FRI Project Record

No. 5915

PROJECTING INVENTORY DATA: REVISED INDIVIDUAL-TREE STATIC HEIGHT EQUATION

R.G. SHULA

REPORT No. 59 **November 1997**

FRI / Industry Research Cooperatives

EXECUTIVE SUMMARY

Stand Growth Modelling Cooperative (SGMC) Report No. 47 (Gordon 1996) documented the initial development of an individual-tree static height equation for radiata pine. The tree-level static height equation is applicable to mid-rotation, post-siliviculture stands, i.e., ≥ 15 years old. Individual-tree height growth is derived indirectly by subtraction (predicted height at time2 minus predicted height at time1).

The objective of this Report is to document revisions to the equation.

The current analyses used the same database used by Gordon (1996), but the analyses were extended to include the investigation of weighted, non-linear regression, and several additional explanatory variables.

The revised equation has significant improvements to adjusted R^2, homogeneous variance of residuals, and significance of parameter estimates; and, includes new approaches/variables to index New Zealand radiata pine productivity potential.

The adjusted R^2 for the revised equation is 0.51. On average across the 7 growth modelling regions and range of tree heights for plantations aged ≥15 years, individual-tree height can be estimated with about ± 5% error.

The revised equations are considered ready for beta-testing in the new generation of individual-tree growth models and any ancillary applications (e.g., GROMARVL) or modelling efforts (e.g., SGMC Work Programme 1997/98: Theme 3 - Crown Development). Nonetheless, formal validation of the prediction equations is warranted and pending (SGMC Work Programme 1997/98: Theme 4, Project 2).

INTRODUCTION

Stand Growth Modelling Cooperative (SGMC) Report No. 47 (Gordon 1996) documented the initial development of an individual-tree static height equation for radiata pine. The tree-level static height equation is applicable to mid-rotation, post-siliviculture stands, i.e., $\geq$ 15 years old. Individual-tree height growth is derived indirectly by subtraction (predicted height at time2 minus predicted height at time1).

The objective of this Report is to document revisions to the equation.

The current analyses used the same database used by Gordon (1996), but the analyses were extended to include the investigation of weighted, non-linear regression, and several additional explanatory variables.

NOTATION

dbh_i	= individual-tree, breast-height (1.4m) diameter (mm)
ht_i	= individual-tree height (m)
MTD	= stand, mean top breast-height diameter (mm)
MTH	= stand, mean top height (m)
S	= site index (m)
N	= stems per hectare

DATA

The current analyses used the same dataset (291 plots) as described in Gordon (1996). In brief, plots were extracted from the F.R.I. Permanent Sample Plot (PSP) system according to the following acceptance criteria:

- first PSP measurement from age 15 to 25 years (inclusive),
- at least 15 trees measured per plot,
- at least 3 or more consecutive measurements per plot,
- only 'normal' levels of mortality (excluding windthrow and poison thinnings, and
- any thinning operations completed prior to the first measurement.

Only trees that had been measured for height were included in the regression analysis dataset. The total number of observations was 5264, and ranged by region from 147 (Canterbury) to 1039 (Kaingaroa).

METHODS

Background

The initial investigation (Gordon 1996) concluded that the direct prediction of individual-tree height growth was not worthwhile. Instead, the direct prediction of future tree height was preferred, via the static prediction of height ratio, individual-tree height (ht_i) to mean top height (MTH). Individual-tree height growth, then, is obtained indirectly by subtraction. Regional specificity was accommodated in a single height ratio prediction equation by the inclusion of regional dummy variables. The current investigation accepted this modelling approach and did not consider the issue further.

The current analyses used SAS (SAS Institute Inc. 1989) weighted, non-linear regression procedure, NLIN, (method=marquardt) to estimate parameter coefficients (α=0.05).

The non-linear equation form that was fit was:

$$\frac{ht_i}{MTH} = \left(\frac{dbh_i}{MTD}\right)^{f(x)} \qquad [1]$$

where,

ht_i/MTH	= height ratio (individual-tree height / mean top height),
dbh_i	= individual-tree diameter breast-height,
MTH	= mean top diameter,
MTD	= mean top diameter,
f(x)	= (a1*R1 + a2*R2 + … + a7*R7)+(b1*X1 + b2*X2 + … + bn*Xn),

and,

R	= regional dummy (0 or 1, as appropriate),
X	= stand- and/or tree-level explanatory variables,
a, b	= coefficients to be determined.

Potential explanatory variables (x_n) included all those tried in the initial investigation, and those newly devised, although used previously in height growth analyses of Douglas-fir (Shula and Knowe 1997). Where re-measurement did not occur 12 months later, polynomial interpolation was used to obtain an annulated observation.

To better ensure homogenous variance of residuals, a variety of weighting schemes were investigated, including:

- the reciprocal (or not) of tree-size attributes (e.g., dbh, height), and
- iterative re-weighting using the reciprocal (or not) of the predicted.

Criteria for judging equation goodness-of-fit, homogeneity of residual variance, and acceptance included:

- adjusted R^2 (Kmenta 1986) and
- Furnival's Index (Furnival 1961).

Adjusted R^2 was used because it considers the number of explanatory variables (p) in an equation in relation to the number of observations (n) in the dataset. Thus, it provides a standardised measure of the predictive ability of equations, differing in n and p, to account for variation from the mean in respective datasets. The benefit of using weighted regression to best ensure homogenous variance of residuals was determined by computing and comparing Furnival's Index from both unweighted and weighted regression. In a comparison of equations, the equation with the 'best' Index will exhibit residuals most normally distributed, most independent, and with most constant standard error.

Explanatory Variables

In addition to the explanatory variables tried in the initial investigation, 2 additional explanatory variables (including transformations) were tried:

- HPIT, and
- chg_pdbh.

Furthermore, a variable that was accepted in the initial investigation was transformed for greater tree-level specificity:

- bal_ratio.

HPIT (Height Potential Index of a Tree). Height Potential Index, HPI, (analogous to site index, S, or mean top height, MTH, at a base-age) was developed for each of the 7 regions to index potential site productivity as a function of height and age (**Appendix 1**). Regional HPI datasets were developed using the 3 tallest trees per plot (plot size approximately 0.04-ha) at the start of each re-measurement period. This replacement sampling method was chosen to accommodate change in tree-dominance through time.

The HPI equation is an algebraic-difference formulation (Clutter *et al* 1983), ADF, of an exponentiated and generalised Schumacher growth equation (Schumacher 1939), and is polymorphic with respect to (w.r.t.) shape. Through algebraic manipulation, the ADF predicts potential tree height given current and future age, and HPI. Herein, HPI base-age is 20 years plantation age, although the ADF is inherently base-age invariant (i.e., in application, any base-age can be specified).

In the current analyses, the appropriate regional HPI equation was applied to each individual-tree in the regional dbh growth datasets, as if it were a MTH-tree, to obtain the 'height potential index of the tree' (HPIT). HPIT, then, represents a particular tree's maximum expected height at base-age, or an index of the tree's potential micro-site height productivity.

Chg_pdbh (change in potential dbh). Analygous to HPI, Diameter Potential Index (DPI) was developed for each of the 7 regions to index potential site productivity as a function of dbh and age (**Appendix 2**). The same regional datasets used in the HPI analyses were used to derive DPI. This approach, to use the most dominant trees based on height, was used to minimise the influence of stand density, and thereby, make DPI less dependent on management regime and to be congruent with HPI. Analogous to the HPIT analyses, the appropriate regional DPI equation was applied to each individual-tree in the regional dbh growth datasets, as if it was a MTH-tree, to obtain the 'diameter potential index of the tree' (DPIT). DPIT, then, represents a particular tree's maximum expected diameter at base-age, or an index of the tree's potential micro-site diameter productivity.

Through algebraic manipulation, the ADF predicts the potential dbh of a tree (PDT) given current and future age, and DPI. Herein, DPI base-age is 20 years plantation age. Collective potential-dbh-by-age paired data produce dbh curves that represent dbh maximum growth trajectories.

The prediction of individual-tree growth often uses a combinatory approach, whereby, maximum expected growth (free-to-grow) is predicted, then, subsequently modified by other explanatory variables pertinent to specific tree-size and competition indices. In the present analyses, maximum expected annual growth or 'change in potential dbh' (chg_pdbh) was derived from calculated annual increments w.r.t. DPIT, PDT (at time2), and initial dbh (at time1). DPIT, chg_pdbh, and transformations thereof, were tried as explanatory variables in combination with other tree- and stand-level variables to predict height ratio.

