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# PROJECTING INVENTORY DATA: REVISED INDIVIDUAL-TREE DIAMETER GROWTH EQUATIONS

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# FRI / Industry Research Cooperatives

### **EXECUTIVE SUMMARY**

Stand Growth Modelling Cooperative (SGMC) Report No. 34 (Gordon and Lawrence 1994) documented the initial development of individual-tree diameter growth equations for radiata pine. The tree-level diameter growth equations are applicable to mid-rotation, post-siliviculture stands, i.e., ≥ 15 years old.

The objective of this Report is to document revisions to the equations.

The current analyses used the same database as described in Gordon and Lawrence (1994), but the analyses were extended to include the investigation of weighted, non-linear regression, and several new, additional explanatory variables.

For all 7 modelling regions, prediction equations were revised relative to the initial investigation, and included new approaches/variables to index New Zealand radiata pine relative stand density and/or productivity potential. In general, revisions resulted in improvements to: adjusted R², homogeneous variance of residuals, and significance of parameter estimates.

Across the modelling regions, adjusted  $R^2$  values ranged from 0.52 (Clays) to 0.77 (Southland). On average across the 7 growth modelling regions and range of initial tree diameters for plantations aged  $\geq$  15 years, tree diameter, one year hence, can be estimated with about  $\pm$  1% error.

The revised equations are considered ready for beta-testing in the new generation of individual-tree growth models and any ancillary applications (e.g., GROMARVL) or modelling efforts (e.g., SGMC Work Programme 1997/98: Theme 3 - Crown Development Processes). Nonetheless, formal validation is warranted and pending (SGMC Work Programme 1997/98: Theme 4, Project 2).

### INTRODUCTION

Stand Growth Modelling Cooperative (SGMC) Report No. 34 (Gordon and Lawrence 1994) documented the initial development of individual-tree diameter growth equations for radiata pine. The tree-level diameter growth equations are applicable to mid-rotation, post-siliviculture stands, i.e.,  $\geq$  15 years old.

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### **NOTATION**

dbh<sub>i</sub> = individual-tree, breast-height (1.4m) diameter (mm) dbh<sub>q</sub> = stand, quadratic mean breast-height diameter (mm) exp(x) = e<sup>x</sup>; e is the base, 2.71828, of the natural logarithm

G = stand, basal area (m²/hectare) log = natural base 2.71828 logarithm

 $log_{10}$  = base 10 logarithm

MTD = stand, mean top breast-height diameter (mm)

MTH = stand, mean top height (m)

S = site index (m)
N = stems per hectare

### **DATA**

The current analyses used the same dataset (291 plots) as described in Gordon and Lawrence (1994). In brief, data was extracted from the F.R.I. Permanent Sample Plot (PSP) system according to the following acceptance criteria:

- first PSP measurement from age 15 to 25 years (inclusive),
- at least 15 trees measured per plot,
- at least 3 or more consecutive measurements per plot,
- only 'normal' levels of mortality (excluding windthrow and poison thinnings, and
- any thinning operations completed prior to the first measurement.

Nearly equal number of observations (approximately 750) were included in each of 7 regional datasets by using random selection with varying probabilities of selection from the total number of observations (n = 65628) meeting the acceptance criteria.

### **METHODS**

# **Background**

The initial investigation (Gordon and Lawrence 1994) concluded that a single equation, across regions, was not appropriate. The current investigation accepted this conclusion and did not consider the issue further.

The current analyses used SAS (SAS Institute Inc. 1989) weighted, non-linear regression procedure, NLIN, (method=marquardt) to estimate parameter coefficients ( $\alpha$ =0.05).

The equation forms that were fitted included:

• linear ..... 
$$y = a_0 + a_1^* x_1 + ... + a_n^* x_n$$
, and [1]

• exponential ... 
$$y = \exp[a_0 + a_1 * x_1 + ... + a_n x_n]$$
. [2]

The dependent variable (y) was individual-tree diameter breast-height (dbh<sub>i</sub>) annual increment (mm). Where re-measurement did not occur 12 months later, polynomial interpolation was used to obtain an annulated observation. Potential explanatory variables (x<sub>n</sub>) included all those tried in the initial investigation, and those newly devised, although used previously in diameter growth analyses of Douglas-fir (Shula and Knowe 1997).

To better ensure homogenous variance of residuals, a variety of weighting schemes were investigated, including:

- the reciprocal (or not) of tree-size attributes (e.g., dbh, height), and
- iterative re-weighting using the reciprocal (or not) of the predicted.

Criteria for judging equation goodness-of-fit, homogeneity of residual variance, and acceptance included:

- adjusted R<sup>2</sup> (Kmenta 1986) and
- Furnival's Index (Furnival 1961).

Adjusted R² was used because it considers the number of explanatory variables (p) in an equation in relation to the number of observations (n) in the dataset. Thus, it provides a standardised measure of the predictive ability of equations, differing in n and p, to account for variation from the mean in respective datasets. The benefit of using weighted regression to best ensure homogenous variance of residuals was determined by computing and comparing Furnival's Index from both unweighted and weighted regression. In a comparison of equations, the equation with the 'best' Index will exhibit residuals most normally distributed, most independent, and with most constant standard error.

### **Explanatory Variables**

In addition to the explanatory variables tried in the initial investigation, 5 additional explanatory variables (including transformations) were tried based upon successful screening for variables from stepwise linear regression:

- SDI.
- RD.
- HPIT,
- chg\_pdbh, and
- bal ratio.

