

**Nelson Study for Internal  
Stem Modelling:  
Evaluation of Relationships  
between Crown, Ring Area and  
Basic Density**

D. Pont

***Report No. 132***

***October 2006***

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NOTE : Confidential to participants of the Stand Growth Modelling Cooperative.  
: This is an unpublished report and must not be cited as a literature reference.

# EXECUTIVE SUMMARY

This report covers the analysis of crown/stem relationships in data collected from experiment NN530/2 Golden Downs. Eight trees, age 26, were destructively sampled from a GF14 (Gwavas seed orchard) seedlot in plot 10/51. Four pairs of trees were selected to have similar DBH but contrasting (high and low) outerwood density.

Crown structure was assessed in terms of foliage mass and average distance to foliage on 8 sample branches selected from each tree to cover the range of branch age and diameter. Stem ring area and basic density was measured by Silviscan analysis of 2 or 3 radial strips cut from discs taken at 7 levels up the stem of each tree. Models estimating ring area and density from crown structure were then evaluated.

The same model form evaluated in the pilot study (FR172/3, SGM Report 131) for stem ring area gave a good fit to the data (R-square 0.78). The relationship between stem basic density and crown structure appears to be more complex. The FR172/3 data set contained four seedlots and showed distinct tree-to-tree variation in the internal pattern of density. The NN530/2 data set was a single seedlot but showed strong year-to-year variation, assumed to be of environmental origin. These sources of variability have required the use of slightly different model forms on each data set to obtain a satisfactory fit. Fitting a model by year gave an R-Square of 0.70. These facts indicate that an accurate model for basic density will probably require incorporation of site and tree effects in addition to crown structure.

# Nelson Study for Internal Stem Modelling: Evaluation of Relationships Between Crown, Ring Area and Basic Density

**D. Pont**

## **BACKGROUND**

This report covers the analysis of crown/stem relationships in data collected from experiment NN530/2 Golden Downs in December 2004 as part of the Internal Stem Modelling theme for the Stand Growth Modelling Cooperative. An earlier report (SGMC Report 131) covers the analysis of the first set of data, collected in experiment 172/3, Kaingaroa.

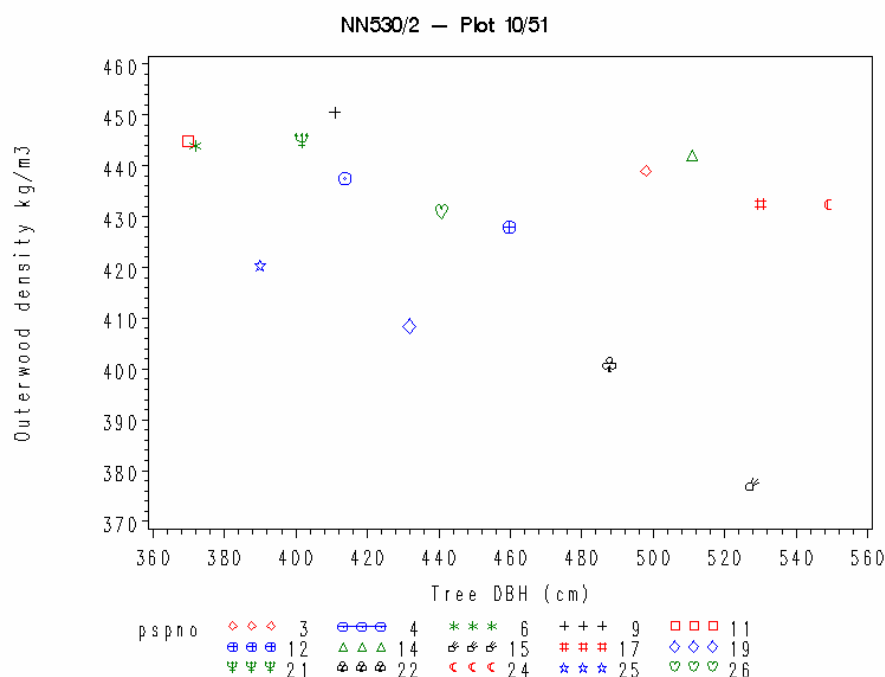
## **MATERIALS**

Data collection was carried out in experiment NN530/2, planted in 1978, compartment 66, Golden Downs forest. Eight trees were destructively sampled from the GF14 Gwavas seed orchard seedlot in plot 10/51. Four pairs of trees were selected to have similar DBH but contrasting (high and low) outerwood density. The trees were felled in December 2004. Table 1 lists the selection characteristics for the sample trees.

**Table 1. Sample trees from experiment NN530/2**

Tree	DBH (cm) August 2004	Density (kg/m <sup>3</sup> )	Subjective field notes
<b>Pair 1</b>			
25	40.0	420.5	Fine branching Leaning a bit uphill
9	42.0	450.5	Bigger branches Reasonably straight
<b>Pair 2</b>			
19	44.5	408.5	Longer internodes Straight
26	44.5	431.0	Fine branches Crown suppressed on one side Butt sweep
<b>Pair 3</b>			
22	50.5	401	Big branches for size Lots of clusters Straight
3	51.0	439	Heavy crown Leaning uphill
<b>Pair 4</b>			
15	53.5	377.5	Some big branches Asymmetrical crown Straight/maybe leaning
24	55.5	432.5	Bigger branches Leaning uphill

In Figure 1 sample tree outerwood density is plotted against DBH, illustrating the range and variation in these two variables for all trees in the plot.



**Figure 1. Relationship between breast height outerwood density and tree DBH.**

## METHODS

All branches on the stem were measured for diameter, and ring counts made below each cluster were used to assign ages to branches. Branch data could not be collected and ring counts were not made in the 5m pruned zone on all trees. A large and small living undamaged branch was selected for foliage measurements in each quarter of living crown (a total of 8 branches for each tree). For each branch total mass and distance from basal end to centre of gravity were measured with and without foliage. Mid-internode discs for SilviScan samples were cut at the following locations below the crown: stem base (approx. 0.5m); breast height (approx. 1.4m); half way between breast height and base of crown; and within the crown: base; 1/4, 1/2 and 3/4 positions. This resulted in 7 discs per tree. 2, or in some cases 3, radial strips were obtained from each disc by cutting a diametral strip passing through the most obvious compression wood and for the larger discs a radial strip was cut at right angles.

## RESULTS

The analysis consists of 3 main stages:

### Crown development

estimating past growth of each branch in the crown at each prior age of tree growth

### Branch foliage

relating branch growth with the amount and average distance to foliage

### Crown / Stem relationships

relating stem wood properties for each growth ring and position on the stem with the foliage carried above that point on the stem at that age

## CROWN DEVELOPMENT

### Branch Growth

Reconstruction of past branch diameters was carried out for all branches using the branch growth functions from TreeBLOSSIM for the Nelson region (SGMC Report 125). The first step was to estimate branch growth potential using the measured branch diameter and age. The branch growth function is not easily rearranged to obtain growth potential given age and diameter so growth potential was solved using an iterative search process. The sample plot received two thinnings: from 1111sph to 600sph at age 8 and to 300sph at age 10. The final stocking was used in estimating growth potentials as it was difficult to correctly incorporate branch thinning response in the iterative solution and it was thought the effect on overall branch growth would be minor. Given the growth potential for a branch its diameter could then be estimated at any age.

## BRANCH FOLIAGE

### Distance to Branch Foliage

Branch average distance to foliage ( $D_{fb}$ ) was calculated from the field branch measurements of mass and distance to centre of mass (CM), with and without foliage (see SGMC Report No. 131 for details on the calculation) for all eight trees combined and plotted against branch diameter (Figure 2a) and branch basal area (Figure 2b). All trees followed a similar trend, comparable with that observed for the FR172/3 data set.

The same model used for the FR172/3 data set was fitted in SAS using PROC NLIN:

$$D_{fb} = aG_b^c \quad \text{Equation 1}$$

where:

$D_{fb}$  is average distance to foliage (mm) for branch  $b$

$G_b$  is basal area ( $\text{mm}^2$ ) for branch  $b$

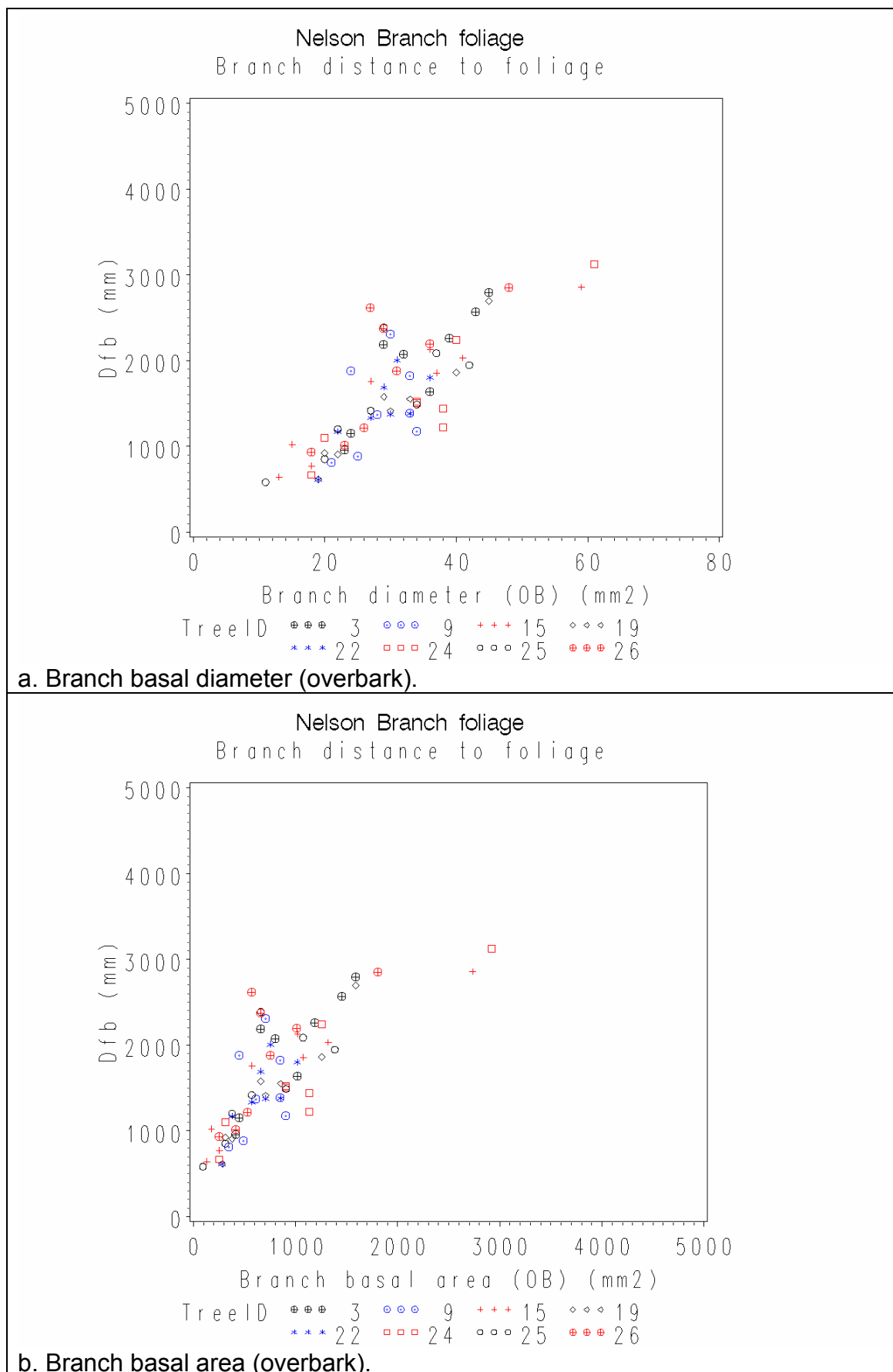
$a$  and  $c$  are model parameters

The SAS output is shown below. The estimates for the parameters  $a$  and  $c$  are 60.9, and 0.50 respectively.

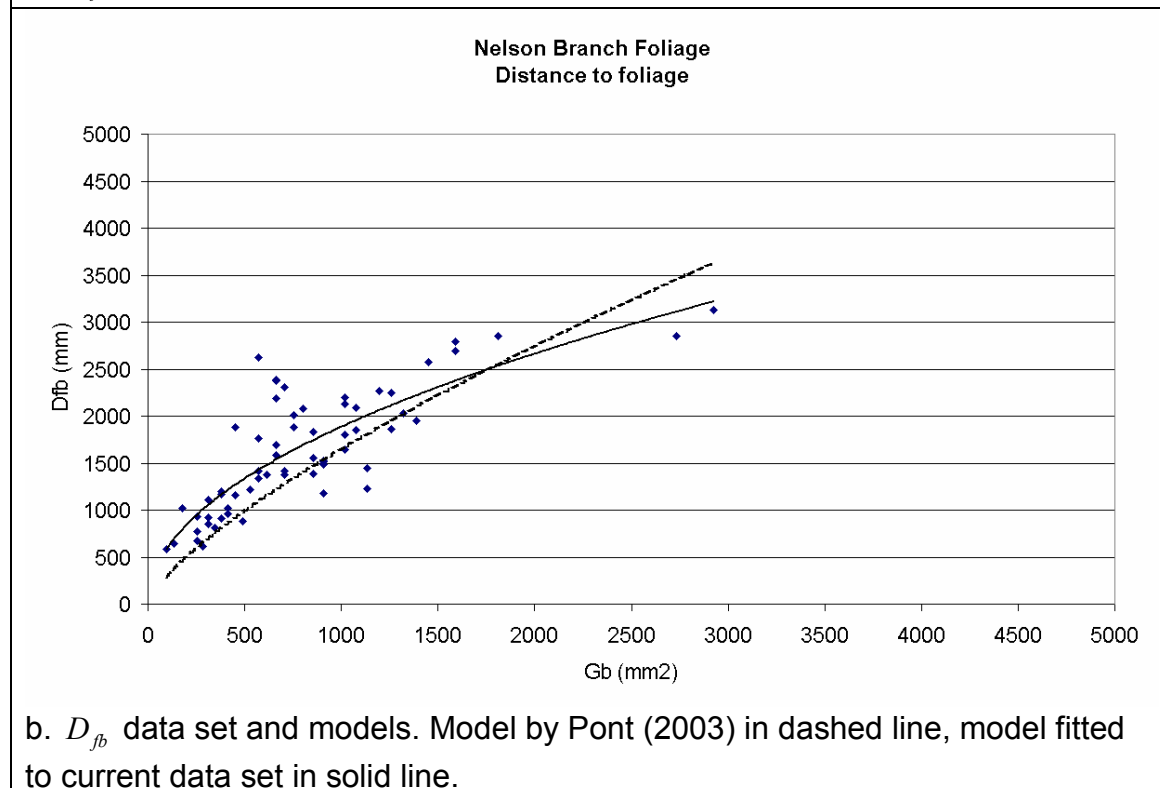
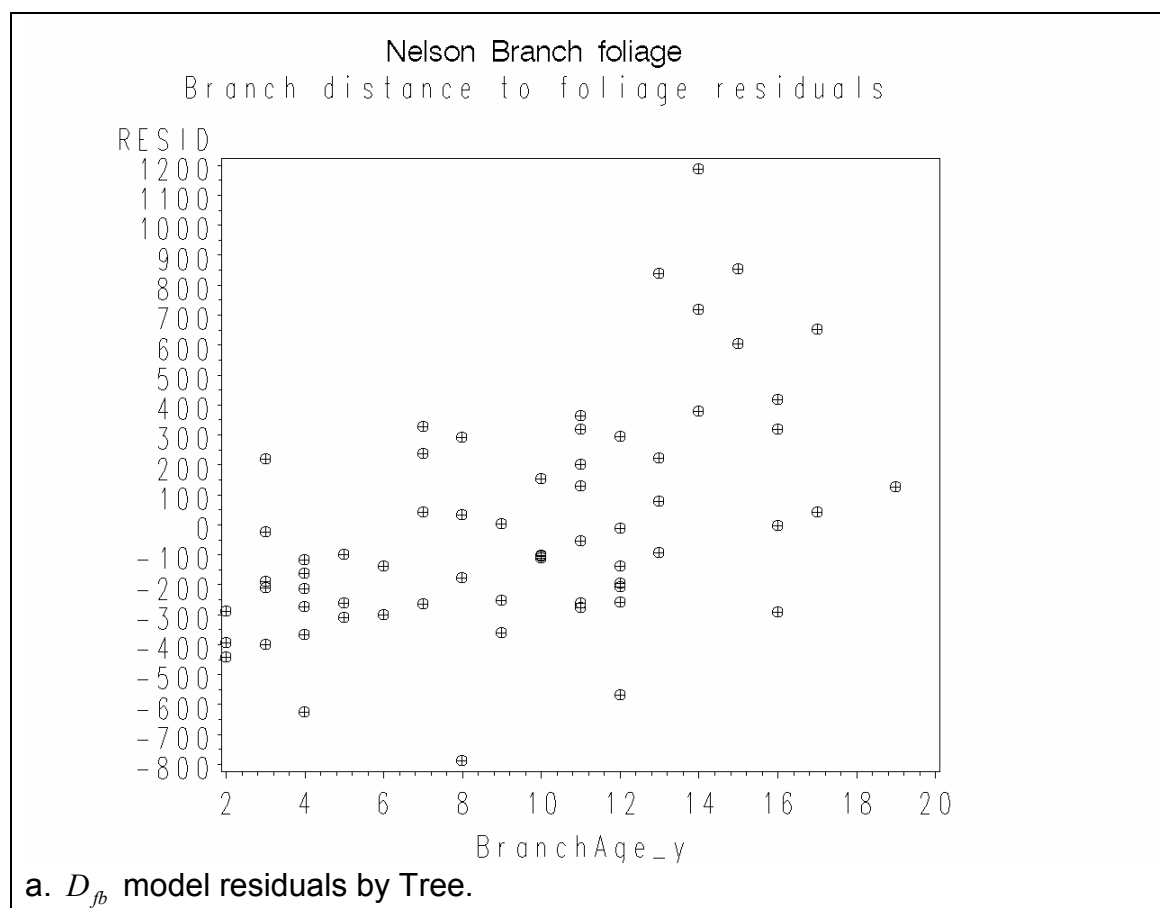
Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	2	1.8399E8	91993017	658.05	<.0001
Residual	62	8667371	139796		
Uncorrected Total	64	1.9265E8			
Corrected Total	63	26249110			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
a	60.9224	19.4364	22.0695	99.7753
c	0.4971	0.0462	0.4047	0.5894

The R-square of this model was 0.67 and the only trend noted in the residuals was for branch age (Figure 3a). Figure 3b shows the data and the fitted regression line for the  $D_{fb}$  model (Equation 1).