Bal_ratio. This variable is the ratio of bal (basal-area-in-trees-larger-than-the-subject-tree) to the subject tree's dbh (dbh_i). This transformation of bal provides greater specificity in implementation because trees from different plots may have an identical bal (identical 'position' in the stand's hierarchy), but have a different dbh (tree-size). Bal_ratio, then, indexes or quantifies intra-specific competition w.r.t. within-plot and between-plot relativity.

RESULTS

General

The revised equation has significant improvements to adjusted R^2, homogeneous variance of residuals, and significance of parameter estimates; and, and includes new approaches/variables to index New Zealand radiata pine productivity potential.

Un-weighted regression provided a better Furnival Index than weighted regression, indicating most constant standard error of prediction, and for the construction of confidence intervals, then, the most asymptotically efficient parameter estimators. All parameter coefficients are significantly different than zero and one (α=0.05).

Fit Statististics and Parameter Coefficients

Table 1. Region, mean residual height ratio, adjusted R^2, and Furnival Index from the regression analyses.

Region (no. obs.)	Mean Residual Height Ratio (standard deviation)	Adjusted R^2	Furnival Index	
			Weighted	Not Weighted
OVER ALL (5264)	0.0015 (0.0530)	0.51	0.05312	0.05311
KANG (1039)	-0.0007 (0.0413)			
GDNS (975)	0.0007 (0.0641)			
HBAY (697)	0.0040 (0.0529)			
CANTY (147)	0.0022 (0.0514)			
CLAYS (423)	0.0037 (0.0556)			
SANDS (963)	0.0017 (0.0547)			
SOUTH (1020)	0.0014 (0.0497)			

Table 2. Region, parameters, and coefficients from the regression analyses using non-linear equation [1]:

$$ht_i \text{ ratio} = (\, dbh_i \,/\, MTD\,)^{(a1*R1\ +\ a2*R2\ +\ \ldots\ +\ a7*R7)+(b1*X1\ +\ b2*X2\ +\ \ldots\ +\ bn*Xn)}$$

Region	Parameter	Coefficient (α=0.05)	Standard Error
KANG	R1	1.79942	0.05514
GDNS	R2	1.57082	0.04900
HBAY	R3	1.54822	0.05478
CANTY	R4	1.54336	0.05165
CLAYS	R5	1.49998	0.05186
SANDS	R6	1.42215	0.04758
SOUTH	R7	1.24644	0.04717
	relspace	-0.11482	0.01026
	bal_ratio	-0.59715	0.08782
	alt_sqd	-0.78126	0.05321
	HPIT	-0.05186	0.00156
	chg_pdbh	0.03083	0.00207

Parameter definitions (not previously described):

ht_i ratio = ht_i / MTH, (m/m)
relspace = 1000 / [MTH x N $^{0.5}$], (m and trees per hectare)
alt_sqd = altitude^2 / 1000000, (m)

Residuals

Appendix 3, Figures 1-7 present height ratio (ht_i/MTH) residuals by stand age for the 7 modelling regions. Residuals are coded w.r.t. the variable, site index class (siclass), which is the integer value of (S / 2.5). No error trends are evident, and the bulk of height ratio residuals are within ± 0.15.

Appendix 3, Figures 8-9 present mean percent error of predicted ht_i (p_ht_i) by actual ht_i (a_ht_i) for the 7 modelling regions. Percent error (PE) was calculated as:

$$PE = \left(\frac{a\_ht_i \text{ - [MTH} \times \text{predicted } ht_i \text{ ratio]}}{a\_ht_i} \right) \times 100$$

Mean percent error of p_ht_i was calculated, by region, on the basis of a_ht_i groups with near equal sample size (i.e, frequency). In Figures 8-9, the dot and star symbols represent 'paired items' which identify 'mean percent error of p_ht_i' (the left vertical axis) and the accompanying 'frequency' (the right vertical axis) upon which the mean was calculated, respectively.

Across regions and a_ht_i groups, mean percent error of p_ht_i averaged ± 5%. By region and within the bulk of a_ht_i groups (usually 20 to 30 m), mean percent error was centred around zero, although at either tail of the actual mean height groups, the trend in errors was towards over- and under-estimation, respectively. This error trend was most pronounced in the regions: GDNS, CLAYS, and SANDS. To check if this trend was simply an artefact of modelling rregional specificity via dummy variables, the GDNS data was re-fit separately. To this end, equation [1] was modified, such that:

$$f(x) = (a0 + b1*X1 + b2*X2 + \ldots + bn*Xn).$$

Upon inspection, the explanatory variable, alt_sqd, was dropped because the coefficient was determined to be not significantly different from zero. **Figure 10a and 10b** present GDNS height ratio residuals by actual height from equation [1] and the re-fit, respectively. A modest improvement to residuals is exhibited in Figure 10b, however, the trend in mean percent error was found to closely mirror the respective error trend presented in Figure 8 (GDNS).

DISCUSSION

The following variables were useful predictors of height ratio and represent new approaches to index New Zealand radiata pine productivity potential, individual-tree competition, and diameter growth, respectively:

- HPIT,
- bal_ratio, and
- chg_pdbh.

Three of the five prediction variables (excluding the regional dummy variables) in equation [1] represent tree-level attributes, and impart unique prediction effects for trees of different size (dbh_i and ht_i). The two remaining prediction variables (relspace and alt_sqd) represent stand-level attributes, and convey the same prediction effects for trees of all sizes.

The ordering of regions by descending magnitude of the dummy variable (R) coefficients (confidence interval with α=0.05):

- KANG 1.80 (1.69 - 1.91)
- GDNS 1.57 (1.47 - 1.67)
- HBAY 1.55 (1.44 - 1.66)
- CANTY 1.54 (1.44 - 1.64)
- CLAYS 1.50 (1.40 - 1.60)
- SANDS 1.42 (1.33 - 1.52)
- SOUTH 1.25 (1.15 - 1.34)

provides a ranking from most to least w.r.t. the departure of ht_i from MTH. That is, if the 5 prediction variables are considered constant at some arbitrary value (i.e., ignored), then equation [1] predicts height ratios that are:

- < 1, and smallest for KANG and largest for SOUTH, for trees with diameter ratios (dbh_i / MTD) < 1, and
- > 1, and largest for KANG and smallest for SOUTH, for trees with diameter ratios > 1.

This interpretation suggests that, provided stand- and tree-level attributes are held constant, predicted height distributions will be least and most centralised around MTH in KANG and SOUTH regions, respectively. Another view is that, relative to the SOUTH region, the KANG region has the propensity for more shorter trees and more larger trees relative to MTH (i.e., greater variation in individual-tree height). The remaining regions follow a similar cascading rank w.r.t. this interpretation of predicted height distributions. It is interesting to note, that the confidence intervals (α=0.05) of the dummy variable coefficients separate the regions into roughly 3 groups:

- KANG;
- GDNS, HBAY, CANTY, CLAYS; and,
- SANDS, SOUTH.

And actually, nearly 4 groups, as SANDS and SOUTH overlap only slightly.

Inspection of the prediction variables in equation [1] reveals that 4 of the 5 variables have a negative coefficient. In practice, then, the collective magnitude of these variables concentrates on the reduction in magnitude of the summation of the equation's power term (inclusive of the regional dummy variables with positive coefficients). In application, then, the relative effects of greater relspace (lower MTH, lower N), alt_sqd (greater elevation), HPIT (greater height index), and bal_ratio (greater competitive index) are:

- to predict a greater height ratio for trees with diameter ratios < 1 (i.e., the majority of trees in a stand), and
- to predict a lesser height ratio, for trees with diameter ratios > 1.

In relation to the discussion on the rank of the regional dummy variables for KANG and SOUTH, above; these foregoing relative effects moderate and accentuate, respectively, the interpretations of predicted height distributions.

It is interesting to note, that the current analysis confers a negative sign to the variable, relspace; while in Gordon (1996), a positive sign was conferred. One explanation is related to the statistical significance of the coefficient in each analysis. In review, when weighted regression (using dbh_i) was used to estimate the parameter coefficients for equation [3] in Gordon (1996), the coefficient on relspace was not significantly different from zero (α=0.05). Therefore, the sign of the coefficient could just as well have been negative. Nonetheless, the coefficient was negative and significantly different from zero at α=0.09. The foregoing discussion, then, suggests the effect of stand density (relative spacing) on height ratio is rather weak and difficult to assign with any clear assurance.

The variable, chg_pdbh, has a positive coefficient, and works to increase the summation of the equation's power term. In application, then, the relative effects of greater chg_pdbh (greater diameter growth) are:

- to predict a lesser height ratio for trees with diameter ratios < 1, and
- to predict a greater height ratio, for trees with diameter ratios > 1.