Furthermore, 3 variables that were tried and/or accepted in the initial investigation were transformed for greater tree-level specificity:

- rain dbh,
- NP\_dbh, and
- SPH rdbh.

**SDI (Stand Density Index)**. SDI (Reineke 1933) provides a relative measure of intra-specific competition, and is a function of quadratic mean dbh (dbh<sub>q</sub>) and tree stocking (N):

SDI = 
$$1.0147*(10) ^ [\log_{10}*N + 1.605*\log_{10}(dbh_a) - 2.25]$$

SDI is the number of trees per hectare, as if, dbh<sub>q</sub> was 25.4 cm (10 inches); and is independent of species, site quality, and age (Reineke 1933).

**RD** (Relative Density). RD (Curtis 1982) was developed for coastal Douglasfir in the Pacific Northwest USA to provide a relative measure of intra-specific competition. RD is a function of dbh<sub>a</sub> and basal area stocking (G):

$$RD = G / (dbh_0)^{0.5}$$

For coastal Douglas-fir, the power coefficient on  $dbh_q$  ranges from 0.45 to 0.50, however, among various species, the coefficient is thought to range from 0.3 to 0.5. In the present analyses, and for the simple purpose of a relative measure of intra-specific competition, the value, 0.5, was accepted.

HPIT (Height Potential Index of a Tree). Height Potential Index, HPI, (analogous to site index, S, or mean top height, MTH, at a base-age) was developed for each of the 7 regions to index potential site productivity as a function of tree height and plantation age (Appendix 1). Regional HPI datasets were developed using the 3 tallest trees per plot (plot size approximately 0.04-ha) at the start of each re-measurement period. This replacement sampling method was chosen to accommodate change in tree-dominance through time.

The HPI equation is an algebraic-difference formulation (Clutter *et al* 1983), ADF, of a exponentiated and generalised Schumacher growth equation

(Schumacher 1939), and is polymorphic with respect to (w.r.t.) shape. Through algebraic manipulation, the ADF predicts potential tree height given current and future age, and HPI. Herein, HPI base-age is 20 years plantation age, although the ADF is inherently base-age invariant (i.e., in application, any base-age can be specified).

In the current analyses, the appropriate regional HPI equation was applied to each individual-tree in the regional dbh growth datasets, as if it were a MTH-tree, to obtain the 'height potential index of the tree' (HPIT). HPIT, then, represents a particular tree's maximum expected height at base-age, or an index of the tree's potential micro-site height productivity.

**Chg\_pdbh**. Analygous to HPI, Diameter Potential Index (DPI) was developed for each of the 7 regions to index potential site productivity as a function of tree dbh and age (**Appendix 2**). The same regional datasets used in the HPI analyses were used to derive DPI. This approach, to use the most dominant trees based on height, was used to minimise the influence of stand density, and thereby, make DPI less dependent on management regime and to be congruent with HPI.

Analogous to the HPIT analyses, the appropriate regional DPI equation was applied to each individual-tree in the regional dbh growth datasets, as if it was a MTH-tree, to obtain the 'diameter potential index of the tree' (DPIT). DPIT, then, represents a particular tree's maximum expected diameter at base-age, or an index of the tree's potential micro-site diameter productivity.

Through algebraic manipulation, the ADF predicts the potential dbh of a tree (PDT) given current and future age, and DPI. Herein, DPI base-age is 20 years plantation age. Collective potential-dbh-by-age paired data produce dbh curves that represent dbh maximum growth trajectories.

The prediction of individual-tree growth often uses a combinatory approach, whereby, maximum expected growth (free-to-grow) is predicted, then, subsequently modified by other explanatory variables pertinent to specific tree-size and competition indices. In the present analyses, maximum expected annual growth or 'change in potential dbh' (chg\_pdbh) was derived from calculated annual increments w.r.t. DPIT, PDT (at time2), and initial dbh (at time1). DPIT, chg\_pdbh, and transformations thereof, were tried as explanatory variables in combination with other tree- and stand-level variables to predict individual-tree dbh annual growth.

**Bal\_ratio**. This variable is the ratio of bal (basal-area-in-trees-larger-than-the-subject-tree) to the subject tree's dbh (dbh<sub>i</sub>). This transformation of bal provides greater specificity in implementation because trees from different plots may have an identical bal (identical 'position' in the stand's hierarchy), but have a different dbh (tree-size). Bal\_ratio, then, indexes or quantifies intra-specific competition w.r.t. within-plot and between-plot relativity.

**Rain\_dbh.** The ratio of total rainfall to dbh<sub>i</sub> was used to add tree-level specificity to this otherwise stand-level (identical for all trees) variable.

NP\_dbh and SPH\_rdbh. The stand-level variables, NP (nitrogen x phosphorus, Hunter *et al* 1976) and tree stocking (N) were given tree-level specificity by multiplication by dbh<sub>i</sub> and relative dbh (dbh<sub>i</sub> / dbh<sub>a</sub>), respectively.

### **RESULTS**

### General

For all 7 modelling regions, prediction equations were revised relative to the initial investigation, and included new approaches/variables to index New Zealand radiata pine relative stand density and/or productivity potential. In general, revisions resulted in improvements to: adjusted R², homogeneous variance of residuals, and significance of parameter estimates.