**Figure 2. Average distance to foliage plotted against branch basal diameter and basal area.**



**Figure 3. Results for fit of  $D_{fb}$  model to Nelson data.**



The parameters are similar to those for the FR172/3 data set and to those of Pont (2003), the latter having a steeper curve (Figure 3b). Due to the limited amounts of data at this early stage of model development separate parameters for the two sites will be retained.

### Branch Foliage Mass

Initial examination of branch foliage mass  $W_{fb}$  showed no clear relationship with branch basal area increment  $\Delta G_b$  (Figure 4a), as used by Pont (2003) and a reasonable relationship with branch basal area  $G_b$  (Figure 4b). A similar issue arose with the FR172/3 data set, resulting in evaluation of alternative models before settling on a model based on  $\Delta G_b$ , albeit with a poorer fit to the data. The relationship in the current data set is so poor that it was not possible to fit a model for  $W_{fb}$  based on  $\Delta G_b$  alone and models forms investigated with the FR172/3 data set were re-evaluated with the current data set. One of the models related branch foliage mass with basal area increment (representing branch growth) and branch basal area (representing maintenance) (Equation 2).

$$W_{fb} = a\Delta G_b + bG_b \quad \text{Equation 2}$$

where:

$W_{fb}$  is weight of foliage (g) for branch  $b$

$\Delta G_b$  is basal area increment ( $\text{mm}^2$ ) for branch  $b$

$G_b$  is basal area ( $\text{mm}^2$ ) for branch  $b$

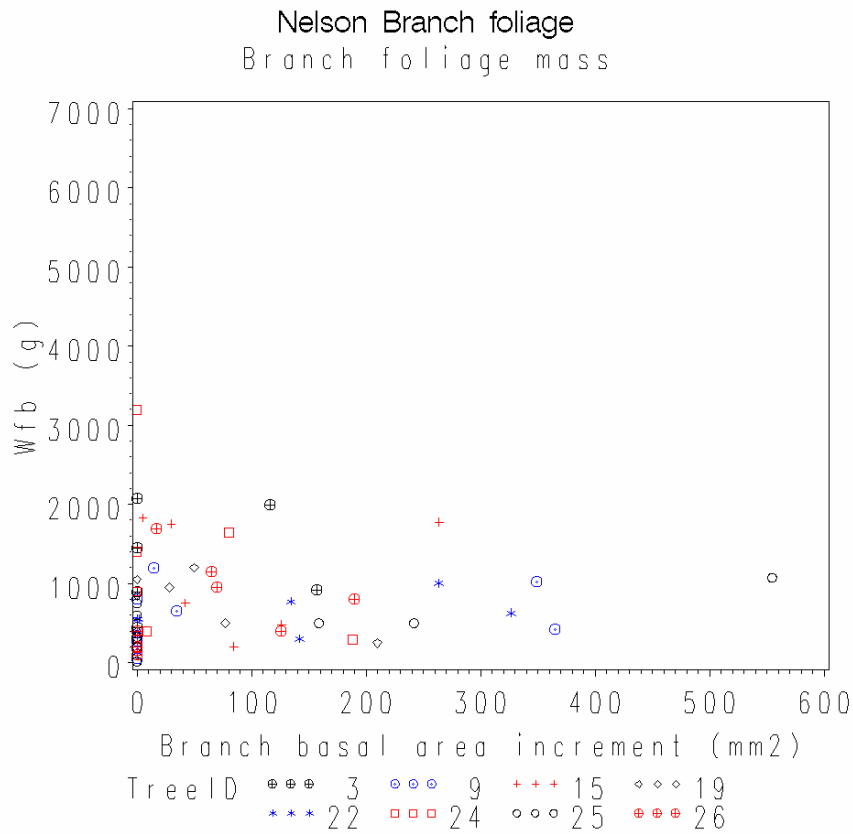
$a$  and  $b$  are model parameters

The model given in Equation 2 was fit to the current data set by utilising branches with no increment to first estimate  $b$  by fitting a simpler model  $W_f = bG_b$ , giving a value of 0.8078 for  $b$  (R-square 0.62). The estimated value for  $b$  was then inserted into Equation 2 and  $a$  estimated.

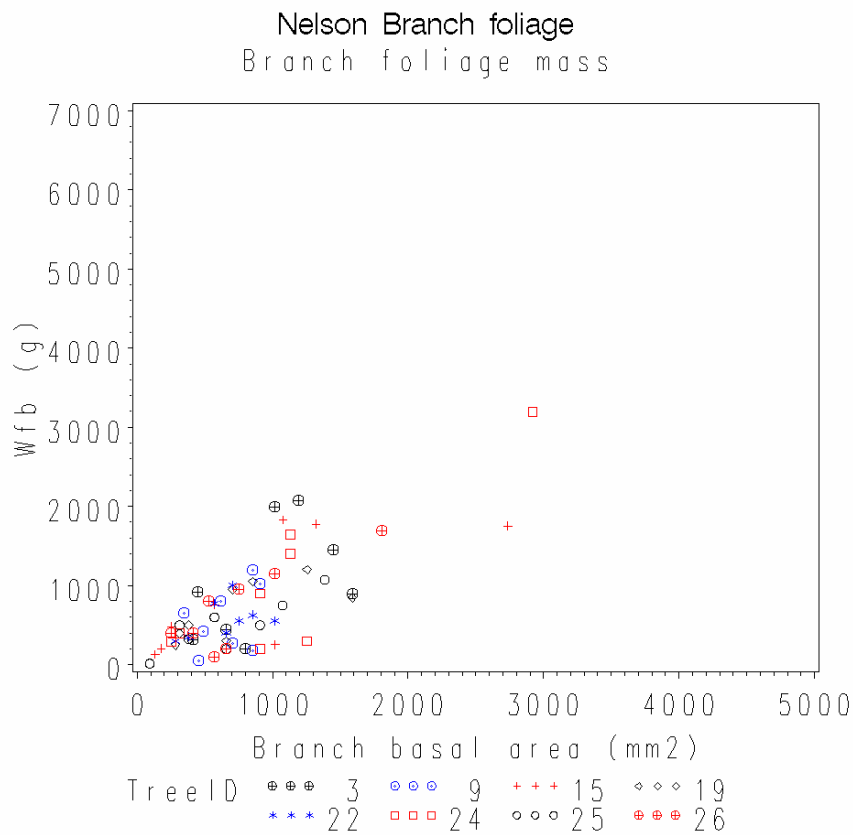
The SAS output is shown below. The estimate for  $a$  is 0.82.

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	1	49348611	49348611		
Residual	63	10023014	159095		
Uncorrected Total	64	59371625			
Corrected Total	63	23168840			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
a	0.8222	0.3715	0.0798	1.5646



a.  $W_{fb}$  plotted against branch basal area increment.



b.  $W_{fb}$  plotted against branch basal area.

**Figure 4. Branch foliage mass ( $W_{fb}$ ) relationships with branch growth.**

The final model had an R-square of 0.57 and accounted for the observed relationship between  $W_{fb}$  and  $G_b$  as well as the expected theoretical relationship between  $W_{fb}$  and  $\Delta G_b$ . Examination of residuals showed a trend associated with  $\Delta G_b$  (Figure 5a), probably a result of the influence of branches estimated to have zero current basal area increment. It was decided to accept the model as the best compromise at this stage. Figure 5b plots estimated  $W_{fb}$  from the fitted model against measured values from the sample branches.