This interpretation suggests that more rapid diameter growth proceeds at the expense of height growth for most trees in a stand (trees with diameter ratios < 1). In relation to the discussion on KANG and SOUTH, above, these foregoing relative effects accentuate and moderate, respectively, the interpretations of predicted height distributions.

Figure 8 and 9 exhibit increasing mean percentage error of predicted height with increasing actual mean height; most pronounced in the GDNS, CLAYS, and SANDS regions. No explanation is at-hand regarding the predilection of these regions to exhibit this trend, while, e.g., the HBAY region exhibits a much more homogenous error trend by actual mean height.

CONCLUSIONS

On average across the 7 growth modelling regions and range of tree heights for plantations aged ≥15 years, the prediction of tree height can be done with about ± 5% error.

The revised equations are considered ready for beta-testing in the new generation of individual-tree growth models and any ancillary applications (e.g., GROMARVL) or modelling efforts (e.g., SGMC Work Programme

1997/98: Theme 3 - Crown Development). Nonetheless, formal validation of the prediction equations is warranted and pending (SGMC Work Programme 1997/98: Theme 4, Project 2).

REFERENCES

Clutter, J.L., Fortson, J.C., Pienaar, L.V., Brister, G.H. and Bailey, R.L. 1983. Timber Management: A Quantitative Approach. John Wiley & Sons, New York. 333 pp.

Furnival, G.M. 1961. An index for comparing equations used in constructing volume tables. *Forest Science* 7(4): 337-341.

Gordon, A.D. 1996. Projecting inventory data: predicting individual tree height growth. FRI/Industry Stand Growth Modelling Cooperative Report No. 47.

Kmenta, J. 1986. Elements of Econometrics. Second Edition, Macmillan Publishing Company, New York. 786 pp.

SAS Institute Inc. 1989. SAS Procedures Guide - Release Edition 6.04. SAS Institute Inc., Cary, NC, USA.

Schumacher, F.X. 1939. A new growth curve and its application to timber-yield studies. *Forestry* 37: 819-820.

Shula, R.G. and Knowe, S.A. 1997. Regional Vegetation Management Model (RVMM) - Version for the Coast Ranges of Oregon and Washington. Forest Research Laboratory, Oregon State University, Corvallis, Oregon. Research Paper (in prep).

APPENDIX 1: HEIGHT POTENTIAL INDEX (HPI) of a TREE (HPIT)

Basic Function: $ht_i = 1.4 + \exp(a_0 + a_1 \times age^{a2})$

where:

Ht_i = subject tree height (m) using 3 tallest trees per plot (plot size approximately 0.04-ha),
exp(x) = e^x, e is the base of the natural logarithm
age = plantation age (years).

Algebraic Difference Formulation - Polymorphic

- Isolate a_2: let shape parameter be site-specific
- Equate height and age at time1 and time2
- Solve for height @ time2: f(age @ time2, height and age @ time1)

$$ht_{i2} = 1.4 + \exp\left\{ a_0 + a_1 \times \exp\left\{ \ln\left[\frac{[\ln(ht_{i1} - 1.4) - a_0]}{a_1} \right] \times \left[\frac{\ln(age_2)}{\ln(age_1)} \right] \right\} \right\}$$

where:

ht_{i1}, ht_{i2} = tree height at time1 and time2
age_1, age_2 = plantation age at time1 and time2
exp(x) = e^x, e is the base of the natural logarithm
ln = natural logarithm, and
a_0, a_1 = coefficients to be determined.

- To estimate height potential index (HPI): Replace ht_{i2} with HPI and age_2 with base-age = 20. Use with each of 3 tallest trees (plot size approximately 0.04-ha), and obtain the average.
- **To estimate height potential index of a tree (HPIT)**: Use preceding, but apply to any tree (without obtaining an average).
- To estimate potential height at time2 (PH_{i2}): Replace ht_{i2} with potential height at time 2 (PH_{i2}) and ht_{i1} with HPI, and, invert the ages.

Parameter Coefficients and Fit Statistics (α=0.05)

Region (no. obs.)	a0	a1	Adjusted r^2
SOUTH (357)	5.26233	-10.23677	0.97
HBAY (330)	4.67375	-11.00566	0.91
KANG (1303)	4.55730	-12.90752	0.98
SANDS (447)	4.05147	-11.35737	0.95
GDNS (582)	5.01187	-9.64185	0.96
CANTY (63)	8.64549	-8.75875	0.93
CLAYS (186)	4.14134	-26.17232	0.90

APPENDIX 2: CHANGE IN POTENTIAL DIAMETER (chg_pdbh)

Basic Function: $dbh_i = \exp(a_0 + a_1 \times age^{a2})$

where:

dbh_i	=	subject tree dbh (mm), using 3 tallest trees per plot (plot size approximately 0.04-ha),
exp(x)	=	e^x, e is the base of the natural logarithm
age	=	plantation age (years).

Algebraic Difference Formulation - Polymorphic

- Isolate a_2: let shape parameter be site-specific
- Equate dbh and age at time1 and time2
- Solve for dbh @ time2: f(age @ time2, dbh and age @ time1)

$$dbh_{i2} = \exp\left\{ a_0 + a_1 \times \exp\left\{ \ln\left[\frac{[\ln(dbh_{i1}) - a_0]}{a_1} \right] \times \left[\frac{\ln(age_2)}{\ln(age_1)} \right] \right\} \right\}$$

where:

dbh_{i1}, dbh_{i2}	=	tree dbh at time1 and time2
age_1, age_2	=	plantation age at time1 and time2
exp(x)	=	e^x, e is the base of the natural logarithm
ln	=	natural logarithm, and
a_0, a_1	=	coefficients to be determined.

- To estimate diameter potential index (DPI): Replace dbh_{i2} with DPI and age2 with base-age = 20. Use with each of 3 tallest trees (plot size approximately 0.04-ha), and obtain the average.
- To estimate diameter potential index of a tree (DPIT): Use preceding, but apply to any tree (without obtaining an average).
- To estimate potential diameter (PD_{i2}): Replace dbh_{i2} with potential diameter at time2 (PD_{i2}) and dbh_{i1} with DPI, and, invert the ages.
- **To estimate change in potential diameter (chg_pdbh)**: Calculate DPIT. Calculate PD_{i2}. Subtract dbh_{i1} from PD_{i2}.

Fit Statististics and Parameter Coefficients

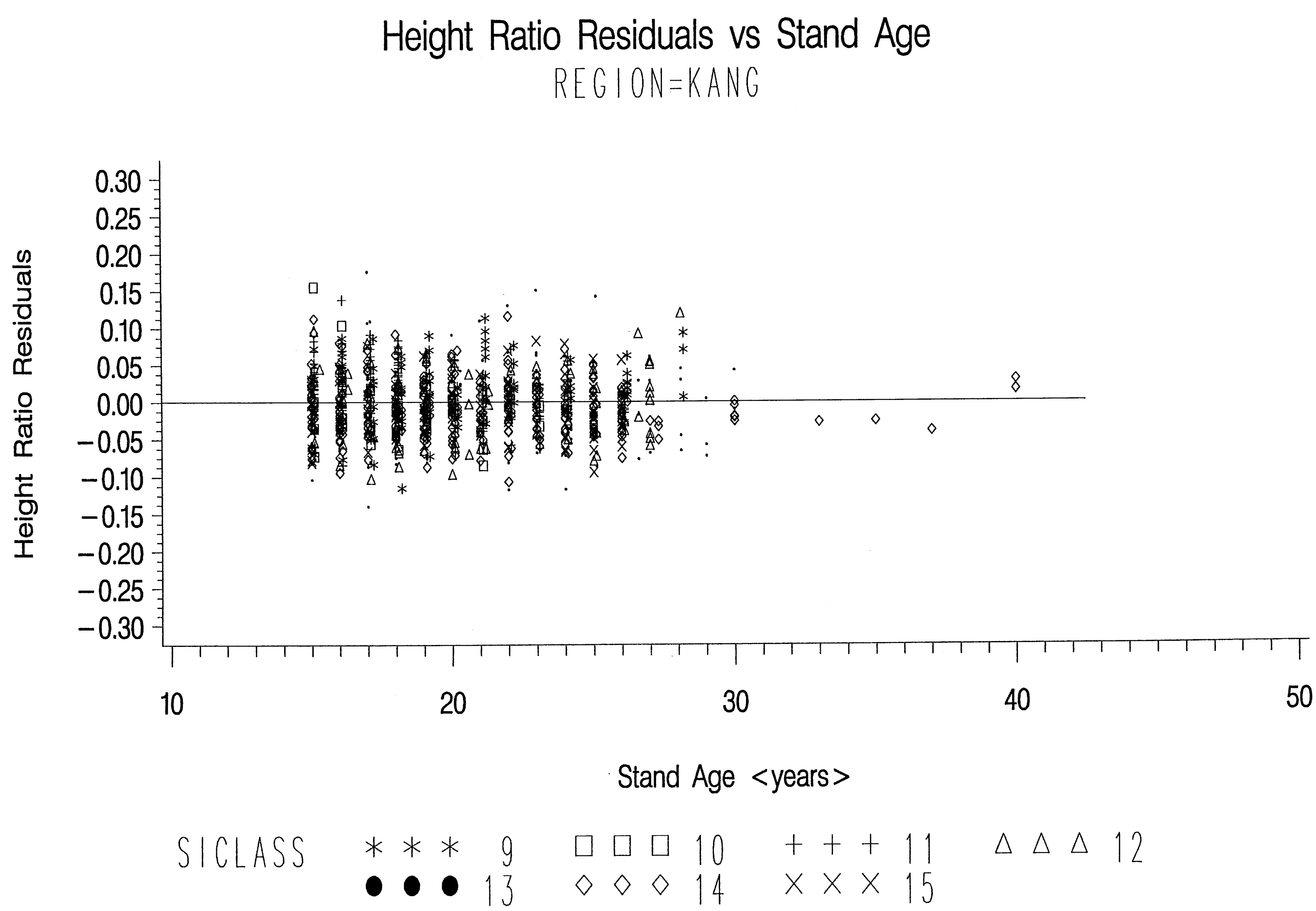
Region (no. obs.)	a0	a1	Adjusted r^2
SOUTH (357)	7.81591	-10.50217	0.98
HBAY (330)	7.45444	-9.62501	0.97
KANG (1303)	7.44302	-5.59127	0.99
SANDS (447)	7.58569	-7.09015	0.98
GDNS (582)	7.64530	-7.65284	0.99
CANTY (63)	8.76768	-5.91372	0.98
CLAYS (186)	6.76975	-17.99992	0.94