For all regions, exponential equation [2] was selected as it provided:

- predicted annual diameter increment greater than zero without the need to statistically bound parameter estimation during fitting procedures,
- · a smooth and continuous approach to a zero growth increment, and
- fit statistics similar to or better than linear equation [1].

For all regions, weighted regression provided a better Furnival Index than unweighted regression, indicating most constant standard error of prediction, and for the construction of confidence intervals, then, the most asymptotically efficient parameter estimators. For all regions (except CLAYS), the weight, 1 / dbh<sub>i</sub>, was the best weighting scheme (provided the best Furnival's Index). In CLAYS, an estimate of individual-tree height, [ (MTD / MTH) x MTH ], was the best weighting scheme.

# Fit Statististics and Parameter Coefficients

Table 1. Region, mean residual, adjusted R<sup>2</sup>, and Furnival's Index from the regression analyses.

Region	Mean Residual (mm)	Adjusted R <sup>2</sup>	Furnival Index	
(no. obs.)	(std. dev.)	IX.		
			Weighted	Not Weighted
SOUTH (747)	0.14 (4.38)	0.77	4.16	4.39
HBAY (711)	0.08 (4.27)	0.75	3.95	4.28
KANG (798)	0.15 (3.81)	0.66	3.59	3.82
SANDS (780)	-0.02 (4.70)	0.57	4.68	4.71
GDNS (745)	-0.07 (4.60)	0.56	4.59	4.61
CANTY (763)	0.01 (3.88)	0.56	3.85	3.89
CLAYS (756)	0.05 (4.61)	0.52	4.61	4.62

Table 2. Region, parameters, and coefficients, and coefficient standard errors from the regression analyses using exponential equation [2]:

y = exp[ 
$$a_0 + a_1^*x_1 + ... + a_nx_n$$
].

(α=0.05)   Error	Region	Parameter	Coefficient	Standard
SOUTH   Intercept   6.1074   0.3870   Ihpit   0.2782   0.1019   bal_ratio²   -21.0419   2.6716   Isdistnd   -0.7782   0.0391   gro_rain   1.4253   0.1198	Region	- aramotor		
Ihpit	SOUTH	intercept	6.1074	
Isdistnd			0.2782	0.1019
BAY		bal ratio <sup>2</sup>	-21.0419	2.6716
HBAY		Isdistnd	-0.7782	0.0391
HBAY		gro rain	1.4253	0.1198
Ichg_pdbh²   0.6593   0.0518     bal_ratio   -5.2904   0.4871     Ird_stnd   -0.3248   0.0314     Ihpit   -0.6865   0.1020     KANG   intercept   5.6203   0.5711     Ichg_pdbh²   0.2362   0.0552     bal_ratio²   -20.3309   2.7452     Isdistnd   -0.6534   0.0610     sph_rdbh   0.0291   0.0043     CANTY   intercept   3.7099   0.6885     chg_pdbh²   0.0022   0.0002     bal_ratio²   -28.0925   2.4009     Isdistnd   -0.2795   0.0947     rain_dbh   0.1743   0.0226     SANDS   intercept   6.9959   0.2561     chg_pdbh²   0.0013   0.0002     hpit   0.0242   0.0049     NP_ldbh   0.0230   0.0029     Isdistnd   -1.0962   0.0411     GDNS   intercept   3.0316   0.3371     Ichg_pdbh²   0.3952   0.0433     bal_ratio²   -8.8797   1.8239     I_sdistnd²   -0.2000   0.0174     CLAYS   intercept   1.1036   0.1740     bal_ratio²   -34.7965   2.9639     nitrogen   0.0293   0.0086	13234			
Ichg_pdbh²   0.6593   0.0518     bal_ratio   -5.2904   0.4871     Ird_stnd   -0.3248   0.0314     Ihpit   -0.6865   0.1020     KANG   intercept   5.6203   0.5711     Ichg_pdbh²   0.2362   0.0552     bal_ratio²   -20.3309   2.7452     Isdistnd   -0.6534   0.0610     sph_rdbh   0.0291   0.0043     CANTY   intercept   3.7099   0.6885     chg_pdbh²   0.0022   0.0002     bal_ratio²   -28.0925   2.4009     Isdistnd   -0.2795   0.0947     rain_dbh   0.1743   0.0226     SANDS   intercept   6.9959   0.2561     chg_pdbh²   0.0013   0.0002     hpit   0.0242   0.0049     NP_ldbh   0.0230   0.0029     Isdistnd   -1.0962   0.0411     GDNS   intercept   3.0316   0.3371     Ichg_pdbh²   0.3952   0.0433     bal_ratio²   -8.8797   1.8239     I_sdistnd²   -0.2000   0.0174     CLAYS   intercept   1.1036   0.1740     bal_ratio²   -34.7965   2.9639     nitrogen   0.0293   0.0086	HBAY	intercept	1.8738	0.4470
bal_ratio		Ichg pdbh <sup>2</sup>	0.6593	0.0518
Ihpit		bal_ratio	-5.2904	0.4871
KANG		Ird stnd	-0.3248	0.0314
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chg_pdbh²         0.0013         0.0002           hpit         0.0242         0.0049           NP_ldbh         0.0230         0.0029           Isdistnd         -1.0962         0.0411           GDNS         intercept         3.0316         0.3371           Ichg_pdbh²         0.3952         0.0433           bal_ratio²         -8.8797         1.8239           I_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086		rain dbh	0.1743	0.0226
chg_pdbh²         0.0013         0.0002           hpit         0.0242         0.0049           NP_ldbh         0.0230         0.0029           Isdistnd         -1.0962         0.0411           GDNS         intercept         3.0316         0.3371           Ichg_pdbh²         0.3952         0.0433           bal_ratio²         -8.8797         1.8239           I_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086			distribution of the second	
chg_pdbh²         0.0013         0.0002           hpit         0.0242         0.0049           NP_ldbh         0.0230         0.0029           Isdistnd         -1.0962         0.0411           GDNS         intercept         3.0316         0.3371           Ichg_pdbh²         0.3952         0.0433           bal_ratio²         -8.8797         1.8239           I_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086	SANDS	intercept	6.9959	0.2561
NP   Idbh   0.0230   0.0029   Isdistnd   -1.0962   0.0411		chg_pdbh <sup>2</sup>	0.0013	0.0002
Isdistnd		hpit	0.0242	0.0049
GDNS         intercept         3.0316         0.3371           lchg_pdbh²         0.3952         0.0433           bal_ratio²         -8.8797         1.8239           l_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086		NP_ldbh	0.0230	0.0029
Ichg_pdbh²   0.3952   0.0433     bal_ratio²   -8.8797   1.8239     I_sdistnd²   -0.2000   0.0174		Isdistnd	-1.0962	0.0411
Ichg_pdbh²         0.3952         0.0433           bal_ratio²         -8.8797         1.8239           I_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086				
bal_ratio²         -8.8797         1.8239           I_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086	GDNS		3.0316	0.3371
I_sdistnd²         -0.2000         0.0174           CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086			0.3952	0.0433
CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086		bal_ratio <sup>2</sup>	-8.8797	1.8239
CLAYS         intercept         1.1036         0.1740           bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086		I_sdistnd <sup>2</sup>	-0.2000	0.0174
bal_ratio²         -34.7965         2.9639           nitrogen         0.0293         0.0086	a de la	100		
nitrogen 0.0293 0.0086	CLAYS		1.1036	0.1740
			-34.7965	2.9639
relspace <sup>2</sup> 0.0888 0.0066			0.0293	0.0086
		relspace <sup>2</sup>	0.0888	0.0066
lchg_pdbh 0.4357 0.0629		lchg_pdbh	0.4357	0.0629

