and error prone especially in an unpruned stand.

Another variable that has been used in numerous studies is form quotient (Hoyer 1985, Bi 1994). Form quotient is the ratio of a diameter at a certain height to the diameter at breast height, as defined by Husch *et al* (1982). Bi (1994) incorporated a lower stem form quotient at 4.5 m above ground in the prediction of volume and showed that the volume was better estimated by 41% in the root mean squared deviation. A lower stem measurement is also easier to measure in the field and should give smaller errors than measuring the height at two-thirds *Dbh*.

This study explores the possibility of improving the volume estimation by including an additional diameter measurement in the volume equation for *Pinus radiata*.

Data Background

Sectional measurements of 817 trees from 9 forests were used in this study. The locations of the forests are shown in Figure 1.

Figure 1. The forest locations

Tree diameters were measured over-bark with diameter tape at 0.15, 0.7, 1.4, 3, 6,... m above ground to within 5m of the tip. All sectional measurements were subjected to a comprehensive set of computer edits to screen out possible measurement and recording errors. Plots of stem profiles were compared with sample averages to select outliers and atypical trees for more detailed checking.

Where diameters at specific heights were missing they were interpolated using a quadratic procedure involving two diameters above and two below the section of interest. A variety of pruning treatments were evidenced in the data but details were not complete enough to include prune height in the analysis.

Trees were selected to cover the *Dbh* range of the sampled stands. The range of the data used is summarised in Tables 1 and 2.

Table 1. Data history

Forests (No. of strata)	No. sampled	Prune height (m)	Age (years)	Last Record of Stocking (stems ha^{-1})
Balmoral (2)	40	5	22	325-370
Golden Downs(10)	92	4.3-6.7	25-29	217-346
Kaingaroa (19)	88	0, 4-6	26-39	190-520
Longwood (1)	96	5.5	30	370
Ngaumu (3)	30	5.5	32	150-250
Riverhead (1)	56	0	29	368
Rotoehu (3)	101	6	28-29	270-320
Te Wera (19)	103	4-6	9-29	200-700
Woodhill (3)	211	0	22-30	200-370

A total of 871 trees were sectionally measured.

Data Ranges

Table 2 shows the range and distribution of breast height diameter (*Dbh*), height (*H*), total stem volume inside bark (*TSVub*) and tree form-factor (*FF*). Breast-height form-factor and form-quotient (*FQ*) using diameter over-bark (*Dob*) at 6 m were calculated as:

$$FF = \frac{40000}{\pi} \frac{TSVub}{Dbh^2 H} \quad (1)$$

$$FQ = \frac{Dob(6m)}{Dbh} \quad (2)$$

Table 2. Descriptive Statistics by forest

Forest	Variable	N	Min	Mean	Max	Std.Dev
Balmoral	DBH	40	16.3000	34.1400	52.1000	8.3533
	HT	40	12.4000	21.7450	27.1000	2.8351
	TSV	40	0.0946	0.7375	1.5586	0.3852
	FF	40	0.2896	0.3404	0.3907	0.0242
	FQ	40	0.6380	0.7700	0.8510	0.0389
GldnDowns	DBH	92	15.9000	41.0207	61.5000	9.4592
	HT	92	19.4000	31.8652	42.5000	4.7426
	TSV	92	0.1547	1.5852	3.5901	0.7374
	FF	92	0.3095	0.3512	0.4269	0.0235
	FQ	92	0.7430	0.8301	0.8947	0.0324
Kaingaroa	DBH	88	21.3000	46.7295	76.8000	10.5504
	HT	88	29.4000	39.2284	49.9000	4.7750
	TSV	88	0.4185	2.4773	5.6910	1.1602
	FF	88	0.2933	0.3491	0.4134	0.0284
	FQ	88	0.7585	0.8409	0.9050	0.0310
Longwood	DBH	96	30.2000	47.1594	67.3000	7.5973
	HT	96	26.9000	34.3344	39.4000	2.5471
	TSV	96	0.8813	2.2643	4.6377	0.8251
	FF	96	0.3162	0.3638	0.4287	0.0237
	FQ	96	0.7701	0.8520	0.9251	0.0329
Ngaumu	DBH	30	46.5000	60.7333	73.1000	7.2574
	HT	30	34.5000	39.4967	42.6000	2.3898
	TSV	30	1.9771	3.9085	5.6096	0.8989
	FF	30	0.3015	0.3385	0.3880	0.0227
	FQ	30	0.7706	0.8631	0.9180	0.0365
Riverhead	DBH	56	22.0000	43.6054	61.8000	9.6233
	HT	56	26.5000	34.8518	40.8000	3.4572
	TSV	56	0.3910	1.8982	3.4530	0.8017
	FF	56	0.2885	0.3463	0.4282	0.0285
	FQ	56	0.7453	0.8183	0.8974	0.0339
Rotoehu	DBH	101	37.1000	51.8158	76.1000	7.5005
	HT	101	36.2000	41.8891	49.8000	2.4832
	TSV	101	1.5579	2.9957	6.6317	0.9197
	FF	101	0.2716	0.3330	0.4146	0.0275
	FQ	101	0.7703	0.8543	0.9267	0.0308
TeWera	DBH	103	19.9000	50.0320	70.2000	11.0057
	HT	103	13.5000	35.8155	49.7000	8.5522
	TSV	103	0.1884	2.6538	5.8045	1.3177
	FF	103	0.2818	0.3437	0.4123	0.0282
	FQ	103	0.6510	0.8398	0.9263	0.0455
Woodhill	DBH	211	20.1000	43.4038	66.7000	8.3385
	HT	211	25.4000	31.9161	37.9000	2.1569
	TSV	211	0.3594	1.8201	4.3847	0.7281
	FF	211	0.3104	0.3695	0.4487	0.0275
	FQ	211	0.7355	0.8342	0.9279	0.0365

The relationship between *Dbh* and *H* is shown in figure 2. Figure 3 shows the *TSVub* against *Dbh* and figures 4-5 show the variation in *FF* and *FQ* with *Dbh.*

Analytical Steps

Sectional volume

The first step of the analysis was to calculate the under-bark sectional volumes from the diameter and bark thickness measurements. The total tree volume was calculated as the sum of all sectional tree volumes. The volume of the tree tip was calculated using the formula for a cone and Smalian's formula was used to calculate the sectional volumes between the breast height (1.4 m) and the tip. The section below breast height was estimated by the formula for a truncated cone. The ground diameter was linearly extrapolated using sectional area of 2 measurement levels between the breast height and ground (e.g at 0.3m and 0.7m). This formula was preferred as it minimises the error in estimating the butt volume (Ellis 1973).

A series of volume equations, both with and without FQ, were then tested to determine their bias and precision when used to predict total stem *TSVub*.

Volume Equations

Table 3 shows the selected volume equations used in the analysis. Equation 1 is the combined-variable volume equation (Spurr 1952 and Husch et al. 1982). Equation 2 is an extension of Schumacher and Hall's (1933) original allometric formulation and Equation 3 is Hann's (1987) non-linear equation. Equations 1-3 use *Dbh* and *H* as predictor variables for the volume prediction and equations 4-6 are the modified versions of the first 3 equations with the additional variable: *FQ*.