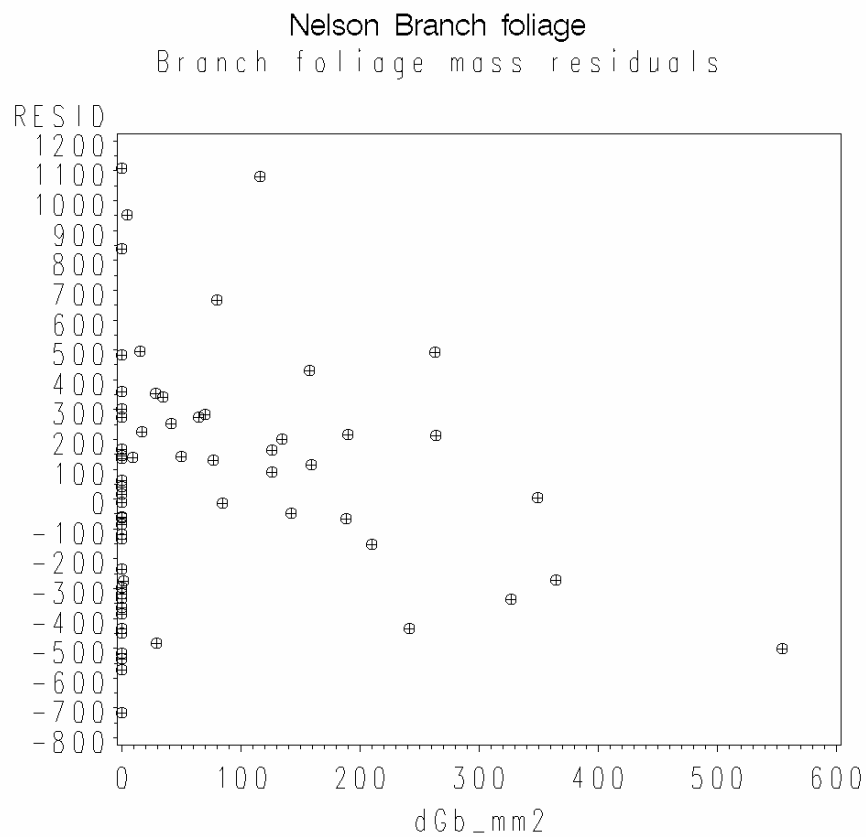
## CROWN / STEM RELATIONSHIPS

Using the relationships derived earlier the foliage mass and average distance to foliage could be estimated for every branch at any given year. Then for each tree the foliage mass  $W_{fi}$  and average distance to foliage  $D_{fi}$  could be calculated for a given year and position up the stem. A data set was produced containing estimates of  $W_{fi}$  (total foliage mass above position  $i$ ) and  $D_{fi}$  (average distance to foliage above position  $i$ ) corresponding to the measures of ring area and basic density obtained from SilviScan analysis of radial strips.

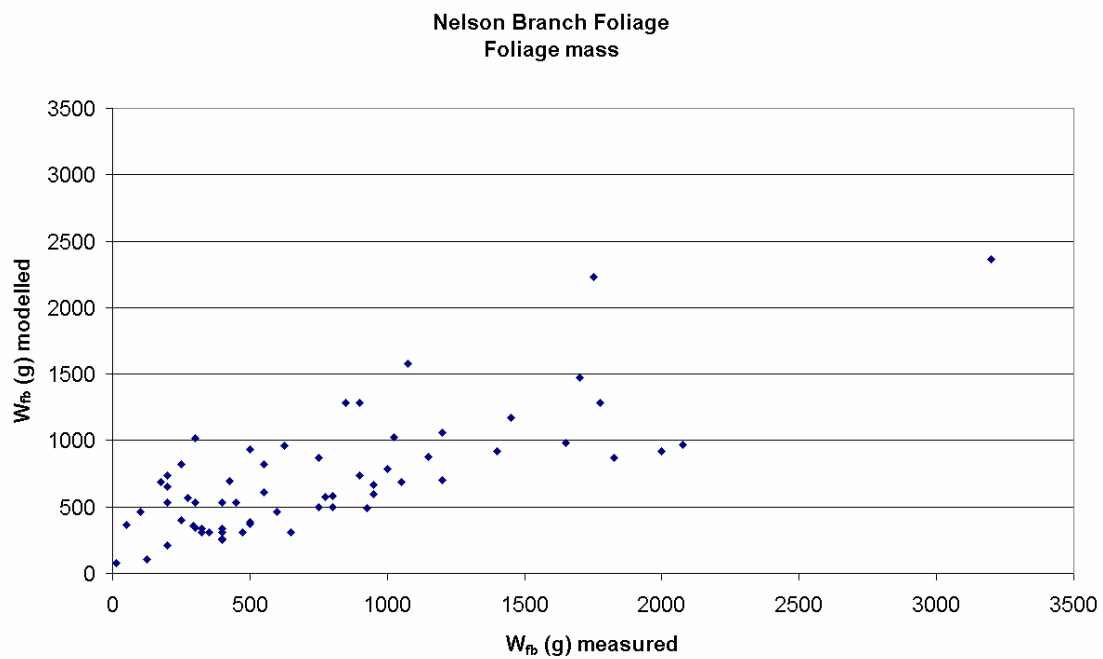
Stem annual ring area increment was calculated from SilviScan ring radii. It should be noted that these radii may be affected by shrinkage resulting from the ethanol extraction and drying carried out on samples in preparation for SilviScan. Annual ring area and basic density for each growth ring were averaged across multiple strip directions (2 or 3 strips) for each disc. Because of the pruning at the base of the trees it was not possible to utilise data from discs collected within the pruned zone (the lower 2 discs). Branch data was missing from the upper half of tree 9 (above cluster 65 at 16.2m). Data from the final year of growth was excluded from the data set as probably being an incomplete growth ring.

### Stem ring area

Examination of Figure 6 shows a slightly curvilinear relationship between ring area and foliage mass (Figure 6a) or  $kp = \frac{W_f}{D_{fi}}$  (Figure 6b), supporting the use of the same model form used in Pont (2003) and with the FR172/3 data (SGMC Report 131).



a. Model residuals by branch basal area increment.



b.  $W_{fb}$  model estimates plotted against measured values.

**Figure 5. Branch foliage mass model.**



**Figure 6. Stem ring area plotted against foliage mass above.**

The following model was fit using SAS PROC NLIN:

$$\Delta G = \frac{aW_f^b}{D_{fT}^c} \quad \text{Equation 3}$$

where:

$\Delta G$  is stem ring area (cm<sup>2</sup>)  
 $W_f$  is weight of foliage (kg) above a given position and at a given age  
 $D_{fT}$  is average distance to foliage (m) at ground level and at a given age  
 $a, b, c$  are the model parameters