### Parameter definitions (not previously described):

```
= annual dbh, increment, (mm)
У
gro rain
            = spring + summer rainfall, (mm)
lchg pdbh = log( chg pdbh ), (mm)
lchg pdbh^2 = log(chg pdbh^2), (mm)
Ihpit
            = \log(hpit), (m)
            = log(RD)
Ird stnd
Isdistnd
            = log(SDI)
I sdistnd^2 = log(SDI^2)
            = Nitrogen index, (Hunter et al.1976)
nitrogen
NP ldbh
            = (Nitrogen x Phosphorus index) x log(dbh<sub>i</sub>)
rain dbh
            = total rainfall / dbh, (mm/mm)
relspace<sup>2</sup> = \{1000 / [MTH \times N^{0.5}]\}^2
sph rdbh = (N/100) \times [\log(dbh_i/dbh_g)]
```

### Residuals

Appendix 3, Figures 1-7 present dbh<sub>i1</sub> growth residuals by stand age for the 7 modelling regions. For direct comparison with Gordon and Lawrence 1994 (Figures 5-11), residuals are coded similarly w.r.t. the variable, site index class (siclass), which is the integer value of (S/2.5). Within the age range of the bulk of the data, error trends are not evident. Nonetheless, at the older ages (> 32 years) in HBAY and KANG, prediction errors tend towards underand over-prediction, respectively.

**Appendix 3, Figures 8-9** present mean percent error of predicted  $dbh_{i2}$  (p\_dbh<sub>i2</sub>) by actual  $dbh_{i2}$  (a\_dbh<sub>i2</sub>) for the 7 modelling regions. Percent error (PE) was calculated as:

$$PE = \left(\frac{a\_dbh_{12} - [a\_dbh_{11} + predicted increment]}{a\_dbh_{12}}\right) \times 100$$

Mean percent error of p\_dbh<sub>i2</sub> was calculated on the basis of a\_dbh<sub>i2</sub> groups with near equal sample size (i.e, frequency). In Figures 8-9, the dot and star symbols represent 'paired items' which identify 'mean percent error of p\_dbh<sub>i2</sub>' (the left vertical axis) and the accompanying 'frequency' (the right vertical axis) upon which the mean was calculated, respectively.

Across regions and a\_dbh<sub>i2</sub> groups, mean percent error of p\_dbh<sub>i2</sub> averages ± 1% without evidence of any serious error trends.

### **DISCUSSION**

The following variables (or transformations, thereof) were useful across most of the 7 modelling regions:

- HPIT.
- bal ratio,

- SDI or RD, and
- chg pdbh.

These variables represent new approaches to index New Zealand radiata pine relative stand density and/or productivity potential. Another new and beneficial approach to growth prediction was to use a 'pre-'estimate of growth itself, chg\_pdbh, in a growth equation. This approach incorporates a predicted pattern of dominant-tree diameter growth to aid in the diameter growth prediction of any subject tree.