Table 3. Description of the equation forms.

Equation	Description
Equation 1:	$Estvol = b_1 + b_2\left(Dbh^2 H\right)$
Equation 2:	$Estvol = e^{b_1} Dbh^{b_2} \left(\frac{H^2}{H-1.4}\right) b_3$
Equation 3:	$Estvol = b_1 + b_2\left(\frac{H}{Dbh}\right)^{b_3} Dbh^2 H$
Equation 4:	$Estvol = FQ\left(b_1 + b_2 Dbh^2 H\right)$
Equation 5:	$Estvol = FQ^{b_0} e^{b_1} Dbh^{b_2} \left(\frac{H^2}{H-1.4}\right)^{b_3}$
Equation 6:	$Estvol = FQ^{b_0} \left\{b_1 + b_2\left(\frac{H}{Dbh}\right)^{b_3} Dbh^2 H\right\}$

b_0, b_1, b_2, and b_3 are the estimated coefficients and all other variables are as previously defined.

Data grouping

The data was divided into 2 groups by random selection, one group was used for fitting the equations and the other group for validation to double-check the accuracy and precision of the selected equations. In summary, 399 sample trees were used for fitting (data set 1) and the remaining 418 trees were used for validation (data set 2).

Parameter Estimates

Parameters were estimated using either linear or non-linear regression. Table 4 shows the parameter estimates for all 6 equations.

Table 4. Parameter estimates

	Parameters			
	b_0	b_1	b_2	b_3
Equation 1		0.160567	0.000250	
		0.020739	*0.000000*	
Equation 2		-9.636325	1.865500	0.891607
		0.118840	*0.026752*	*0.035058*
Equation 3		0.167317	0.000025	-0.024342
		0.022412	*0.000000*	*0.030707*
Equation 4		0.263632	0.000029	
		0.018748	*0.000000*	
Equation 5	1.464400	-8.921250	1.823649	0.808246
	0.067165	*0.085596*	*0.018078*	*0.024019*
Equation 6	1.478856	0.347770	0.000030	-0.069575
	0.072662	*0.023469*	*0.000000*	*0.022846*

Note: The standard errors of the parameter estimates were presented in italic numbers. Coefficient b_3 in equation 3 is not significantly different from zero.

Comparative Statistics

Several statistics were calculated to compare the accuracy and precision of the equations:

Index of fit (I^2), a substitute for coefficient of determination (R^2) as most of equations are not linear, is a measure of the amount of variability in estimated volume accounted for by the predictor variables in the regression.

$$I^2 = 1 - \frac{\sum_i \left(TSVub_i - EstVol_i\right)^2}{\sum_i \left(TSVub_i - \overline{TSVub}\right)^2} \quad \textbf{where } \overline{TSVub} = \textbf{mean of } \boldsymbol{TSVub}. \qquad (3)$$

A software procedure that tests the accuracy of predictions, ATEST (Rauscher 1985), was run to check the differences between the actual and predicted values. ATEST calculates bias and standard deviation, mean square error *(MSE)*, prediction interval and tolerance interval. The test also checks the normality of the residuals. The mean of 100 predicted errors will fall within the prediction interval (*PI*) with 95% confidence. Tolerance interval (*TI*) gives the interval that 95% of future errors will fall within over a long term period, with 95% confidence. The intervals range is calculated as the bias plus or minus of the prediction or tolerance value. However it is important to note that this test is only accurate when applied to the population from which the sample was drawn (Reynolds 1984).

To check for trends in the volume estimate error with tree size, the individual tree percentage volume errors were plotted over *Dbh* to examine the errors by forest in relation to the errors between trees for Equations 2 and 5 (Figures 6-9). The percentage error in the total volume was calculated to compare the accuracy and precision of the volume equation being tested in predicting volume aggregated by forest. *Percentage Error* was calculated as:

$$PercentageError = \frac{100(\sum EstVol - \sum TSVub)}{\sum EstVol} \qquad (4)$$

where *EstVol* is estimated volume under bark,
TSVub is sectional stem volume under bark
and the summation is over all trees in a forest.

The distribution of percentage error among the equations was checked using box-and-whisker plots[1] as shown in figures 10-11.

Table 8 shows the Root Mean Squared Deviation of Volume Prediction (*RMSDVp*) introduced by Bi (1994) which is calculated as follows:

$$RMSDVp = \sqrt{\frac{\sum_i (TSVub_i - EstVol_i)^2}{N}}, \qquad (5)$$

where N is the total number of sample trees

Results

The results when the equations were applied to the fitting data set 1 and the validation data set 2 are shown in tables 5-6.

[1] A box-and-whisker plot consists of a box, whiskers, and outliers. The line across the box is the median of the data. The bottom of the box is at the first quartile (Q1) and the top is at the third quartile (Q3). The whiskers are the lines that extend from the top and bottom of the box to the adjacent values, the lowest and highest observations still inside the region defined by the lower limit Q1 - 1.5 (Q3 - Q1) and the upper limit Q3 + 1.5 (Q3 - Q1). Outliers are points outside the lower and upper limits (Minitab 1996).

Table 5. Results of the comparative statistics for data set 1.

	Comparative Statistics for Data Set 1				
	r^2 (%)	MSE	Bias (%)	PI %(Bias +/-)	TI %(Bias +/-)
Equation 1	96.78%	0.0369	1.0511	2.6412	25.0365
Equation 2	96.97%	0.0346	1.6319	1.9293	18.2886
Equation 3	96.78%	0.0374	2.0630	2.7131	25.7182
Equation 4	98.23%	0.0204	1.5188	2.4031	22.7797
Equation 5	98.63%	0.0157	0.6202	1.1977	11.3535
Equation 6	98.43%	0.0181	0.7856	2.5358	24.0375

Table 6. Results of the comparative statistics for data set 2.

	Comparative Statistics for Data Set 2				
	r^2 (%)	MSE	Bias (%)	PI %(Bias +/-)	TI %(Bias +/-)
Equation 1	97.30%	0.0353	-0.1054	3.1022	29.5011
Equation 2	97.51%	0.0328	0.3841	1.7661	16.7950
Equation 3	97.35%	0.0349	0.8670	3.2081	30.5088
Equation 4	98.37%	0.0215	0.8435	2.8816	27.4031
Equation 5	98.78%	0.0160	0.0160	1.1174	10.6262
Equation 6	98.55%	0.0191	0.2333	3.0178	28.6985

The percentage errors in aggregate volume by forest are shown in tables 7-8. Table 9 shows the improvement in *RMSDVp* when *FQ* is incorporated to the volume equations and Table 10 presents the complete pairwise comparison between equations with form quotient and without it.

Table 7. Percentage Error in aggregate volume for data set 1 (per forest).