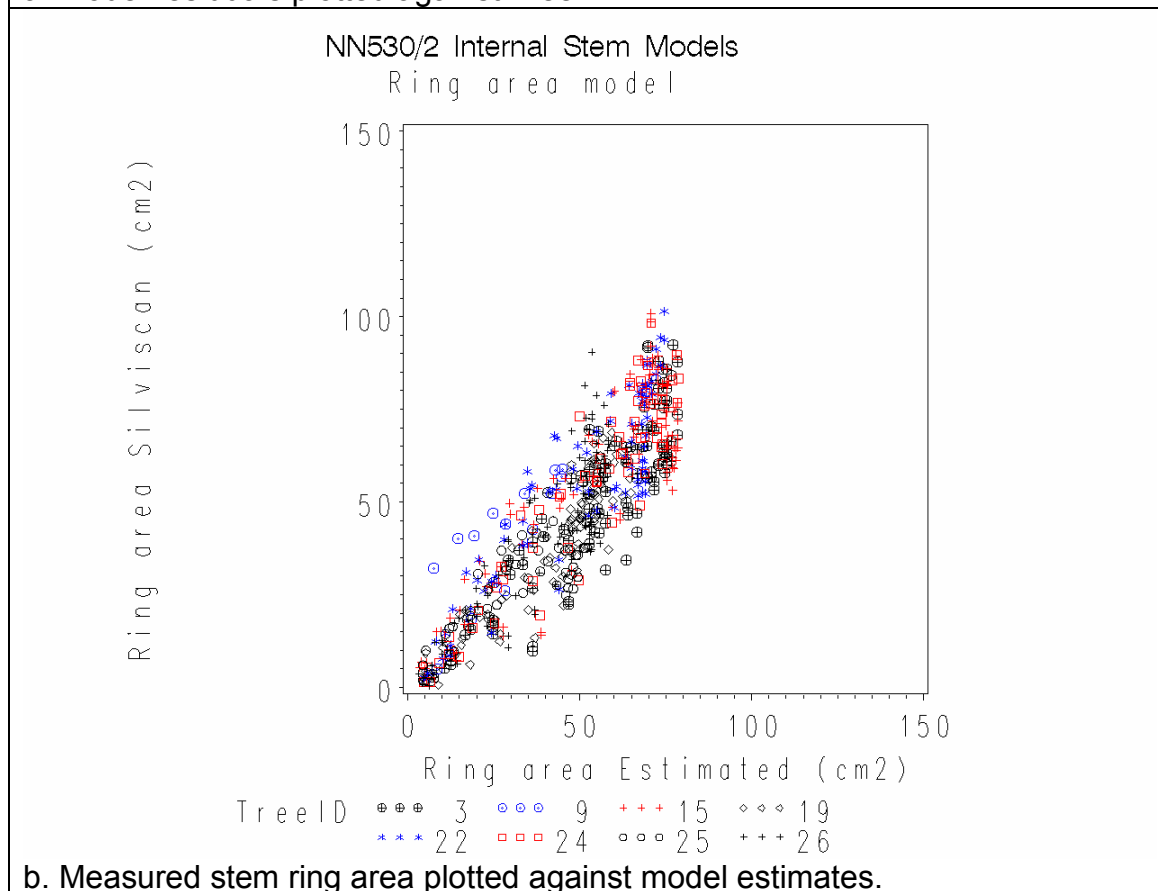
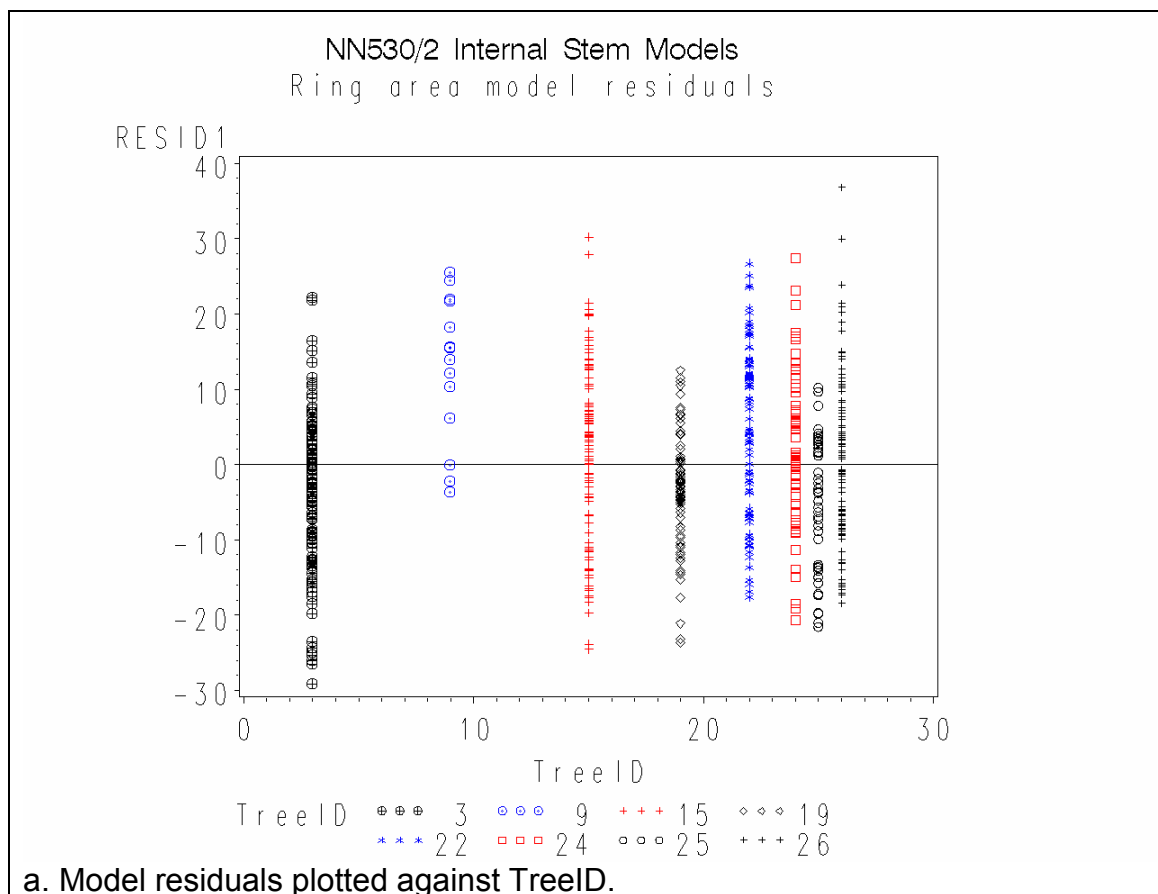
The SAS output is shown below. The estimates for the parameters  $a$ ,  $b$  and  $c$  are 29.9, 0.57 and 0.52 respectively.

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	3	1686301	562100	4523.89	<.0001
Residual	612	76041.9	124.3		
Uncorrected Total	615	1762343			
Corrected Total	614	347920			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
a	29.8856	1.9417	26.0724	33.6988
b	0.5732	0.0185	0.5368	0.6096
c	0.5212	0.0310	0.4603	0.5820

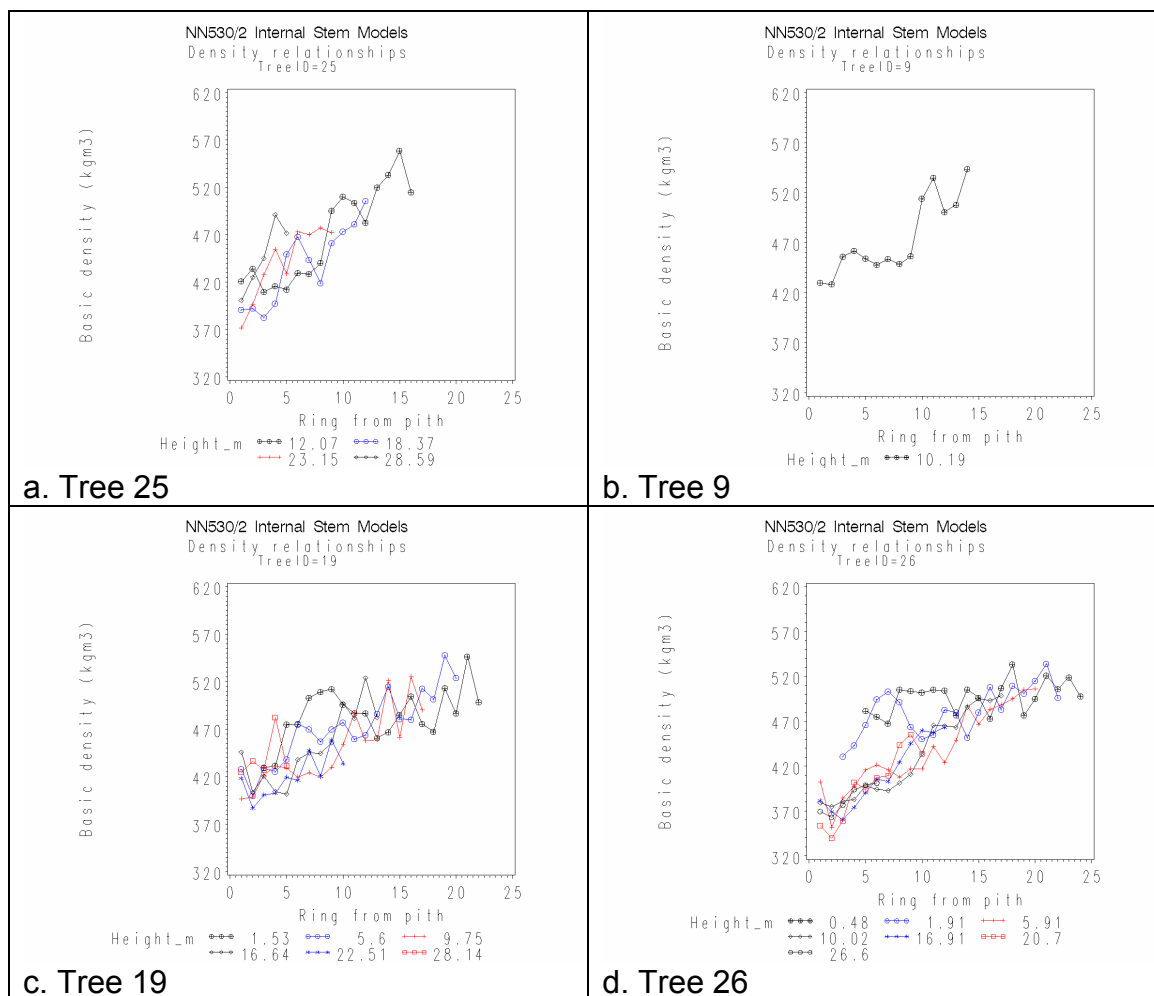
The R-square for the model of stem ring area was 0.78. Examination of residuals showed a slight association of errors with TreeID (see Figure 6 and 7a). A trend by tree was also noted with the FR172/3 data set and the decision was made to retain the model fitted to all trees for the current data set. Figure 7b shows measured stem ring area plotted against estimated ring area, showing the model predicts reasonably well.