For the most part, prediction variables relate to tree- or stand-level size/yield attributes, however, rainfall and foliar nutrient indices were useful in the development of equations for the regions: Southland (gro\_rain), Canterbury (total rainfall), Sands (nitrogen x phosphrus), and Clays (nitrogen). While a particularly influential nutrient in the Clays region is phosphorus (Shula 1987), no definitive explanation is at-hand regarding the greater usefulness and inclusion of the nitrogen index, and not the phosphorus index in the Clays' diameter growth equation. The Clays region dataset represents fertilised and unfertilised stand conditions, so one explanation is simply that phosphorus deficiency had been routinely moderated, moreso than nitrogen deficiency.

The sign of the coefficients for the explanatory variables in equation [2] conform with anticipated effects on diameter growth. For example, the negative effects on diameter growth attributable to increased competition (Isdistnd and bal\_ratio²), and the positive effects attributable to rainfall (gro\_rain) and Diameter Potential Index (Ichg\_pdbh²). For 2 of 3 regional equations that include the explanatory variable, HPIT (or transformations thereof), the sign of the coefficient is positive, as would be expected. However, in HBAY Region, the sign of the coefficient is negative. No explanation for this reversal is at-hand.

### **CONCLUSION**

On average across the 7 growth modelling regions and range of initial tree diameters for plantations aged  $\geq$  15 years, tree diameter, one year hence, can be estimated with about  $\pm$  1% error.

The revised equations are considered ready for beta-testing in the new generation of individual-tree growth models and any ancillary applications (e.g., GROMARVL) or modelling efforts (e.g., SGMC Work Programme 1997/98: Theme 3 - Crown Development). Nonetheless, formal validation of the prediction equations is warranted and pending (SGMC Work Programme 1997/98: Theme 4, Project 2).

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# APPENDIX 1: HEIGHT POTENTIAL INDEX (HPI) of a TREE (HPIT)

Basic Function:  $ht_i = 1.4 + exp(a_0 + a_1 \times age^{a_2})$ 

where:

Ht, = subject tree height (m) using 3 tallest trees per plot

(plot size approximately 0.04-ha),

 $exp(x) = e^x$ , e is the base of the natural logarithm

= plantation age (years).

### Algebraic Difference Formulation - Polymorphic

- Isolate a<sub>2</sub>: let shape parameter be site-specific
- Equate height and age at time1 and time2
- Solve for height @ time2: f(age @ time2, height and age @ time1)

$$ht_{z} = 1.4 + exp \left\{ a_{0} + a_{1} \times exp \left\{ ln \left[ \frac{[ln (ht_{1} - 1.4) - a_{0}]}{a_{1}} \right] \times \left[ \frac{ln (age_{2})}{ln (age_{1})} \right] \right\} \right\}$$

where:

ht<sub>i1</sub>,ht<sub>i2</sub> = tree height at time1 and time2 age<sub>1</sub>, age<sub>2</sub> = plantation age at time1 and time2

exp(x) = e<sup>x</sup>, e is the base of the natural logarithm In = natural logarithm, and = coefficients to be determined.

- To estimate height potential index (HPI): Replace ht<sub>i2</sub> with HPI and age<sub>2</sub> with base-age = 20. Use with each of 3 tallest trees (plot size approximately 0.04-ha), and obtain the average.
- To estimate height potential index of a tree (HPIT): Use preceding, but apply to any tree (without obtaining an average).
- To estimate potential height at time2 (PH<sub>i2</sub>): Replace ht<sub>i2</sub> with potential height at time 2 (PH<sub>i2</sub>) and ht<sub>i1</sub> with HPI, and, invert the ages.

### Parameter Coefficients and Fit Statistics ( $\alpha$ =0.05)

Region (no. obs.)	a0	a1	Adjusted r <sup>2</sup>
SOUTH (357)	5.26233	-10.23677	0.97
HBAY (330)	4.67375	-11.00566	0.91
KANG (1303)	4.55730	-12.90752	0.98
SANDS (447)	4.05147	-11.35737	0.95
GDNS (582)	5.01187	-9.64185	0.96
CANTY (63)	8.64549	-8.75875	0.93
CLAYS (186)	4.14134	-26.17232	0.90

# APPENDIX 2: CHANGE IN POTENTIAL DIAMETER (chg pdbh)

Basic Function:  $dbh_i = exp(a_0 + a_1 \times age^{a^2})$ 

where:

dbh; = subject tree dbh (mm) using 3 tallest trees per plot

(plot size approximately 0.04-ha),

 $exp(x) = e^x$ , e is the base of the natural logarithm

= plantation age (years). age

### Algebraic Difference Formulation - Polymorphic

• Isolate a2: let shape parameter be site-specific

Equate dbh and age at time1 and time2

Solve for dbh @ time2: f(age @ time2, dbh and age @ time1)

$$dbh_{12} = exp \left\{ a_0 + a_1 \times exp \left\{ ln \left[ \frac{[ln (dbh_{11}) - a_0]}{a_1} \right] x \left[ \frac{ln (age_2)}{ln (age_1)} \right] \right\} \right\}$$

where:

dbh<sub>i1</sub>, dbh<sub>i2</sub> = tree dbh at time1 and time2

age<sub>1</sub>, age<sub>2</sub> = plantation age at time1 and time2

 $\exp(x)$  =  $e^x$ , e is the base of the natural logarithm In = natural logarithm, and  $a_0$ ,  $a_1$  = coefficients to be determined.