FOREST	Percentage of Error					
	Eqn 1	Eqn 2	Eqn 3	Eqn 4	Eqn 5	Eqn 6
Balmoral	15.273	14.273	15.990	11.782	7.589	11.834
GldnDowns	2.894	3.308	2.949	3.388	3.629	3.644
Kaingaroa	0.499	-0.013	0.323	-0.183	-1.628	-1.091
Longwood	-5.968	-4.927	-5.906	-3.567	-1.340	-2.379
Ngaumu	-2.360	-2.666	-2.298	-0.374	0.300	0.637
Riverhead	3.717	4.108	3.653	0.478	-1.006	-1.379
Rotoehu	0.824	0.287	0.546	1.497	0.807	0.948
TeWera	2.922	2.742	2.941	2.722	2.280	2.551
Woodhill	-3.728	-2.658	-3.527	-3.904	-2.907	-3.529
ALL	0.000	0.137	0.000	0.101	0.038	0.044

Table 8. Percentage Error in aggregate volume for data set 2 (per forest).

FOREST	Percentage of Error					
	Eqn 1	Eqn 2	Eqn 3	Eqn 4	Eqn 5	Eqn 6
Balmoral	13.864	12.006	15.382	11.881	6.628	12.087
GldnDowns	2.284	2.573	3.315	3.547	2.943	3.414
Kaingaroa	-0.034	-0.614	0.663	0.662	-1.317	-0.902
Longwood	-4.393	-3.497	-3.258	-2.319	-1.490	-2.108
Ngaumu	-0.236	-0.824	1.081	0.416	-0.739	0.439
Riverhead	1.400	1.679	2.339	0.761	-0.645	-0.643
Rotoehu	2.553	2.067	3.313	3.722	2.349	2.638
TeWera	-1.500	-2.004	-0.492	0.485	-0.452	0.305
Woodhill	-6.105	-4.822	-4.896	-4.672	-3.815	-4.529
ALL	-1.1465	-1.031	-0.1326	0.049	-0.582	-0.387

Table 9. Root Mean Squared Deviation of Volume Prediction and the comparison between equations.

	RMSDVp (m^3)		Improvement		
	Data Set 1	Data Set 2	Equation	Difference	Percentage
Equation 1	0.1858	0.1822	Eqn 4(D1) - Eqn 1(D1)	-0.0542	-29.15%
Equation 2	0.1820	0.1775	Eqn 5(D1) - Eqn 2(D1)	-0.0650	-35.71%
Equation 3	0.1886	0.1845	Eqn 6 (D1) - Eqn 3(D1)	-0.0609	-32.31%
Equation 4	0.1316	0.1390	Eqn 4(D2) - Eqn 1(D2)	-0.0431	-23.68%
Equation 5	0.1170	0.1217	Eqn 5(D2) - Eqn 2(D2)	-0.0559	-31.46%
Equation 6	0.1276	0.1326	Eqn 6 (D2) - Eqn 3(D2)	-0.0518	-28.09%

Note: D1 and D2 under the 'Equation' column refer to the data set used in this comparison.

Table 10. Pairwise comparisons of volume equations.

	I^2 (%)		MSE		Bias (%)		PI (Bias +/-)		TI (Bias +/-)	
	Difference	% Improve.	Difference	% Improve.	Difference	% Improve.	Difference	% Improve.	Difference	% Improve.
Data Set 1										
Eqn 4 - Eqn 1	0.0145	1.50%	-0.0165	-44.77%	0.4677	N/A	-0.2381	-9.01%	-2.2568	-9.01%
Eqn 5 - Eqn 2	0.0166	1.71%	-0.0189	-54.62%	-1.0117	-62.00%	-0.7316	-37.92%	-6.9351	-37.92%
Eqn 6 - Eqn 3	0.0164	1.70%	-0.0193	-51.60%	-1.2774	-61.92%	-0.1773	-6.53%	-1.6807	-6.54%
Data Set 2										
Eqn 4 - Eqn 1	0.0107	1.10%	-0.0138	39.09%	0.9489	N/A	-0.2206	-7.11%	-2.0980	-7.11%
Eqn 5 - Eqn 2	0.0127	1.30%	-0.0168	51.22%	-0.3681	-95.83%	-0.6487	-36.73%	-6.1688	-36.73%
Eqn 6 - Eqn 3	0.0120	1.23%	-0.0158	-45.27%	-0.6337	-73.09%	-0.1903	-5.93%	-1.8103	-5.93%

Discussions

Tables 7 and 8 show that the inclusion of *FQ* in the volume equations improved the prediction ability. Equation 5 performed generally better than the rest. The largest error in the total forest volumes was a 7.59% over-estimate in data set 1 and 6.63% over-estimate in data set 2 for Balmoral forest using equation 5 compared with 14.27% in data set 1 and 12% in data set 2 using equation 2 (Tables 7 and 8). Figures 6-9 show the percentage errors were scattered and the mean errors were not significantly different from zero. However it was also noticed that the percentage errors from Equation 5 were in smaller range than the ones from Equation 2. Both figures from Data set 1 or 2 show that the range of percentage errors from Equation 5 are ±20% while the percentage errors for Equation 2 vary between ±30%. When *RMSDVp* is used to measure the improvement level (Table 9), equation 5 reduced the statistic by over 30% compared to equation 2.

The distributions of individual tree percentage error for Equations 1, 3, 4, and 6 appeared to be highly skewed (Figures 10-11). There were a large number of outliers at one end of tail and they increased the variability of the distributions greatly. However the percentage error distributions for Equations 2 and 5 were fairly symmetric for both data sets. This sampling distribution is useful as a rough indicator of the population shape of percentage error when the particular equation is used for volume prediction.

Most of the variation in volume is accounted for by *Dbh* and *H* alone ($I^2 \approx 97\%$, table 5). Table 10 shows that *FQ* improved I^2 in the volume equations by approximately 1.1-1.7%. *MSE* was improved by a minimum of 39%. Bias, standard deviation, prediction interval and tolerance interval were also improved by good percentages. Equation 5 was the best equation in overall performance.

Conclusion

Incorporating the lower form quotient into the volume equation showed a significant improvement in volume estimation. The ratio of this diameter to *Dbh* should monitor the change of stem taper and form over time which in turn affects the volume estimation as the cubic volume is always mathematically related to form-factor (Rustagi 1990). The improved volume estimation also offers more precision with which to study the response in volume growth to silviculture treatments like thinning, pruning and site improvements.

In our sample of 817 *Pinus radiata* trees, the incorporation of lower stem diameter at 6 m above the ground level resulted in over 30% reduction in *RMDVp* for under-bark stem volume prediction. Bi (1994) incorporated lower stem form quotient at 4.5 m above the ground level into the volume equations for *Eucalyptus fastigata*. Bi's results showed a 41% reduction in this statistic for over-bark stem volumes prediction and 12.5% reduction for under-bark stem volumes.

One drawback to using an upper stem diameter at a fixed height is that trees at, or below this height can not be processed by volume and taper equations which require the upper diameter as a parameter.