APPENDIX 3:

- Figures 1 - 7: Height ratio (ht_i / MTH) residuals by stand age
- Figures 8 - 9: Mean percent error of predicted ht_i by actual mean ht_i
- Figure 10: GDNS height ratio residuals by actual ht_i

  a) equation [1] with regional dummy variables

  b) modified equation [1]: without regional dummy variables and the explanatory variable, alt_sqd



Height Ratio Residuals vs Stand Age
REGION=KANG
Height Ratio Residuals
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
10
20
30
40
50
Stand Age <years>
SICLASS
9
10
11
12
13
14
15

Figure 1.

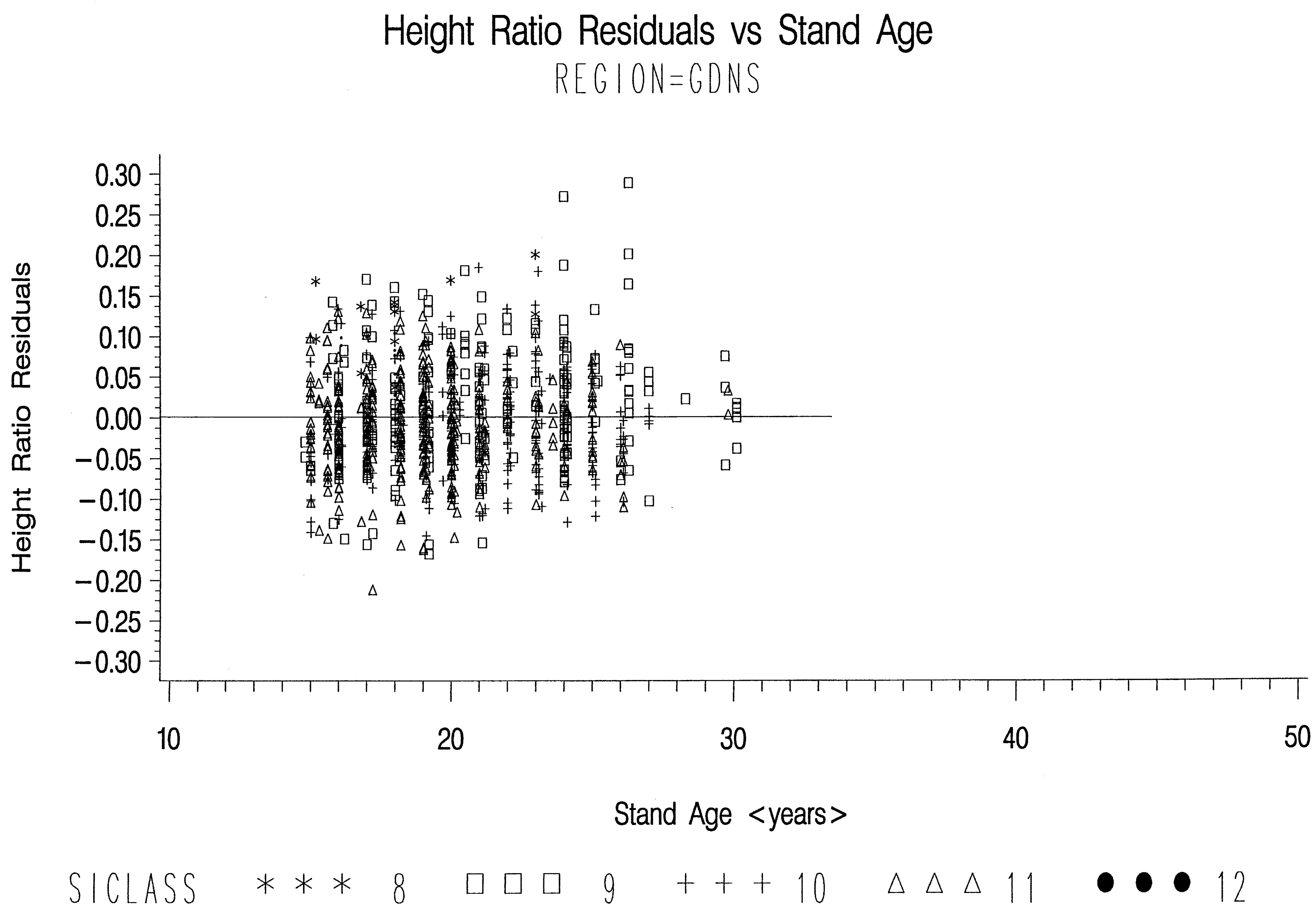
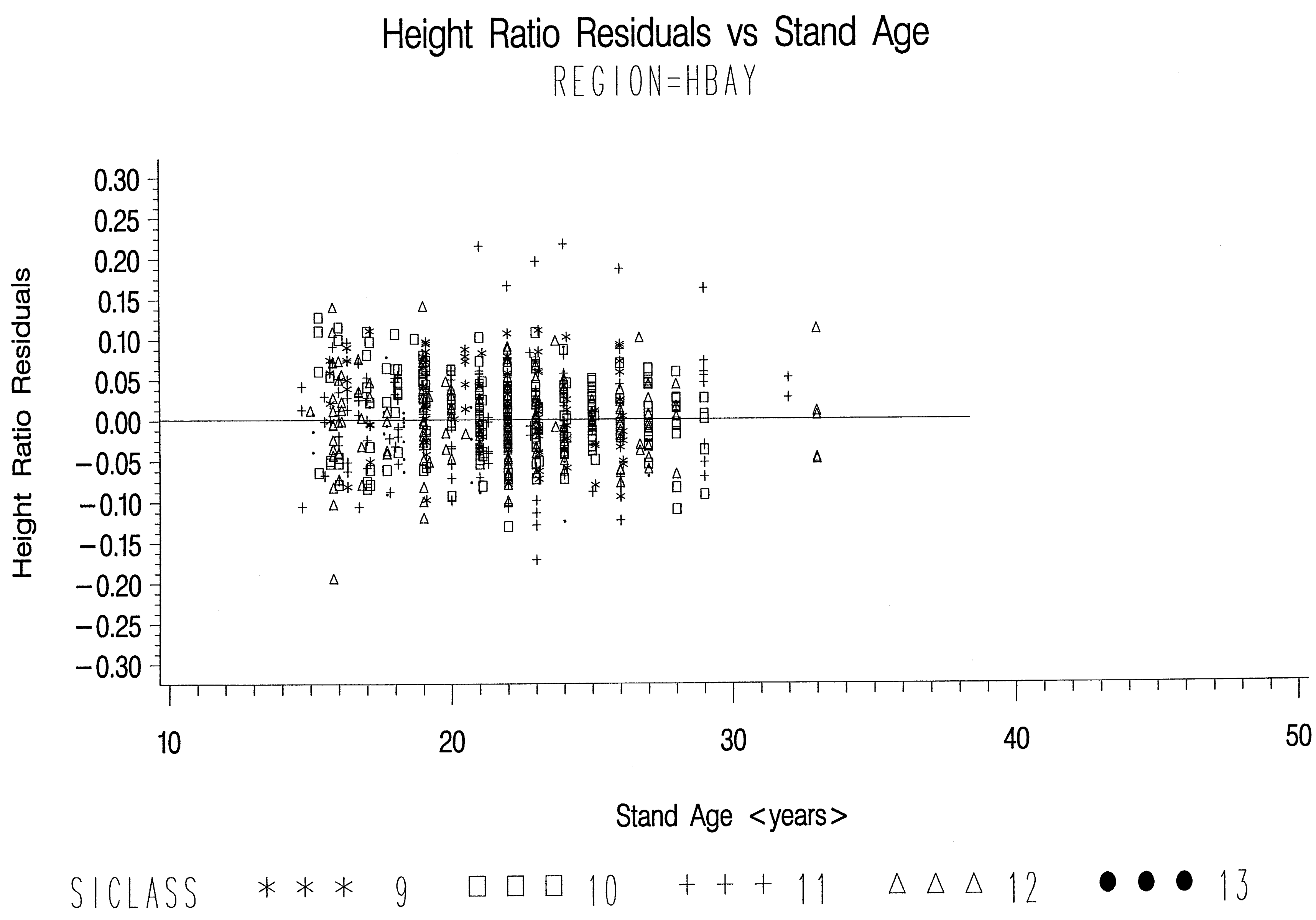
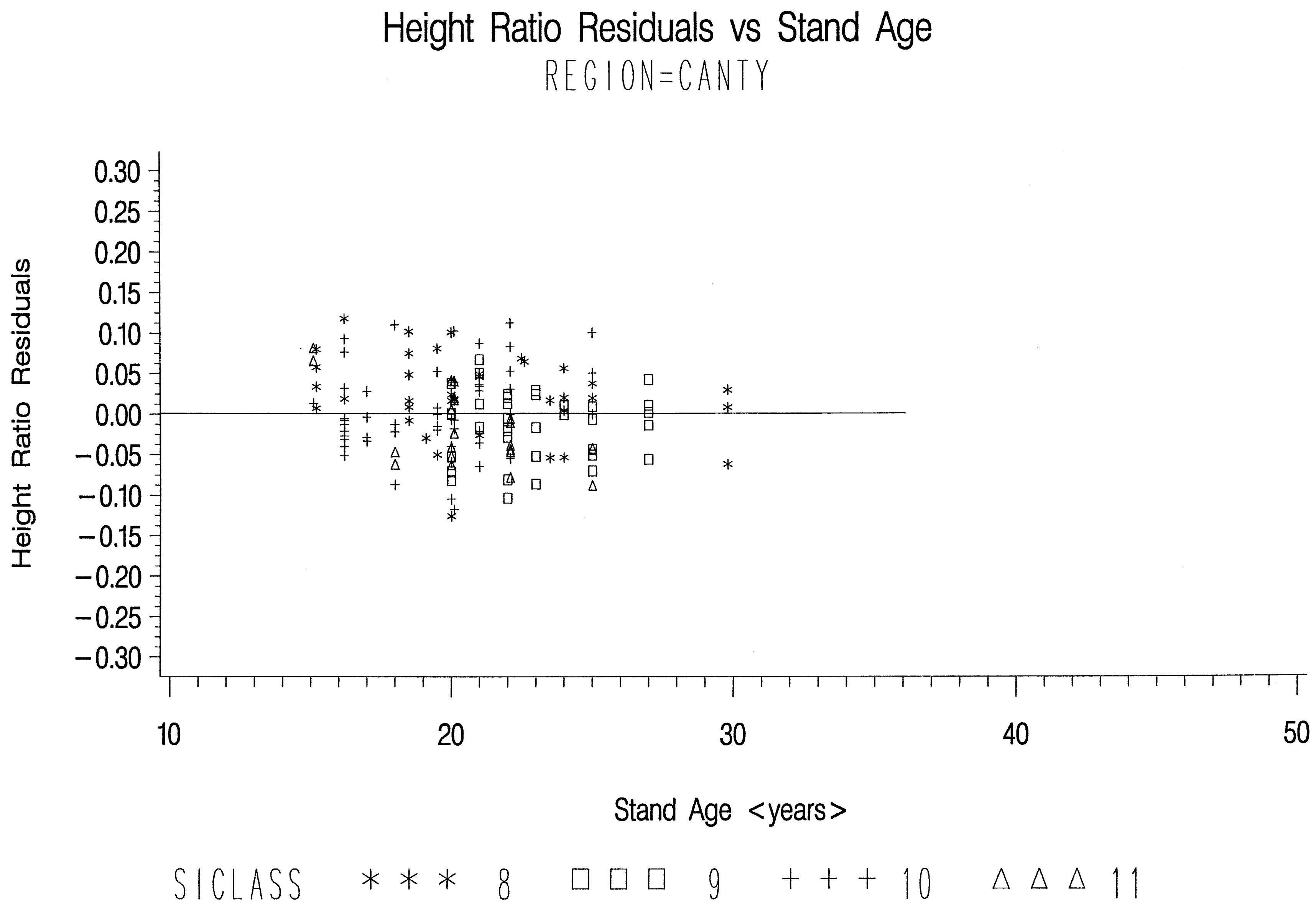