- To estimate diameter potential index (DPI): Replace dbh<sub>12</sub> with DPI and age2 with base-age = 20. Use with each of 3 tallest trees (plot size approximately 0.04-ha), and obtain the average.
- To estimate diameter potential index of a tree (DPIT): Use preceding, but apply to any tree (without obtaining an average).
- To estimate potential diameter (PD<sub>i2</sub>): Replace dbh<sub>i2</sub> with potential diameter at time2 (PD<sub>i2</sub>) and dbh<sub>i1</sub> with DPI, and, invert the ages.
- To estimate change in potential diameter (chg\_pdbh): Calculate DPIT. Calculate PD<sub>i2</sub>. Subtract dbh<sub>i1</sub> from PD<sub>i2</sub>.

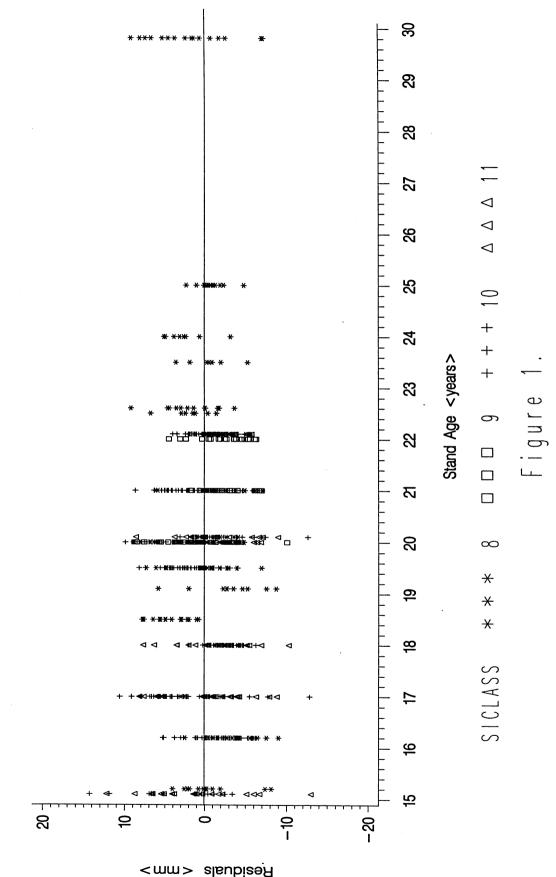
### Fit Statististics and Parameter Coefficients

Region (no. obs.)	a0	a1	Adjusted r <sup>2</sup>
SOUTH (357)	7.81591	-10.50217	0.98
HBAY (330)	7.45444	-9.62501	0.97
KANG (1303)	7.44302	-5.59127	0.99
SANDS (447)	7.58569	-7.09015	0.98
GDNS (582)	7.64530	-7.65284	0.99
CANTY (63)	8.76768	-5.91372	0.98
CLAYS (186)	6.76975	-17.99992	0.94

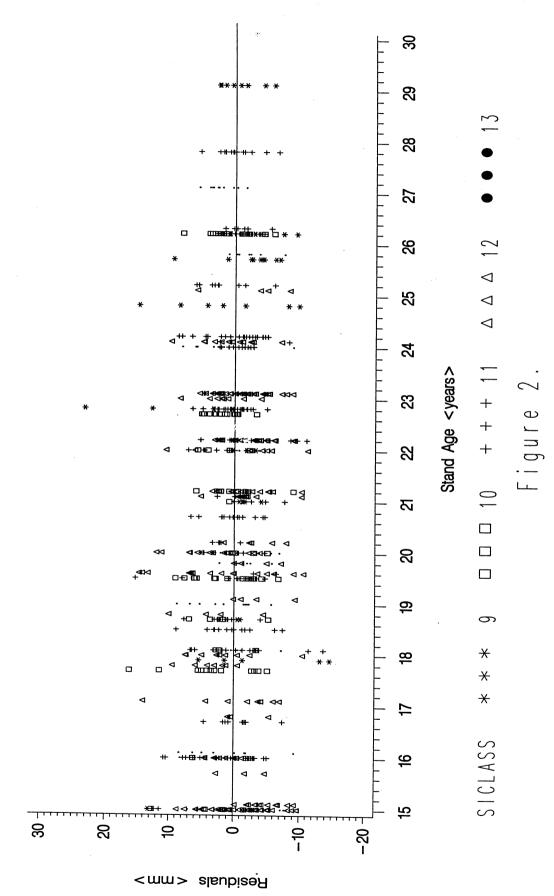
# **APPENDIX 3:**

- Figures 1 7: Dbh<sub>i1</sub> growth residuals by stand age
- $\bullet~$  Figures 8 9: Mean percent error of predicted  $dbh_{i2}$  by actual mean  $dbh_{i2}$