The improvements in accuracy shown here indicate that worthwhile gains can be made by measuring an additional stem diameter. The next step is to ensure that a flexible taper equation which incorporates *Dob(6m)* can be constructed using a method such as the composite equations of Gordon *et al* (1995). While measuring the *Dob(6m)* on every tree may be feasible in permanent plots, is unlikely to be cost-effective in routine inventory, and stand simulation systems will need methods for predicting form quotient. Further work is then required to look at the relationship between *FQ* and stand, tree and site variables that are available to predict *FQ* in circumstances where it is not possible, or not cost-effective, to measure upper stem diameters.

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Figure 2. Relationship between Total Height and Dbh

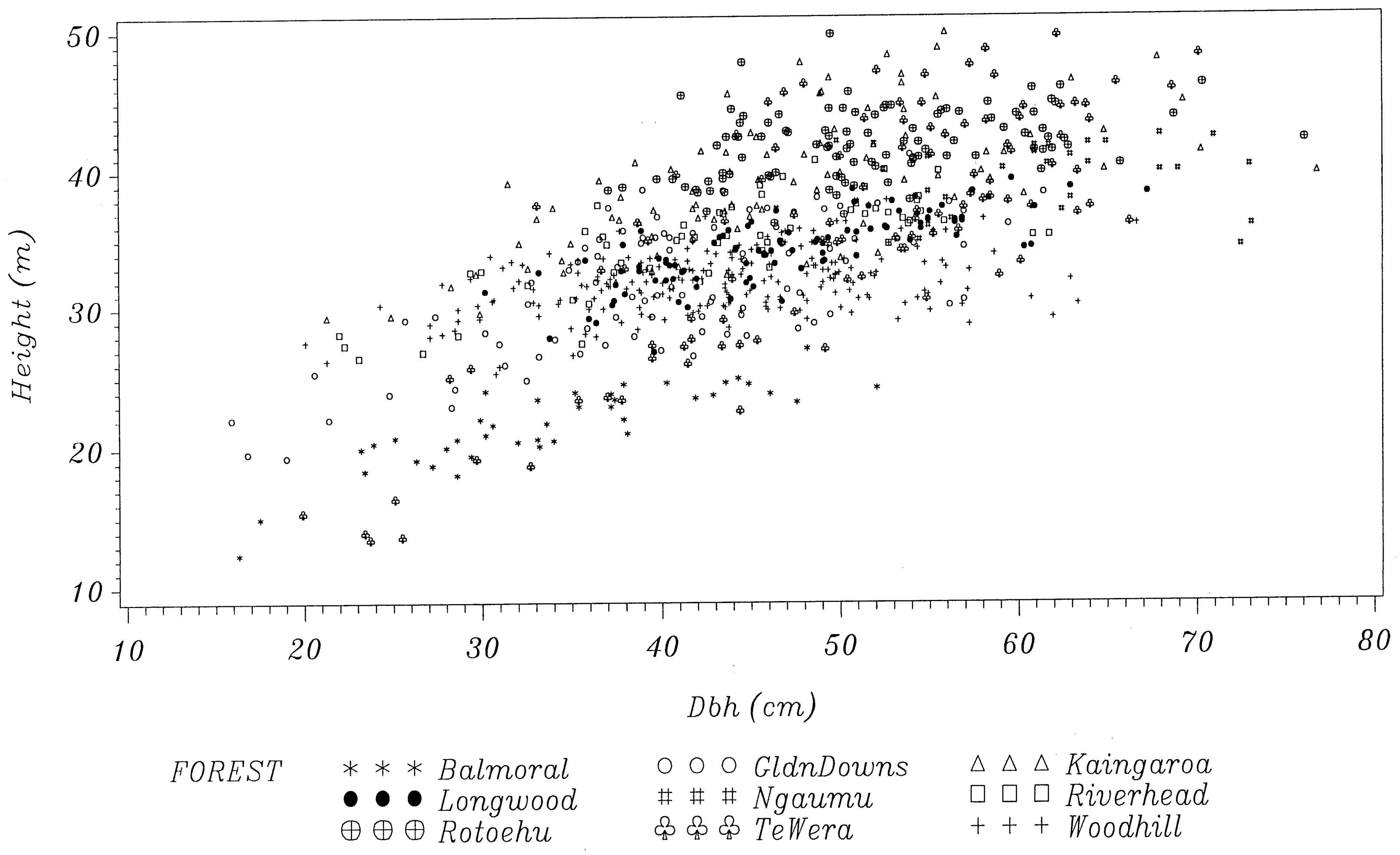


Figure 3. Relationship between TSVub and Dbh

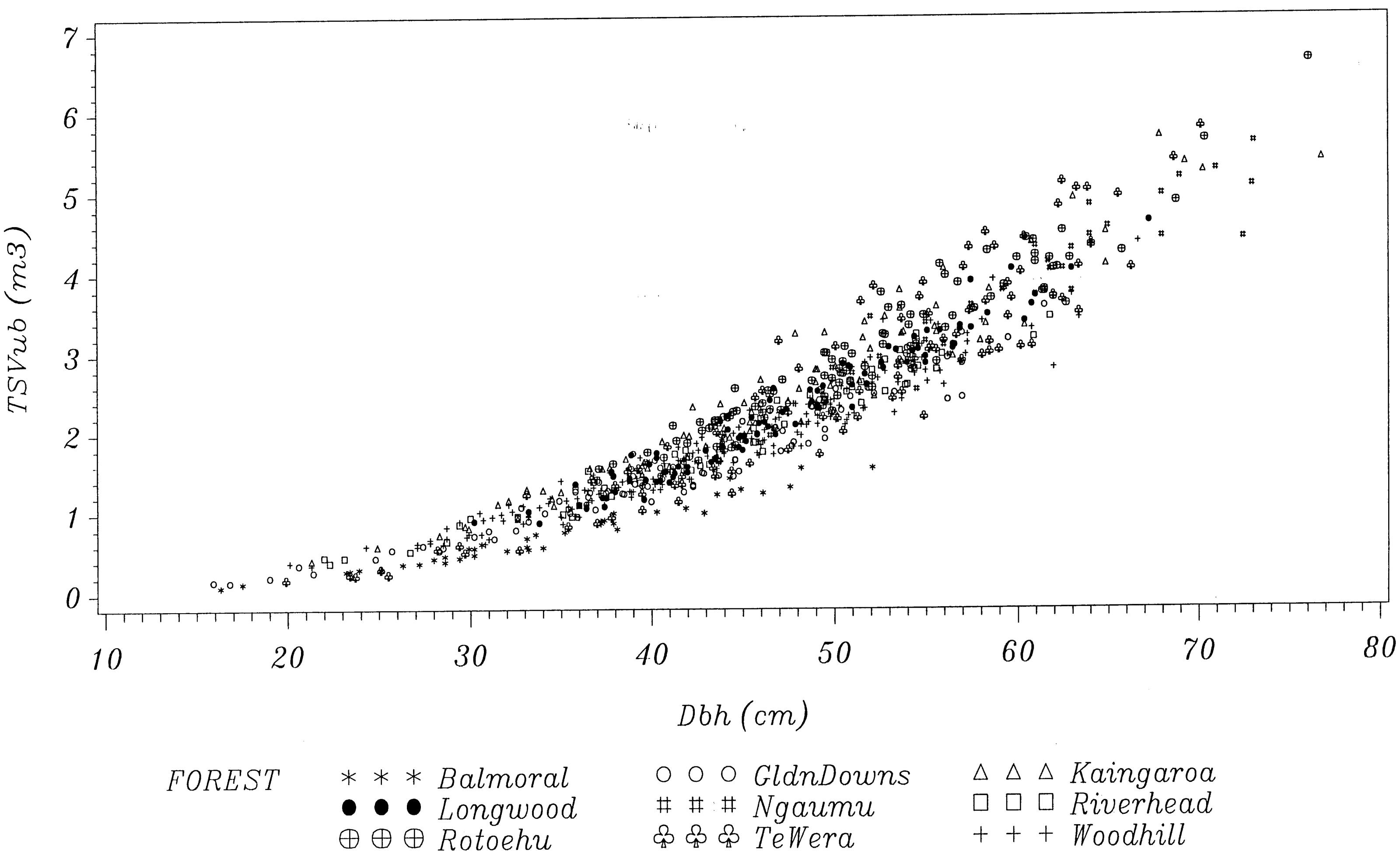


Figure 4. Relationship between Form-Factor and Dbh

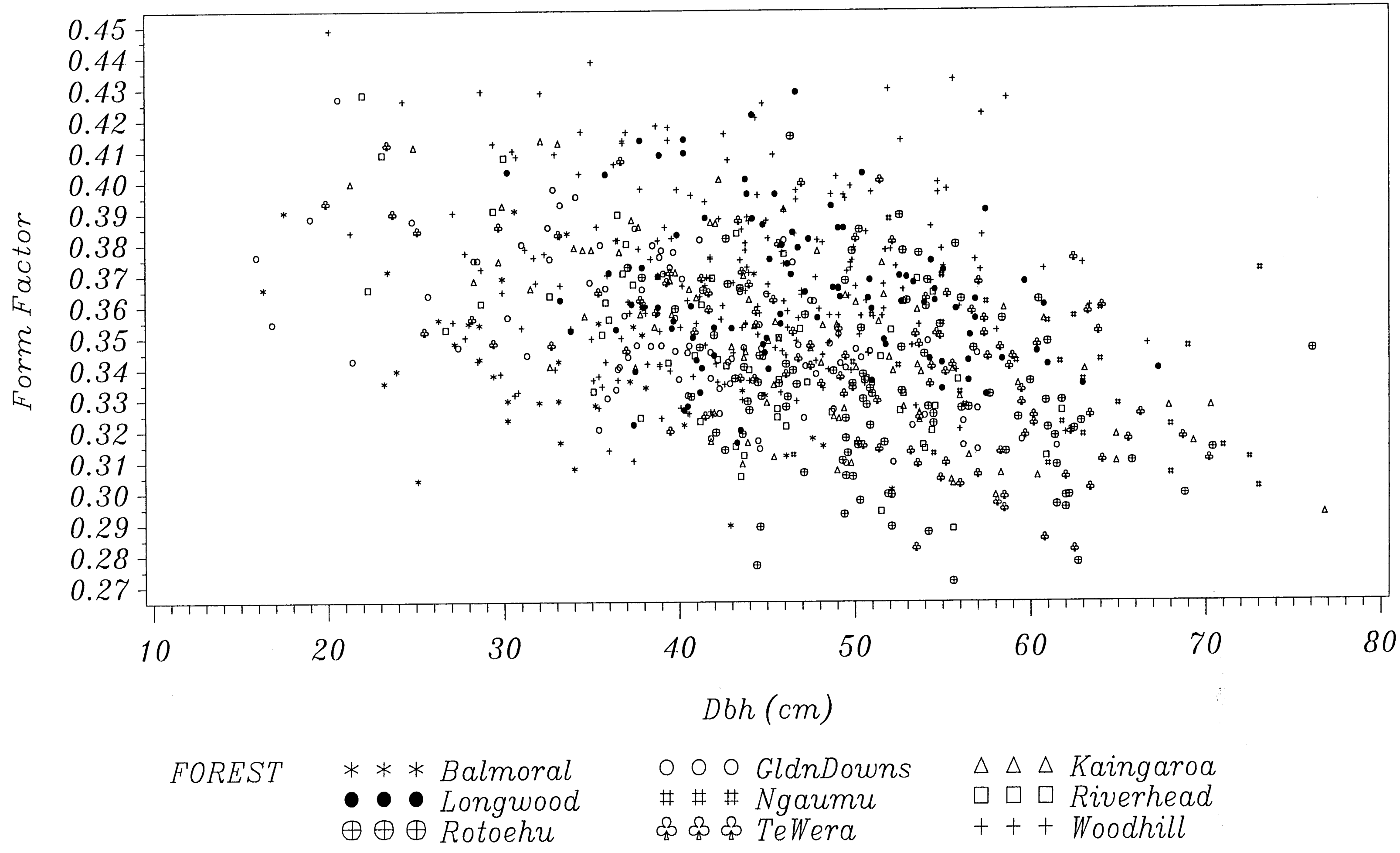


Figure 5. Relationship between Form-Quotient and Dbh

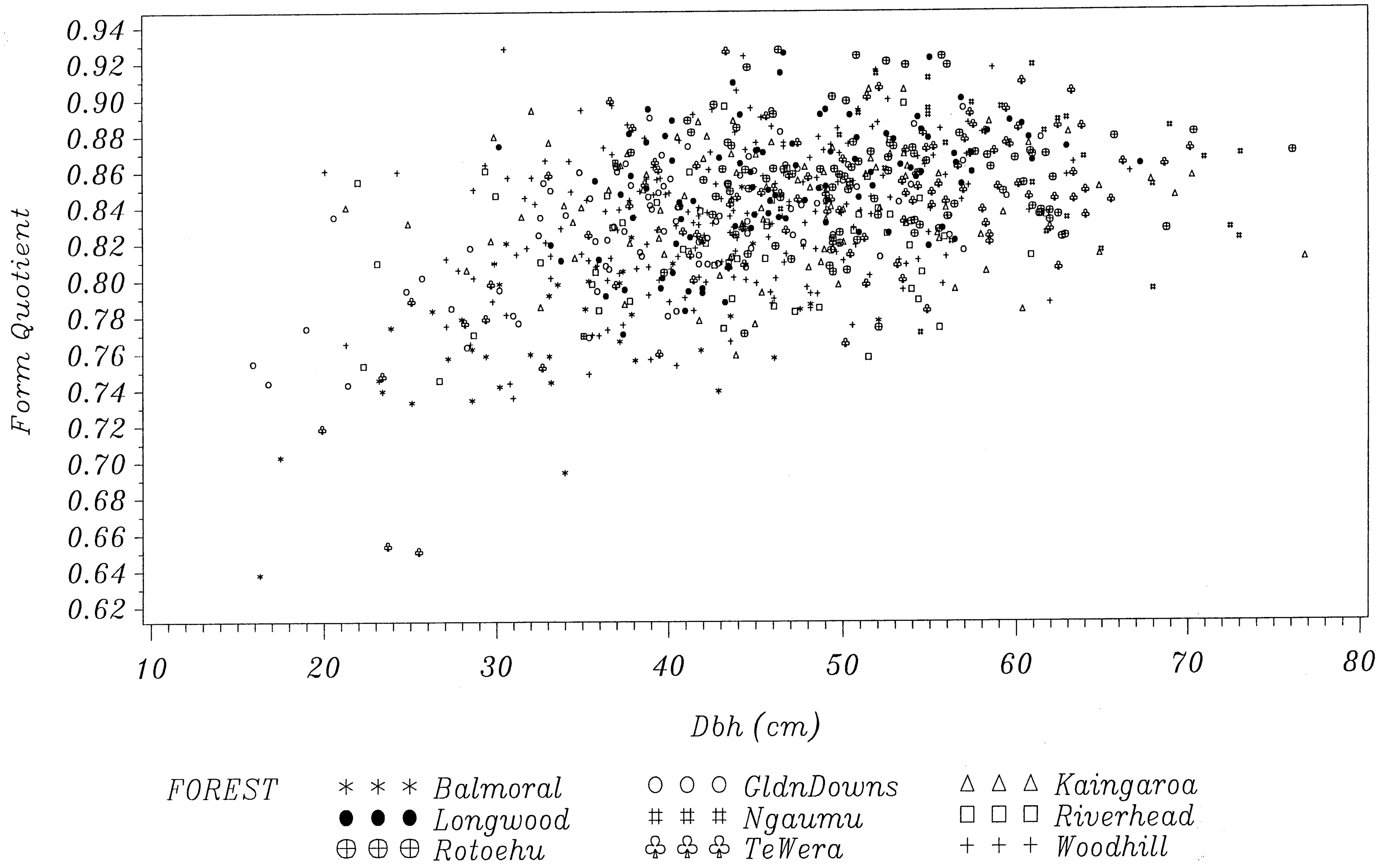


Figure 6

Percentage Error in volume estimation against Dbh
Data Set 1. Equation 2