Height Ratio Residuals vs Stand Age
REGION=GDNS
Height Ratio Residuals
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
10
20
30
40
50
Stand Age <years>
SICLASS * * * 8 □ □ □ 9 + + + 10 △ △ △ 11 ● ● ● 12

Figure 2.



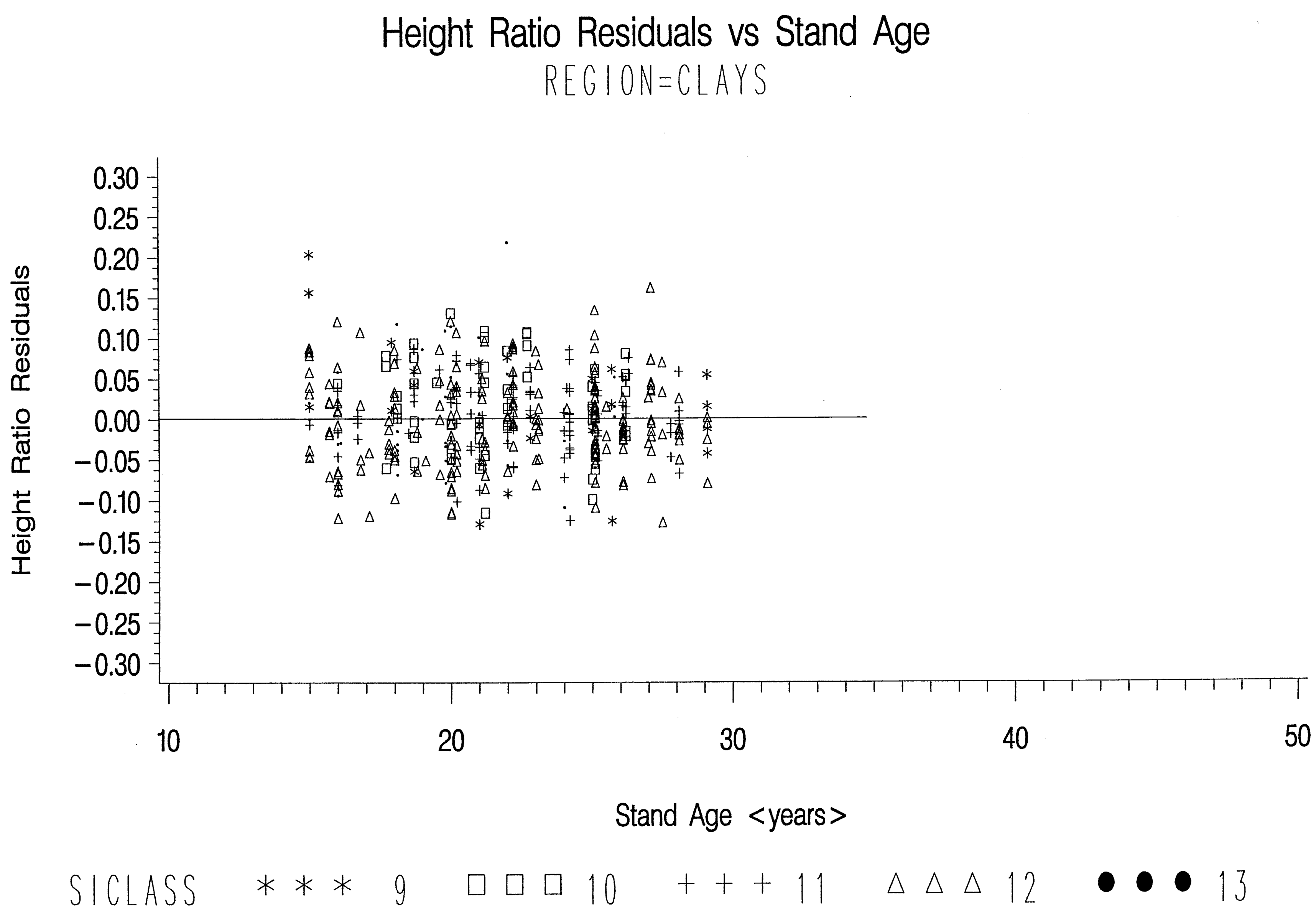
Height Ratio Residuals vs Stand Age
REGION=HBAY
Height Ratio Residuals
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
10
20
30
40
50
Stand Age <years>
SICLASS * * * 9 □ □ □ 10 + + + 11 △ △ △ 12 ● ● ● 13