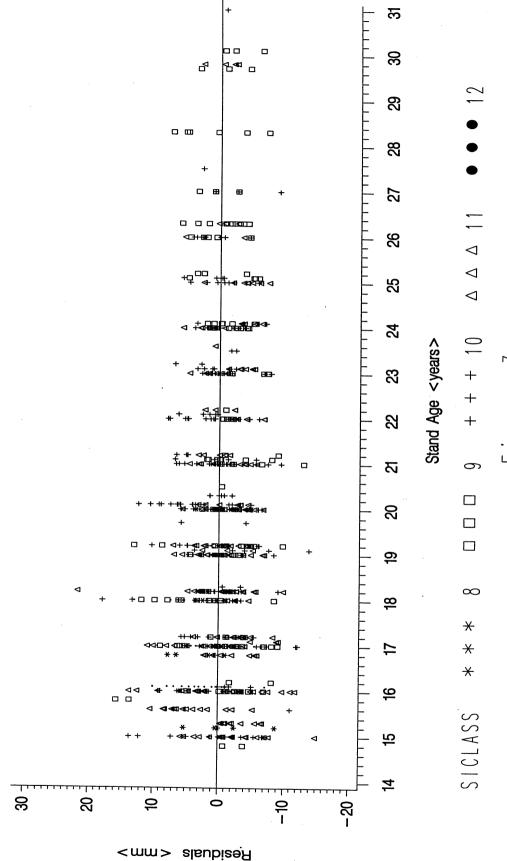
Dbh Increment Residuals vs Stand Age REGION=CANTY



**Dbh Increment Residuals vs Stand Age**REGION=CLAYS



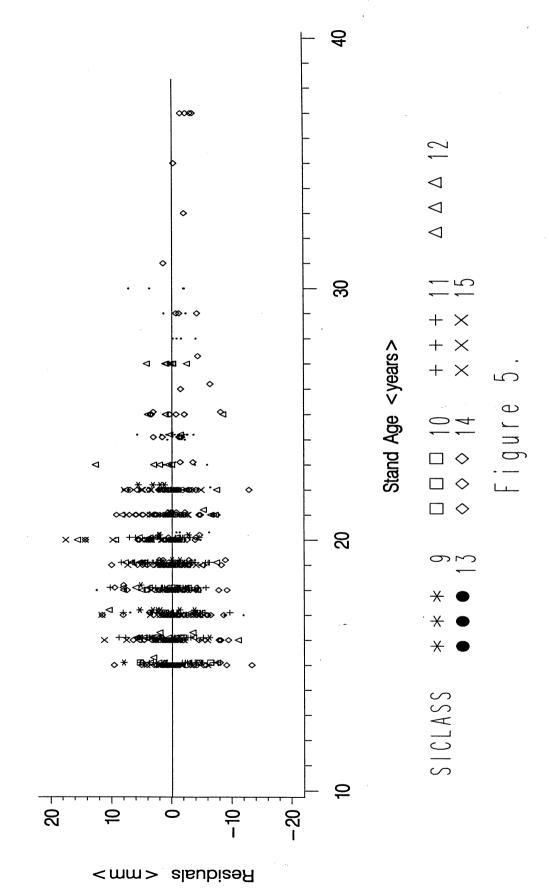
Dbh Increment Residuals vs Stand Age REGION=GDNS



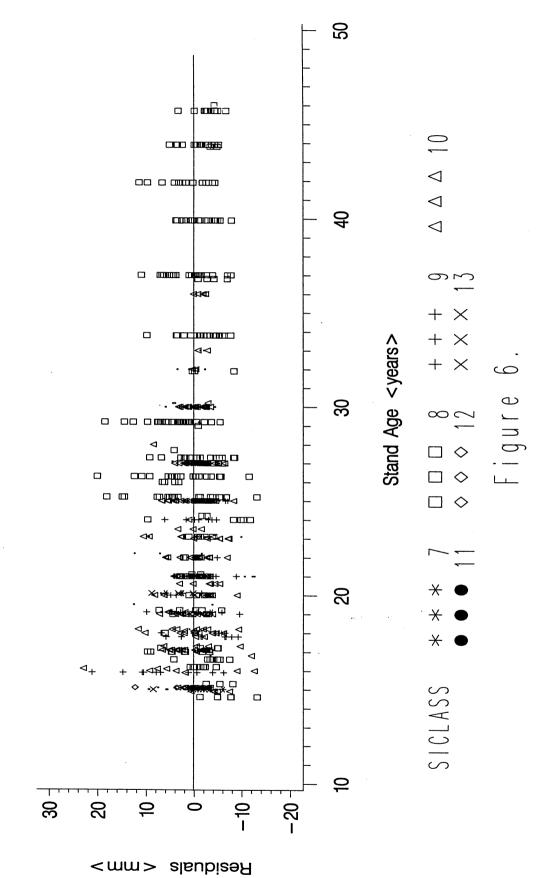
33 32 स्र ဗ္တ 83 8 27 ⊲ 28 ध्य 24 ೫  $\aleph$ ফ 8 9 0 0 5  $\infty$  $\times$ \* 9 d\*∞ === 5 8 9 0 유 Residuals < mm>