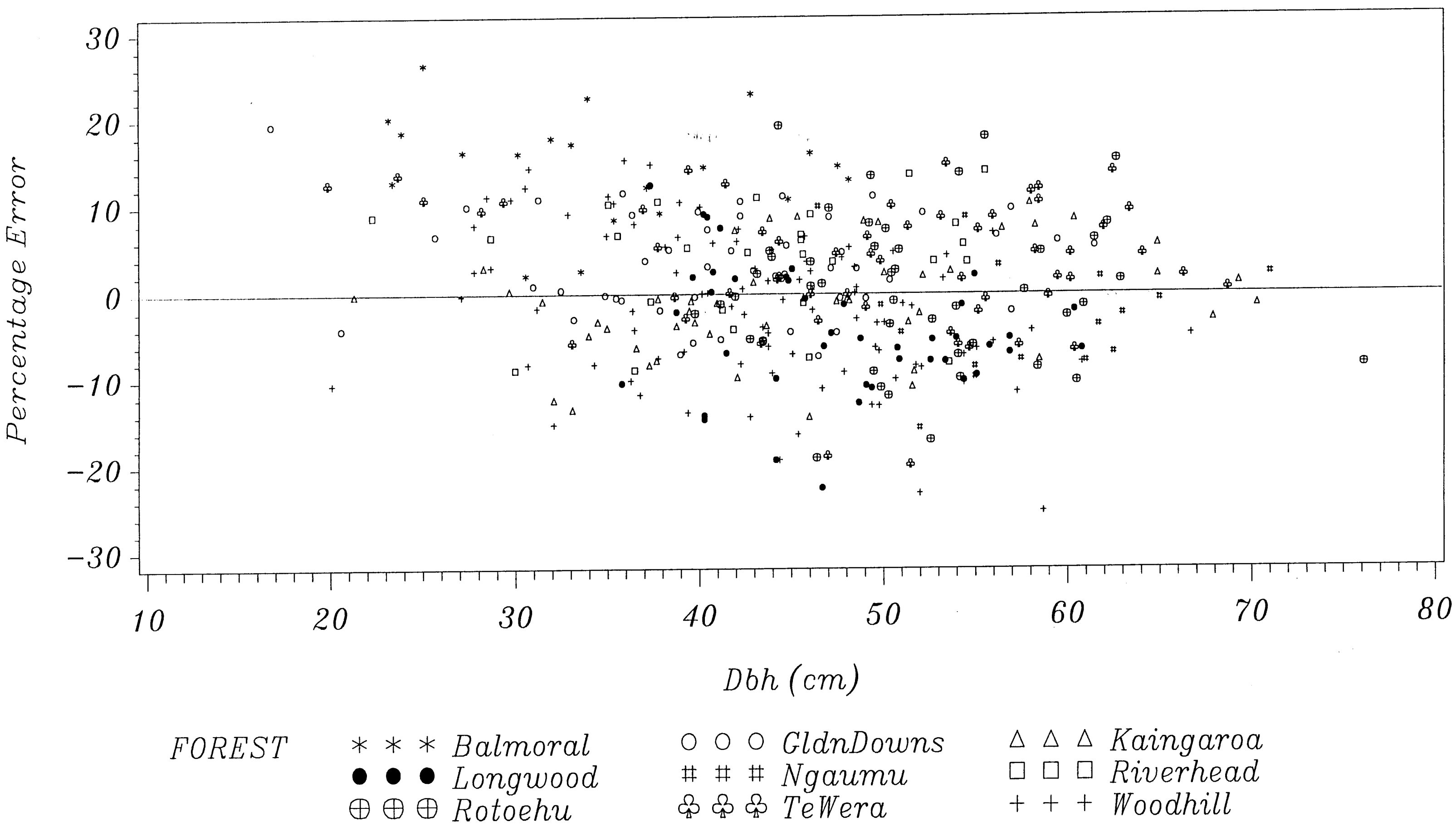


Figure 7

Percentage Error in volume estimation against Dbh
Data Set 1. Equation 5

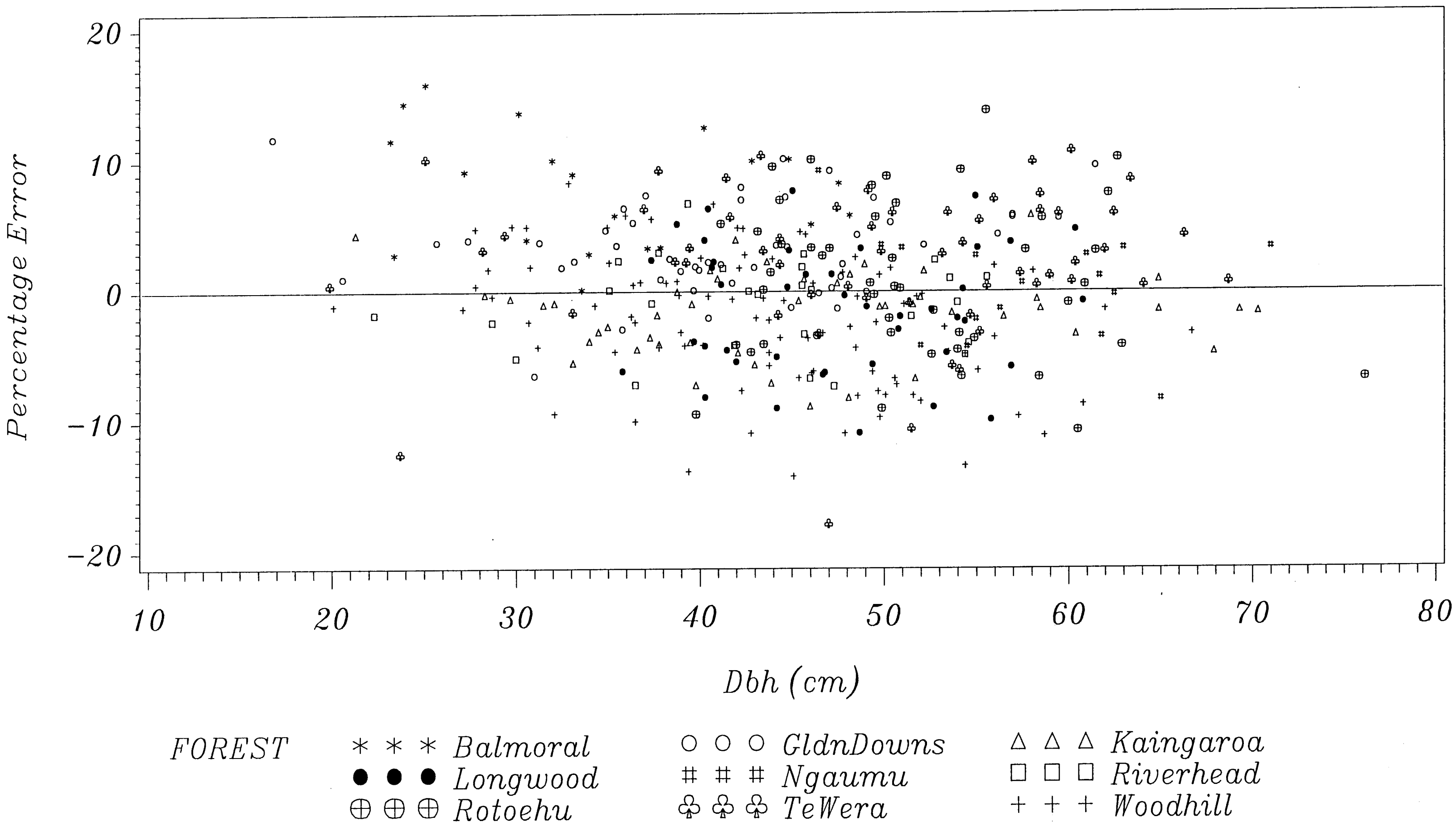


Figure 8

Percentage Error in volume estimation against Dbh
Data Set 2. Equation 2

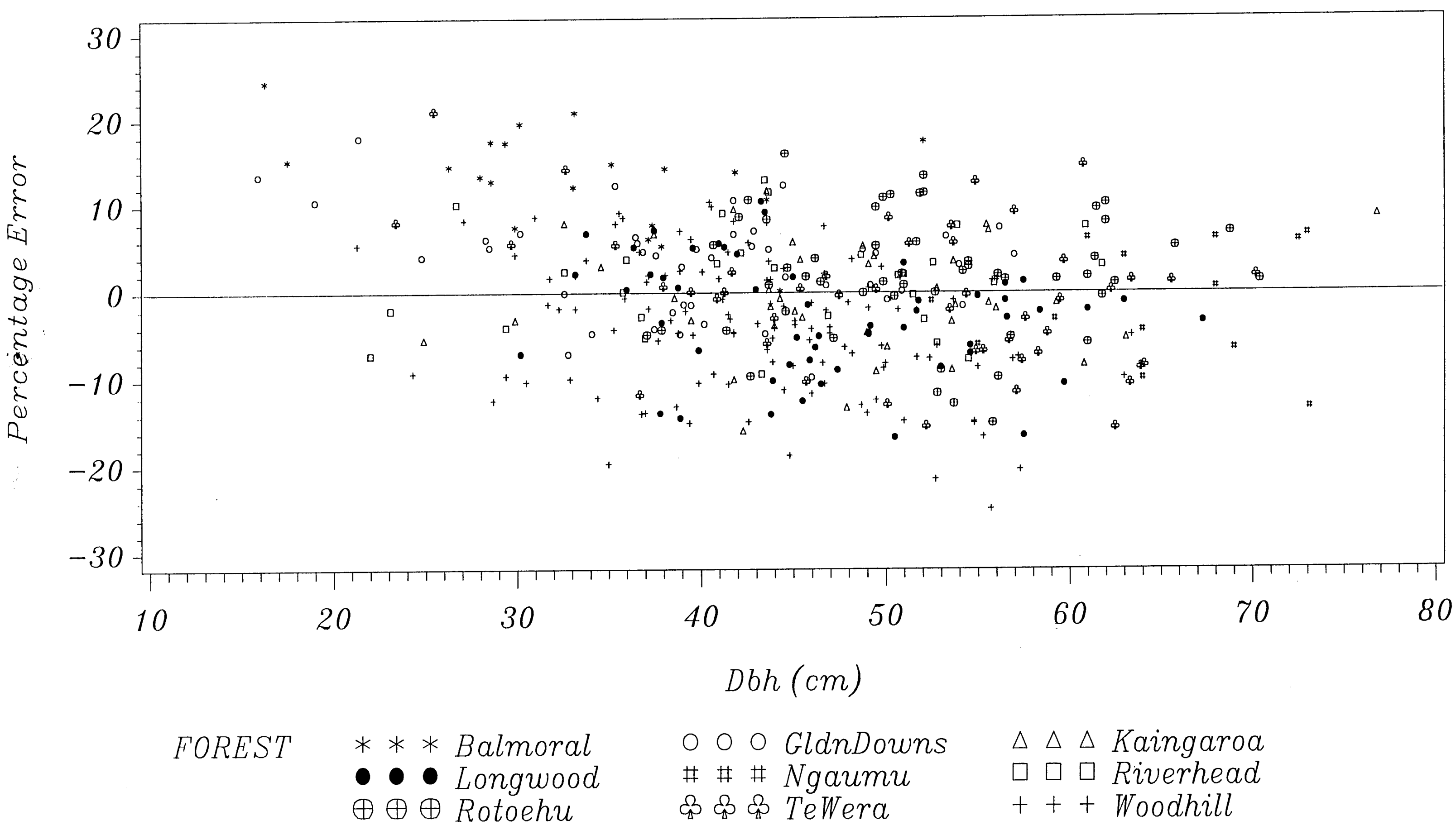


Figure 9

Percentage Error in volume estimation against Dbh
Data Set 2. Equation 5

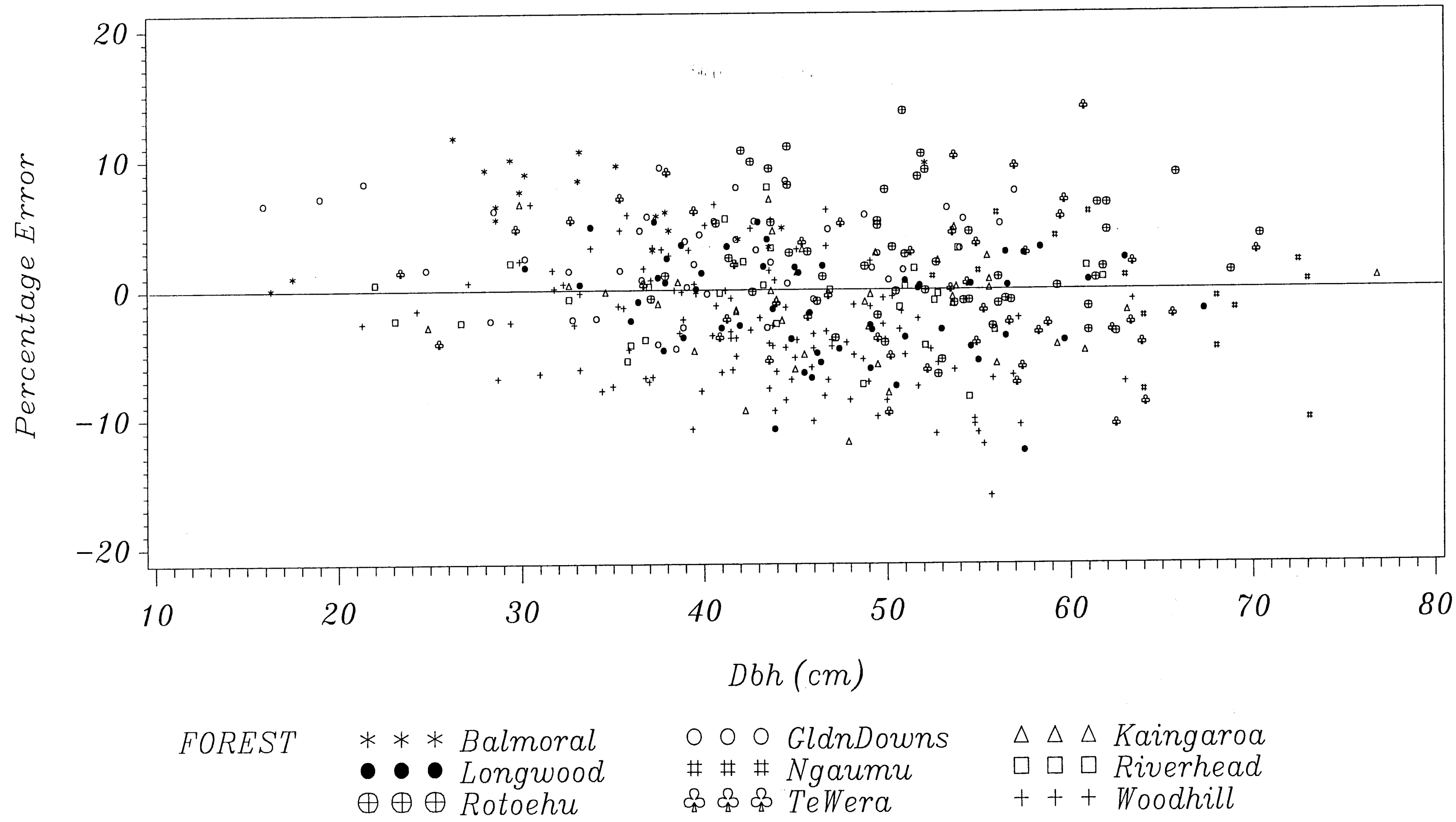