Figure 3.



Height Ratio Residuals vs Stand Age
REGION=CANTY
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
Height Ratio Residuals
10
20
30
40
50
Stand Age <years>
SICLASS * * * 8 □ □ □ 9 + + + 10 △ △ △ 11

Figure 4.



Height Ratio Residuals vs Stand Age
REGION=CLAYS
Height Ratio Residuals
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
10
20
30
40
50
Stand Age <years>
SICLASS * * * 9 □ □ □ 10 + + + 11 △ △ △ 12 ● ● ● 13

Figure 5.

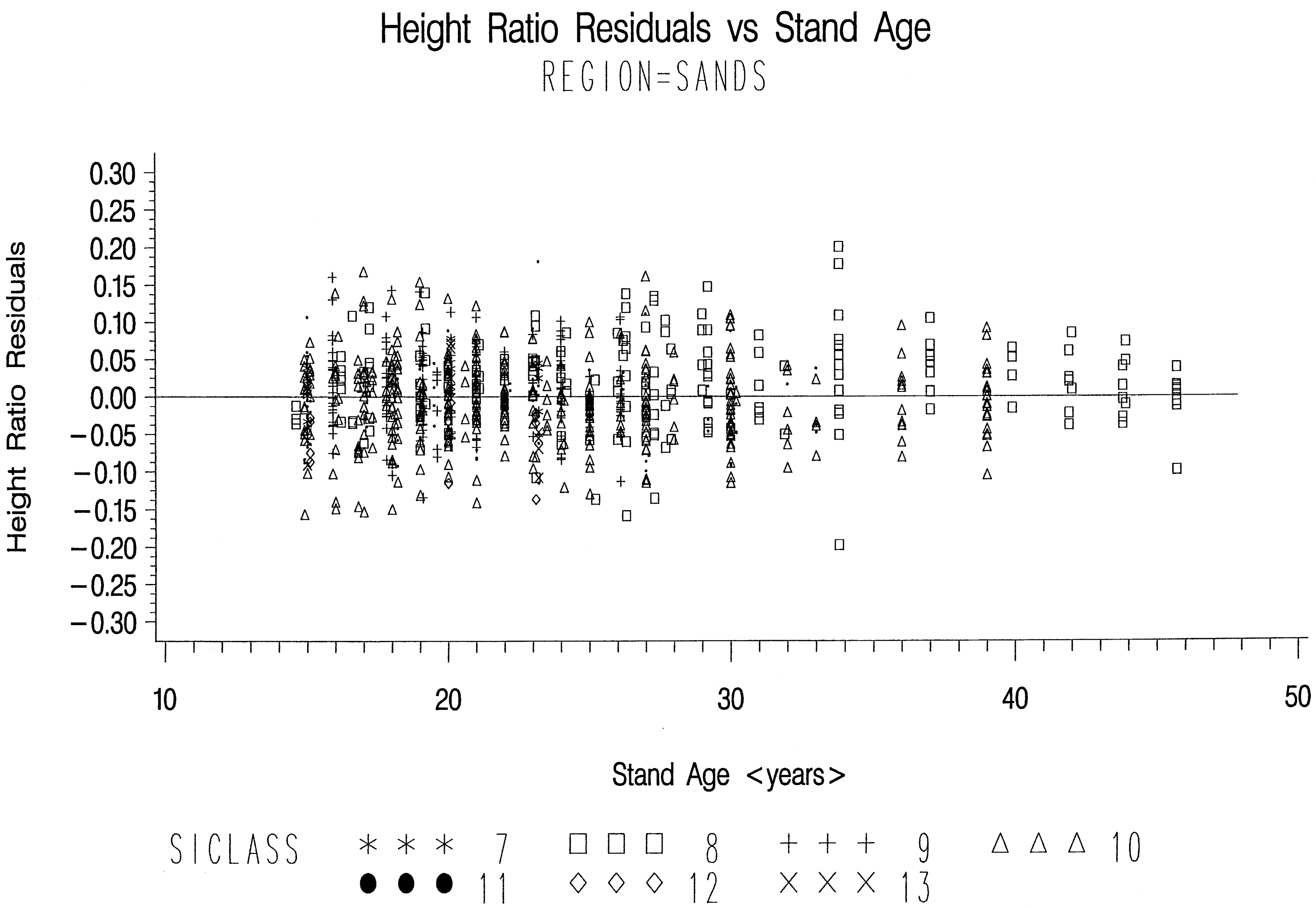

Height Ratio Residuals vs Stand Age
REGION=SANDS
Height Ratio Residuals
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
10
20
30
40
50
Stand Age <years>
SICLASS
7
8
9
10
11
12
13

Figure 6.