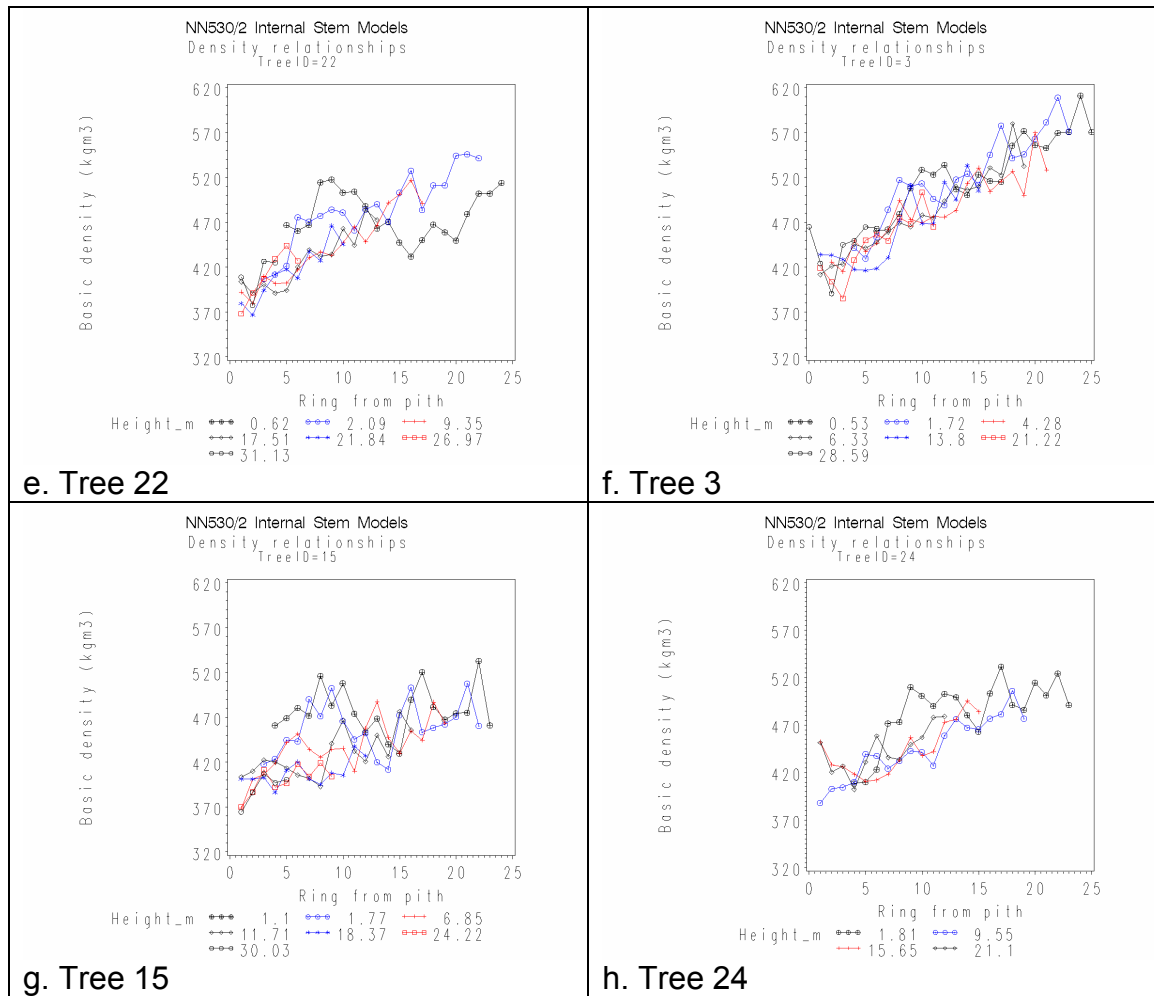
**Figure 7. Stem ring area model.**

## Stem Ring Basic Density

Initial analysis of the basic density data showed the trees exhibited different patterns of within stem variation, although not to the same degree as the FR172/3 data set. Figure 8 (a-h) presents the pith-to-bark series for each tree and examination shows elevated density values occur for some trees at the lowest disc level and near the pith, as noted for FR172/3, but less obvious because the lower part of the stem (approx. 5m) is absent. Excluding the lowest level the pith to bark series at different levels within a tree tend to be remarkably similar, with significant ring-to-ring variability.







**Figure 8. Pith to bark density profiles from SilviScan analysis of strips at different positions in the stem for each tree.**

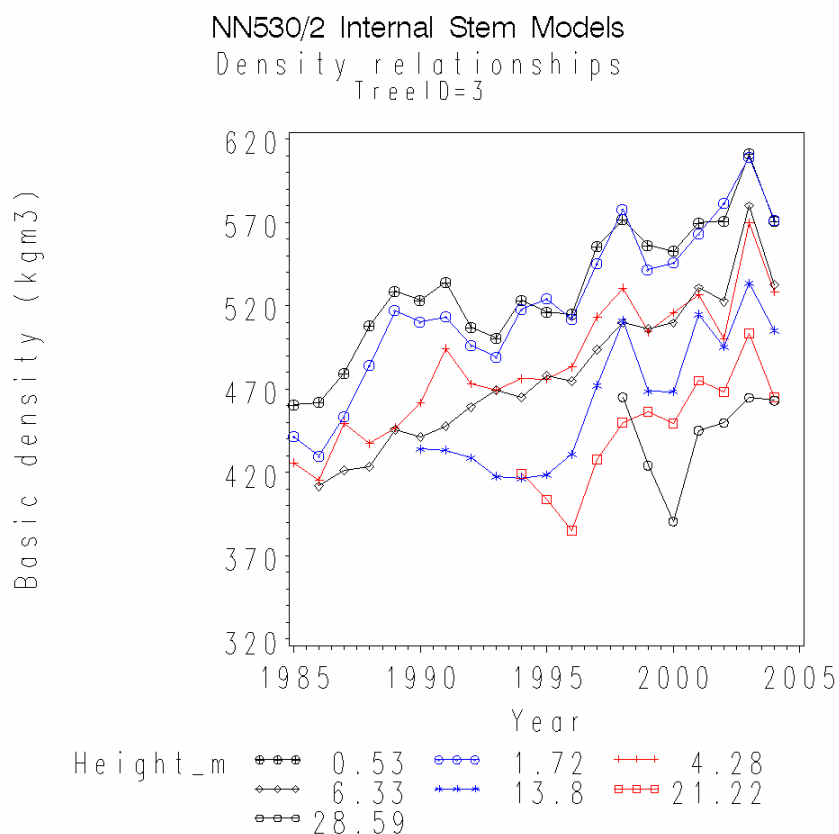
Figure 9 shows the pith to bark series for Tree 3 (Fig. 9a) and Tree 15 (Fig. 9b) plotted against year the ring was formed, illustrating the fact that the variability at different positions up the stem is synchronised, indicating a strong environmental influence on basic density. This effect is visible across all trees (for example 1998 and 2003 have elevated density, see Fig. 9a and 9b), but trees also have differing responses. There is no obvious effect associated with the two thinnings (age 8 in 1986, age 10 in 1988).

Initially a model of the form used by Pont (2003) was fit to the basic density data (Equation 4), relating basic density to the ratio of the average distance to foliage above and the total foliage mass for the tree.

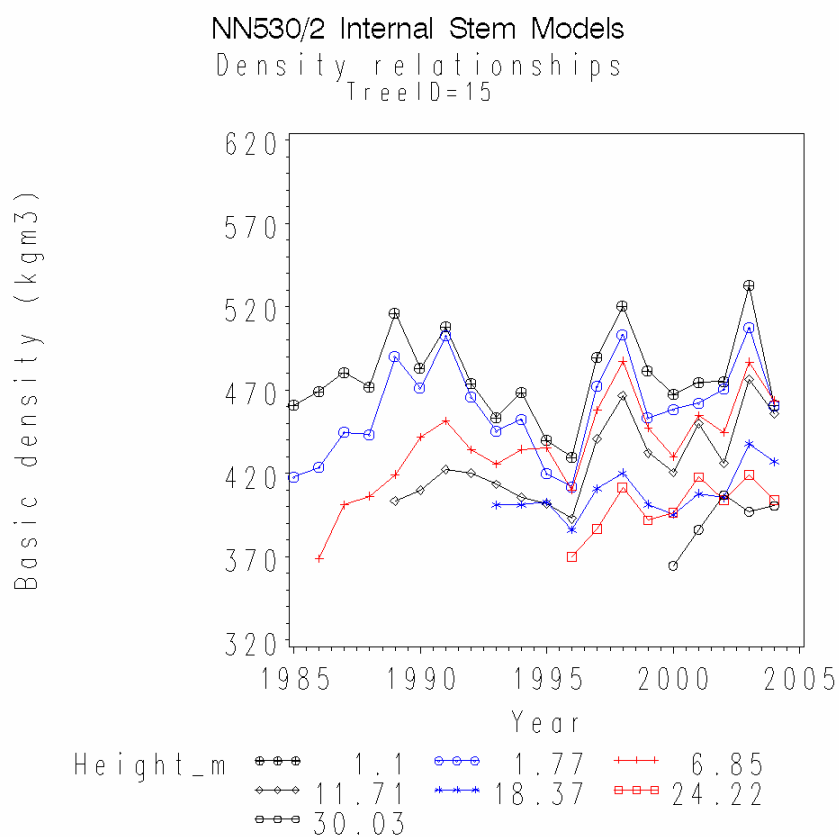
$$\rho = a \left( \frac{D_f}{W_{fT}} \right)^b \quad \text{Equation 4}$$

where:

- $\rho$  is stem ring basic density ( $\text{kgm}^{-3}$ )
- $D_f$  is average distance to foliage (m) above a given position and at a given age
- $W_{fT}$  is whole-tree foliage mass (kg) at a given age
- $a$  and  $b$  are the model parameters



a Tree 3.



b Tree 15.