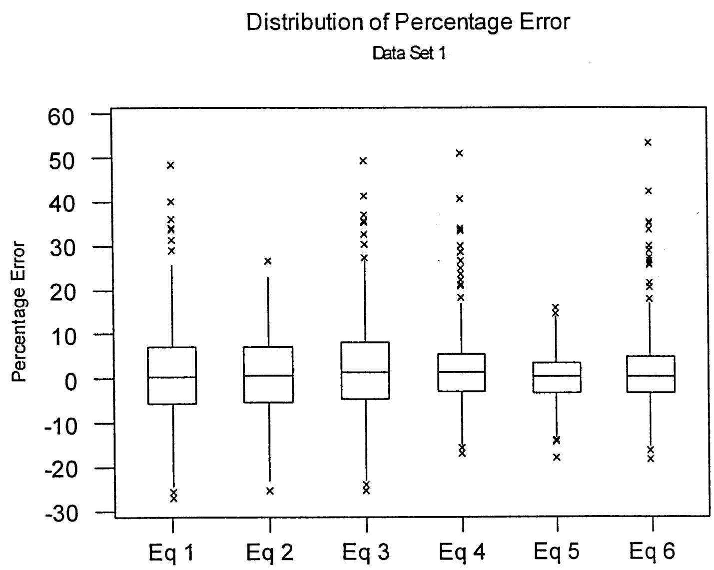
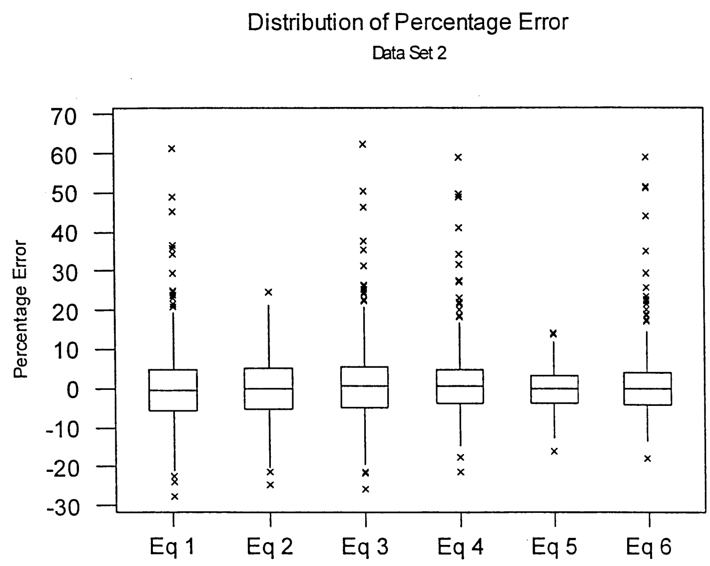


Figure 10. Box and Whiskers plot for distribution of individual tree percentage error (Data set 1)



PS: Crosses outside the IQ range symbolise the outliers of each boxplot

Figure 11. Box and Whiskers plot for distribution of individual tree percentage error (Data set 2)



PS: Crosses outside the IQ range symbolise the outliers of each boxplot