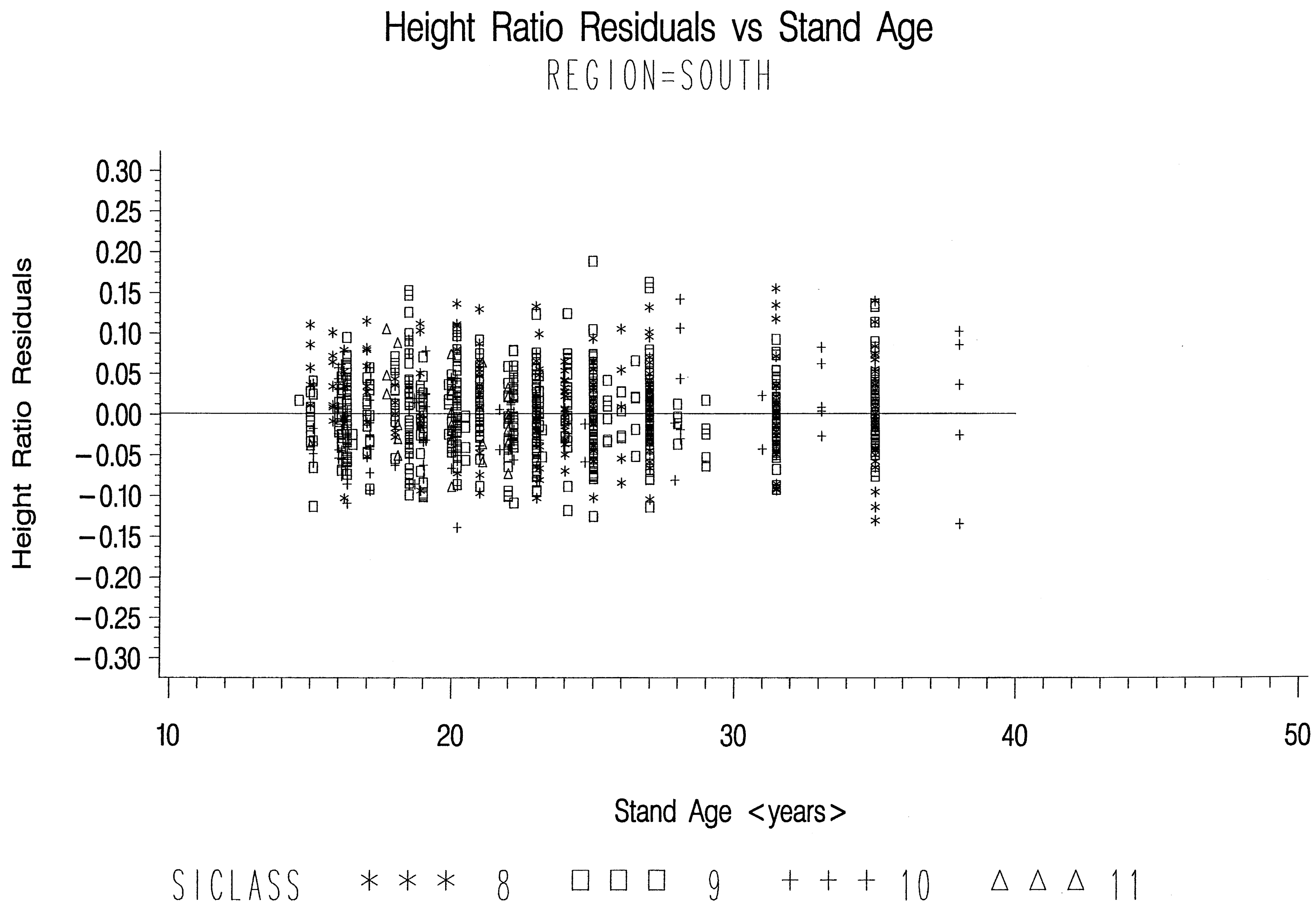
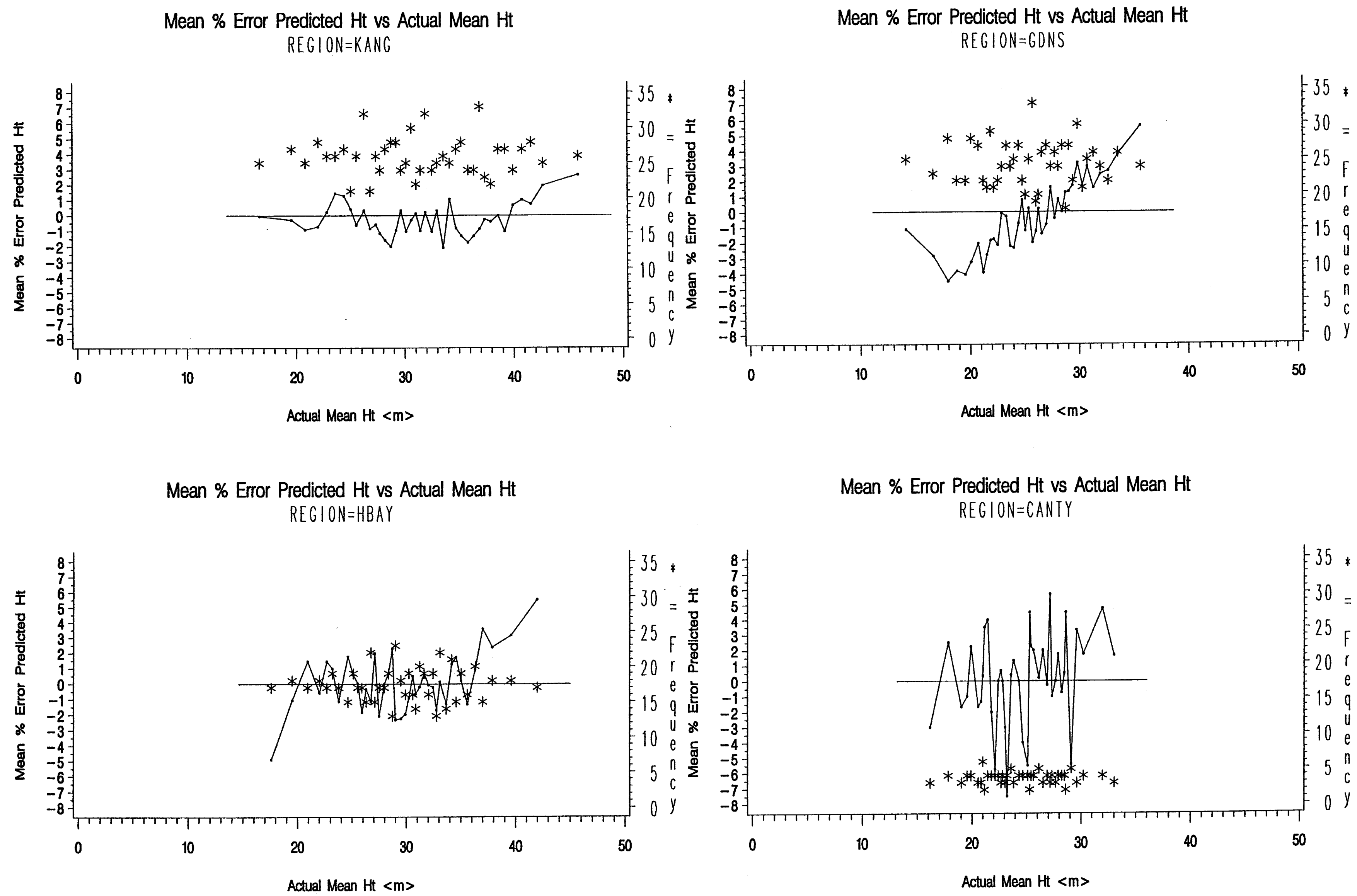
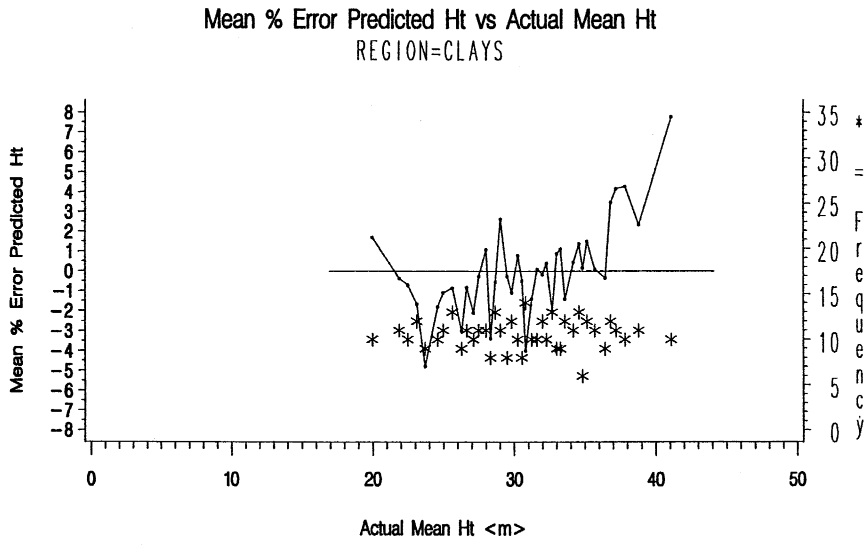

Height Ratio Residuals vs Stand Age
REGION=SOUTH
Height Ratio Residuals
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
10
20
30
40
50
Stand Age <years>
SICLASS * * * 8 □ □ □ 9 + + + 10 △ △ △ 11

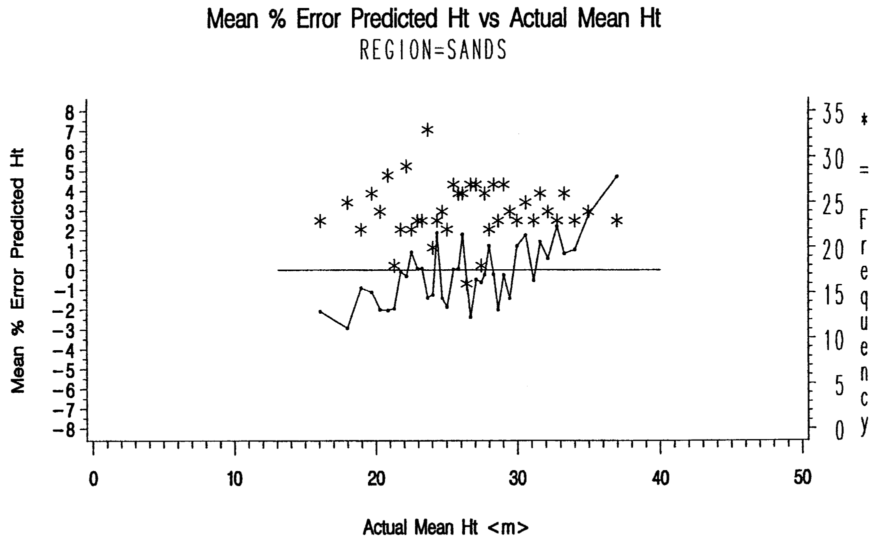
Figure 7.