**Figure 9. Pith to bark density profiles from SilviScan analysis of strips at different positions in the stem, plotted against Year.**

This model gave a poor fit to the data for all trees (R-square 0.31). Residuals showed strong patterns associated with a number of variables. Incorporating ring number from the pith into the model gave the following (Equation 5), as used with the FR172/3 data set:

$$\rho = aR^c \left( \frac{D_f}{W_{fT}} \right)^b \quad \text{Equation 5}$$

where:

$\rho$  is stem ring basic density ( $\text{kgm}^{-3}$ )  
 $D_f$  is average distance to foliage (m) above a given position and at a given age  
 $W_{fT}$  is whole-tree foliage mass (kg) at a given age  
 $R$  is ring number from the pith  
 $a, b, c$  are the model parameters

This model showed a slightly better fit (R-square 0.43) but the best fit was obtained with a simplification of Equation 4:

$$\rho = a + bD_f^c \quad \text{Equation 6}$$

where:

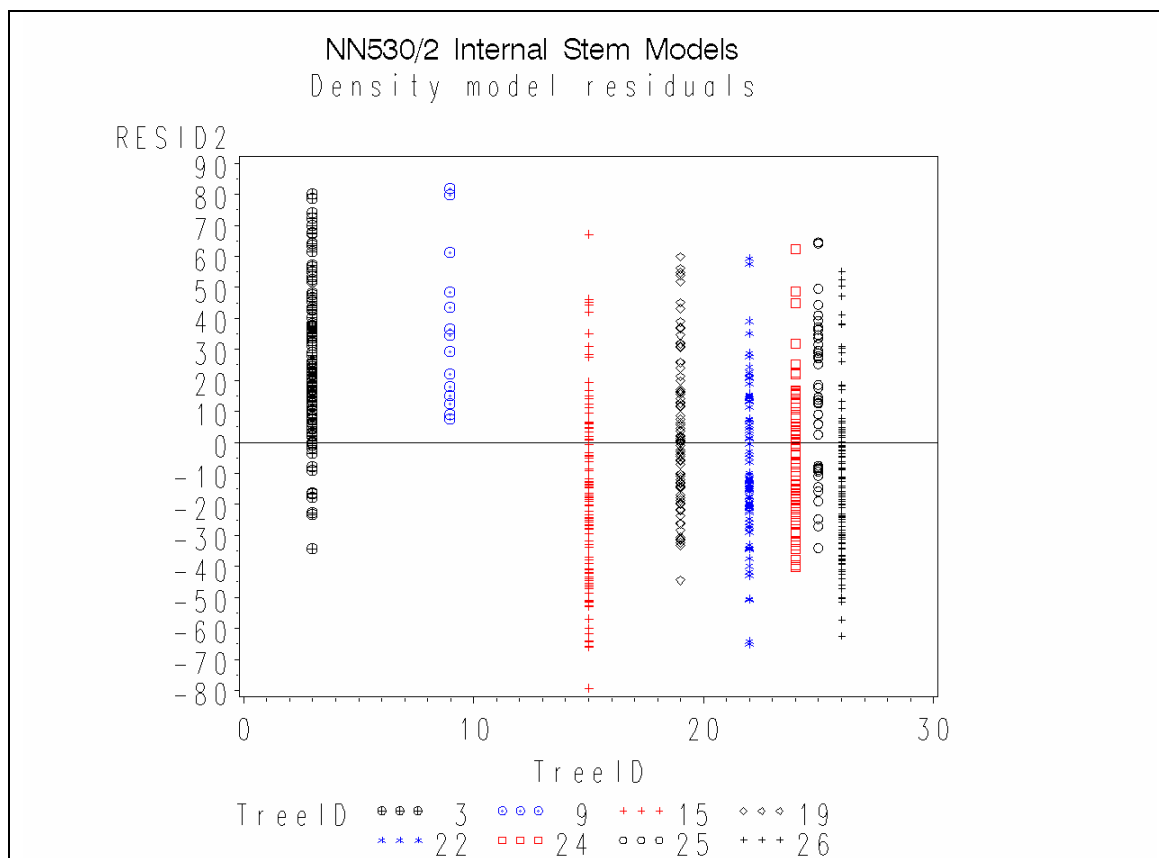
$\rho$  is stem ring basic density ( $\text{kgm}^{-3}$ )  
 $D_f$  is average distance to foliage (m) above a given position and at a given age  
 $a, b, c$  are the model parameters

The SAS output is shown below. The estimates for the parameters  $a$ ,  $b$  and  $c$  are 353.2, 47.5 and 0.39 respectively.

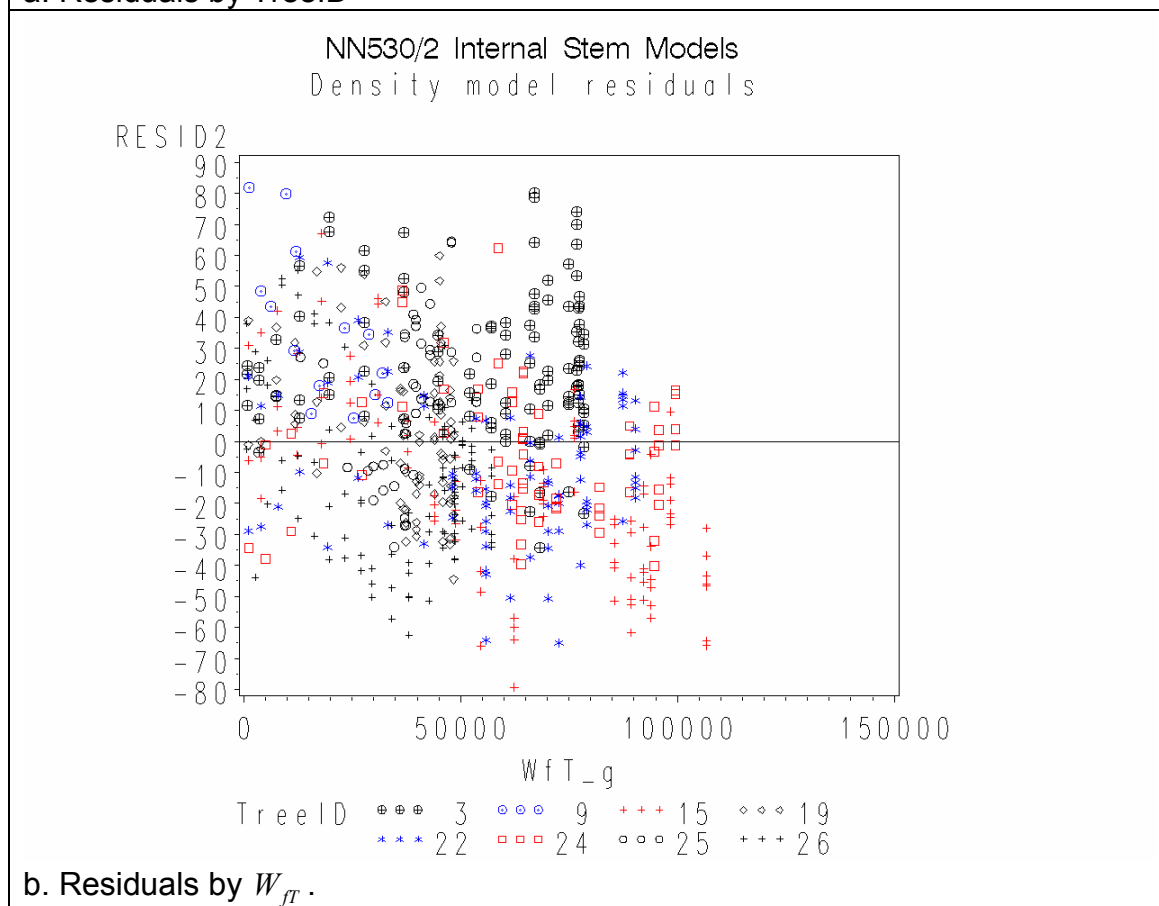
Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	3	1.2961E8	43204399	465.49	<.0001
Residual	612	524754	857.4		
Uncorrected Total	615	1.3014E8			
Corrected Total	614	1323010			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
a	353.2	15.3642	323.1	383.4
b	47.5228	13.3259	21.3523	73.6933
c	0.3905	0.0627	0.2673	0.5137