Dbh Increment Residuals vs Stand Age REGION=HBAY

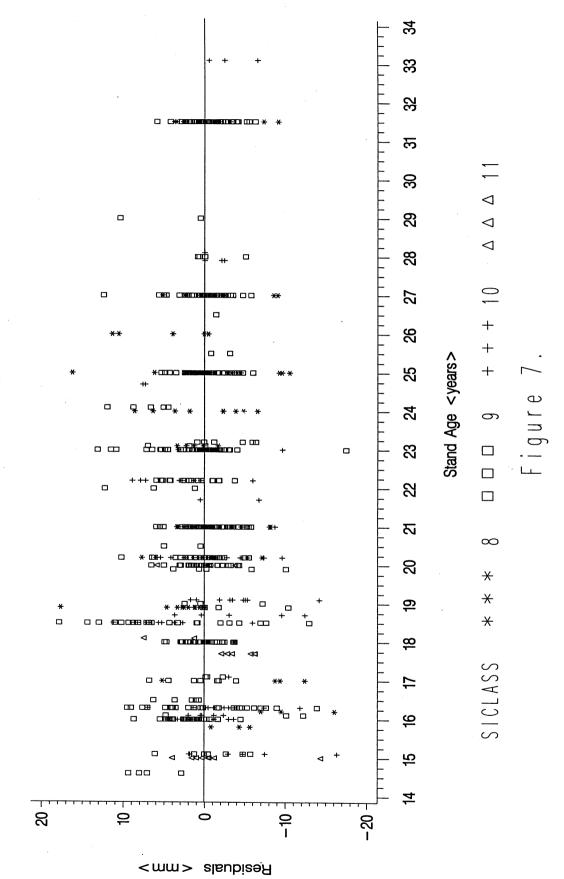
Dbh Increment Residuals vs Stand Age REGION=KANG

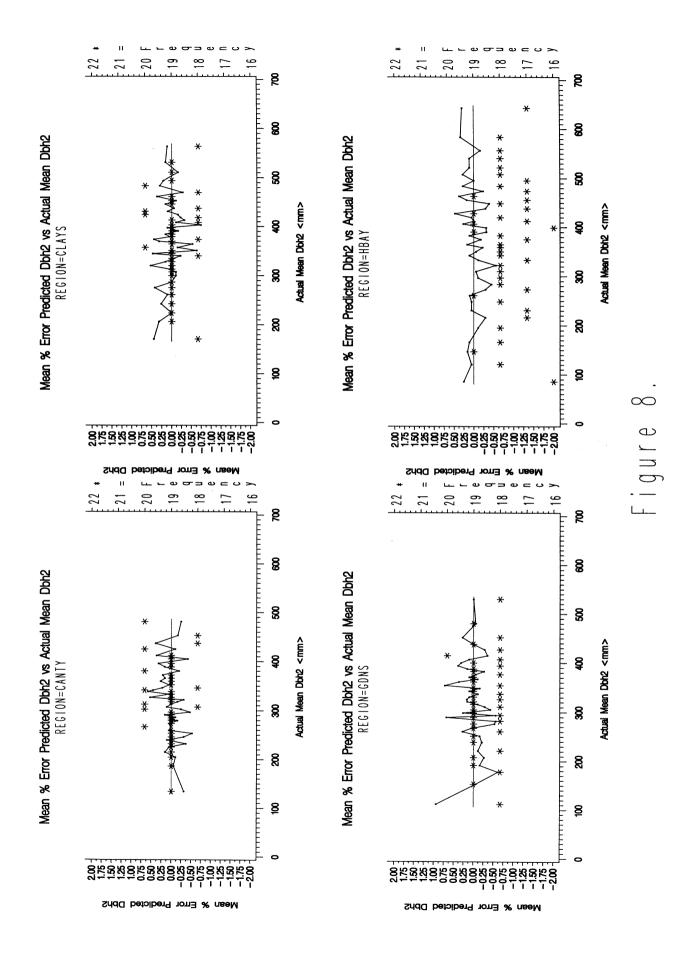


Dbh Increment Residuals vs Stand Age REGION=SANDS



# Dbh Increment Residuals vs Stand Age REGION=SOUTH





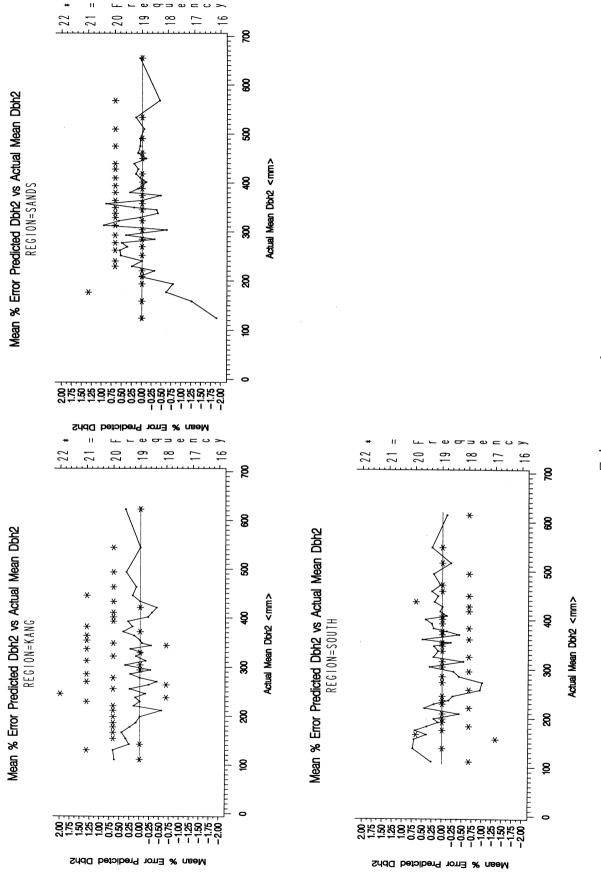


Figure 9.