Mean % Error Predicted Ht vs Actual Mean Ht
REGION=KANG
Mean % Error Predicted Ht vs Actual Mean Ht
REGION=GDNS
Mean % Error Predicted Ht vs Actual Mean Ht
REGION=HBAY
Mean % Error Predicted Ht vs Actual Mean Ht
REGION=CANTY
Mean % Error Predicted Ht
Actual Mean Ht <m>
* = Frequency

Figure 8.


Mean % Error Predicted Ht vs Actual Mean Ht
REGION=CLAYS
Mean % Error Predicted Ht
Actual Mean Ht <m>
* = Frequency


Mean % Error Predicted Ht vs Actual Mean Ht
REGION=SANDS
Mean % Error Predicted Ht
Actual Mean Ht <m>
* = Frequency

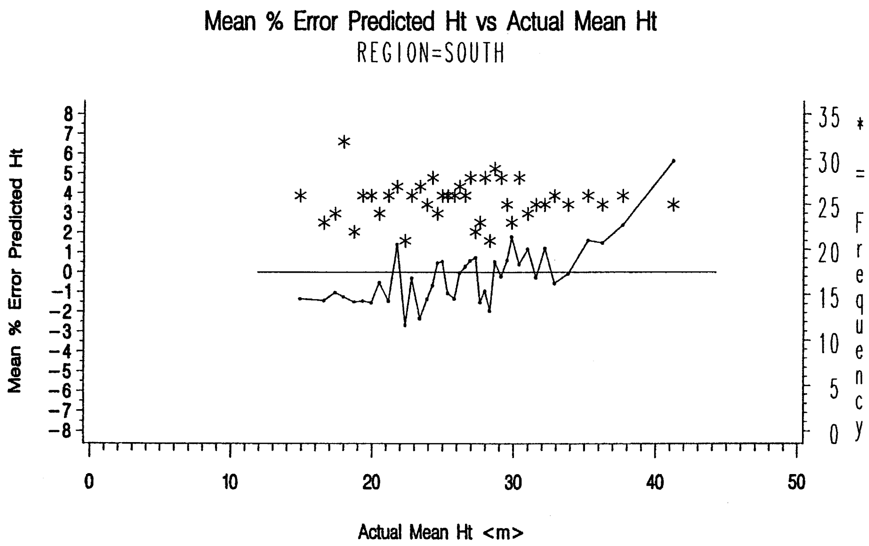
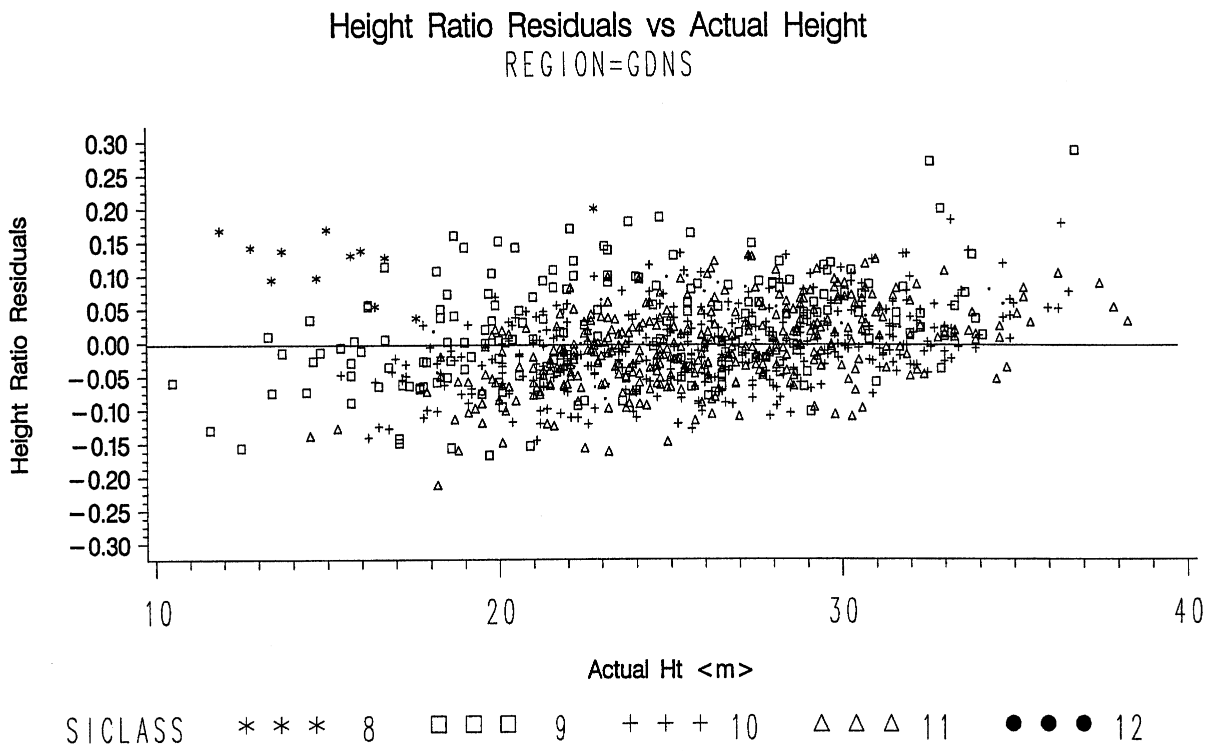

Mean % Error Predicted Ht vs Actual Mean Ht
REGION=SOUTH
Mean % Error Predicted Ht
Actual Mean Ht <m>
* = Frequency

Figure 9.



a) with dummy variable

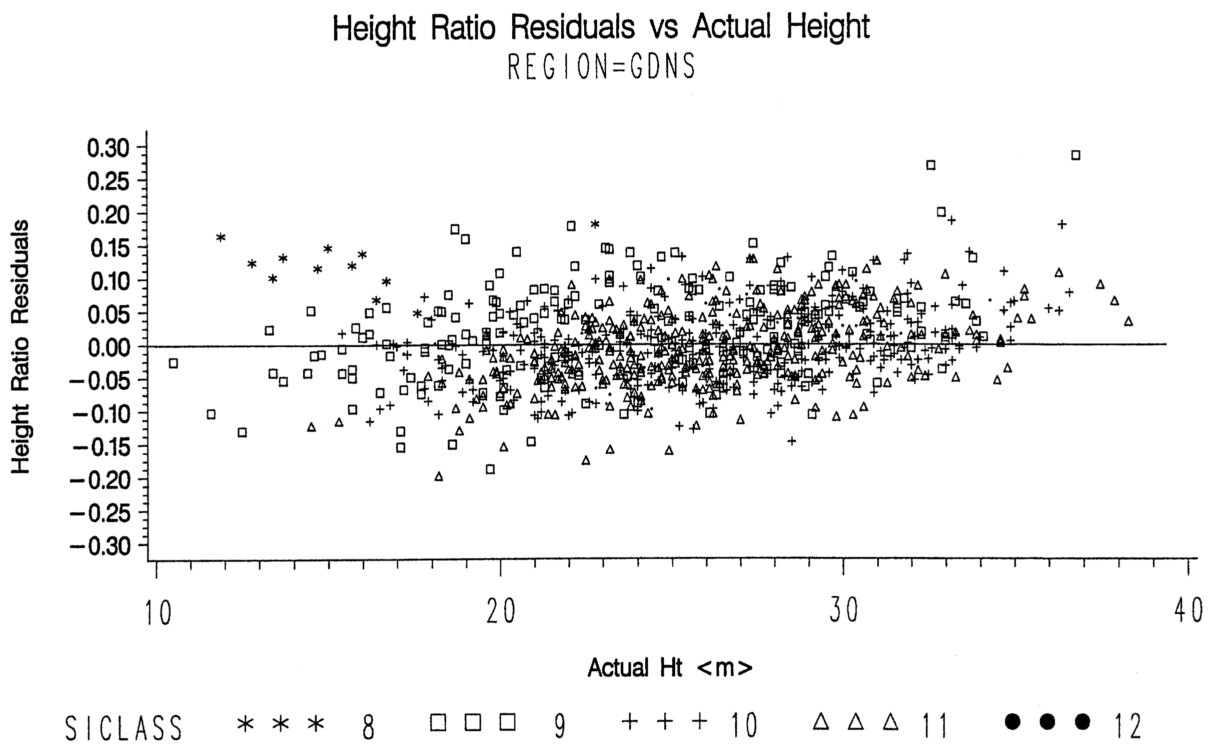


Figure 10. b) without dummy variable