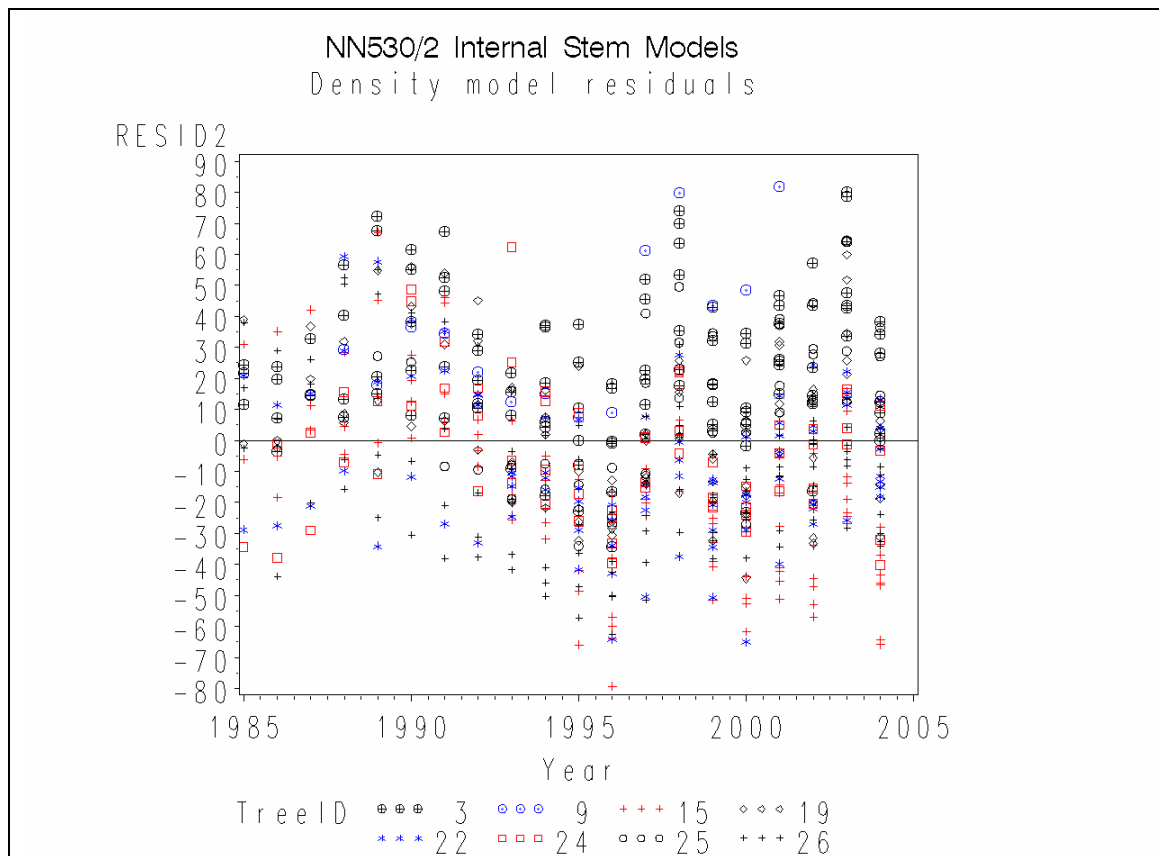
The R-square of this model was 0.60. Residual plots (Figure 10a-c) showed variation by tree and year and a slight trend by  $W_{fT}$ . Removing the year effect by subtracting mean residual (actual-predicted) by year increased the R-square to 0.70. Fitting the model by tree further increased the R-square to 0.82 on average, but the value estimated for the  $b$  parameter in the model was not significant for most trees. The final model, fitted to all trees with the year effect removed predicts basic density reasonably well. Examination of Figure 10d shows some consistent variation associated with individual trees.



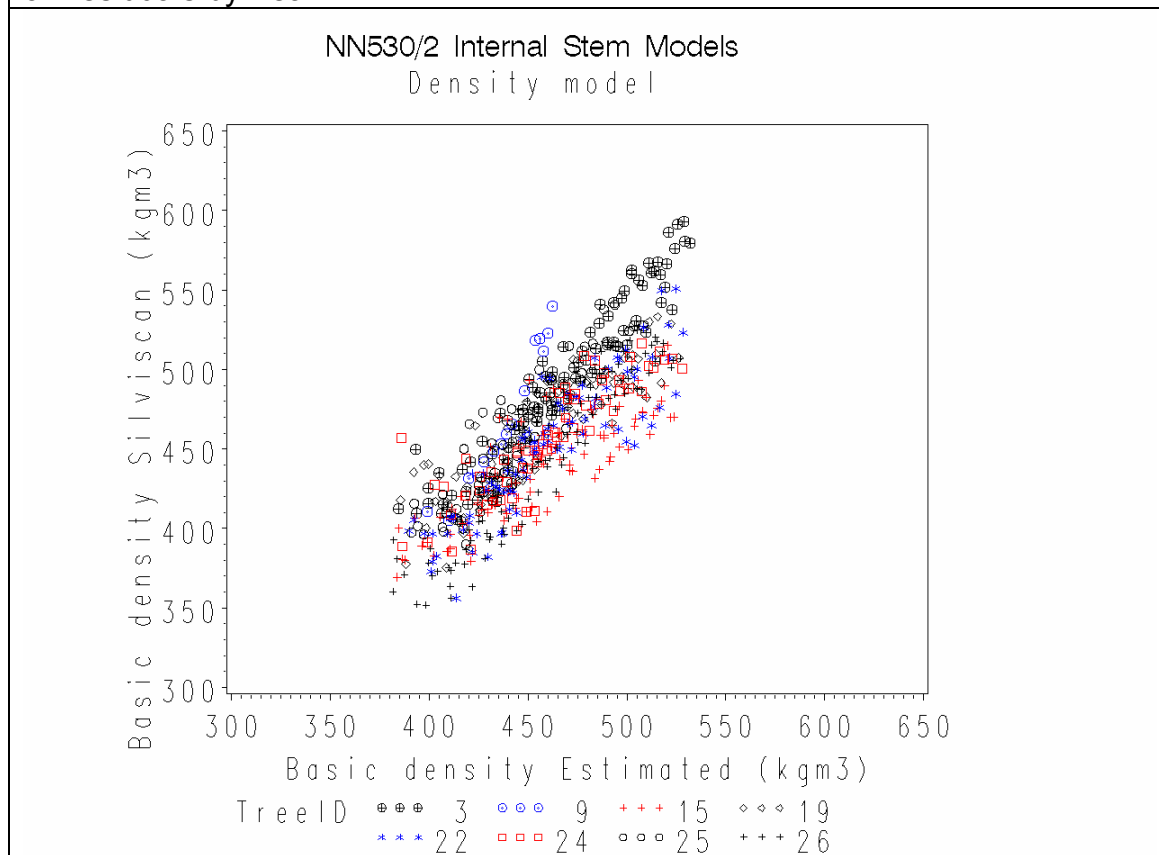
a. Residuals by TreeID



b. Residuals by  $W_{fT}$ .



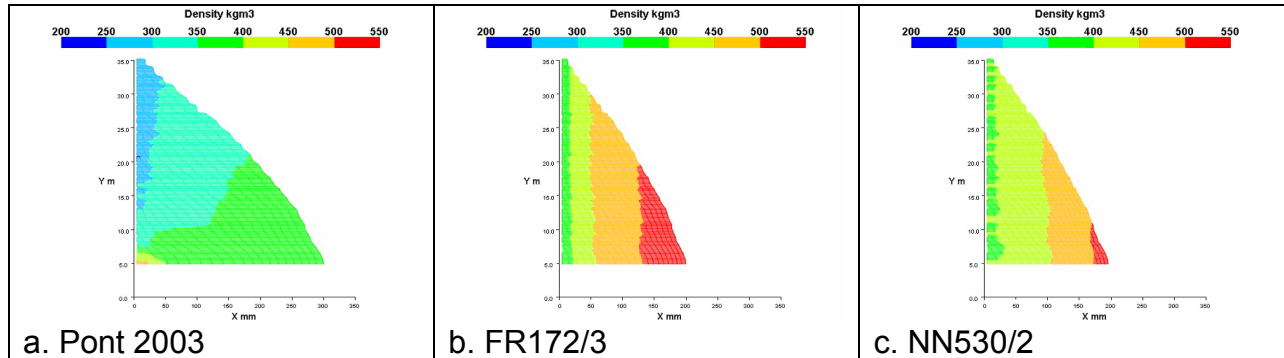
c. Residuals by Year.



d. Measured density plotted against estimated values.

**Figure 10. Density model (Equation 6) residuals and plot of actual values against model estimates.**

Figure 11 allows comparison of the ring area and density patterns produced by the models fitted to date. The models fitted to the SGMC data sets (FR172/3 Figure 11b and NN530/2 Figure 11c) are quite similar. It should be noted that the FR172/3 model was fitted to much younger trees and is therefore being used to extrapolate well beyond the range of the basic data. The model of Pont (2003) (Figure 11a) was fitted to data from a single tree, which is shown to have relatively high ring area and low density.



**Figure 11. Diagrams of ring area and density model predictions for Tree 22 from 3 models: a - Pont 2003, b - FR172/3, c - NN530/2.**

## DISCUSSION

The same model forms have been applicable for the relationships between branch foliage and growth across all three studies (Pont 2003, FR172/3 and NN530/2) with small differences in parameters. The model for branch foliage mass was less accurate than that for average distance to foliage and may need reviewing in the future.

The same model form for stem ring area has also been useful across the three studies, with a consistently good fit to the data. The relationship between stem basic density and crown structure appears to be more complex. The FR172/3 data set contained four seedlots and showed distinct tree-to-tree variation in the internal pattern of density. The NN530/2 data set was a single seedlot but showed strong year-to-year variation, assumed to be of environmental origin. These sources of variability have required the use of slightly different model forms on each data set to obtain a satisfactory fit. These facts indicate that an accurate model for basic density can not be based on crown structure alone and will probably require incorporation of site and tree effects. An alternative for forestry applications could be to use empirical fits to sample sets of trees representing particular seedlots and sites to obtain 'averaged' patterns of internal stem properties.

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