FRI Project Record

No. 5179

BRANCH DIAMETER GROWTH IN RADIATA PINE IN EXPERIMENT RO 696

J.C. GRACE

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EXECUTIVE SUMMARY

This report contains a detailed account of the analyses undertaken to develop an equation to predict branch diameter growth in radiata pine.

This function will be used in a model of crown development.

BRANCH DIAMETER GROWTH IN RADIATA PINE IN EXPERIMENT RO696

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OBJECTIVE

The objective of this study is to develop equation(s) to model how the branch diameter adjacent to the stem changes over time. At the end of the rotation, the developed equation(s) will allow one to estimate the branch diameter within the stem for each year that the stem has grown.

In a previous study (Grace, 1994), branch diameter was predicted as a function of age. However a more flexible approach woud be to predict branch diameter increment knowing its current diameter and age.

In this study these two different approaches have been investigated.

PREVIOUS WORK

Data (from destructive sampling) indicated that over time branch diameter, adjacent to the stem, increases rapidly to a maximum and then decreases slightly in diameter (Grace, 1994). The decrease in diameter is assumed to occur once the branch has stopped growing, and the stem wood is being laid down over a tapering branch.

Eqn. 1 was fitted to branch diameter data from individual branches on two trees, one from Kaingaroa (Experiment RO905) and one from Tikitere.

$$D = A / (a1 + b1 * A^2) \tag{1}$$

where:

D is the branch diameter
A is the age of the branch
a1 and b1 are model coefficients

This equation predicts branch diameter as a function of age and allows the branch diameter to reach a maximum and then decrease.
The coefficients a1 and b1 were found to be functions of the maximum branch diameter of the individual branch (Grace, 1994).

NEW DATA AVAILABLE

In spring 1994, twelve 26-year-old radiata pine trees were felled from 4 plots in Experiment RO696 in Kaingaroa Forest. These were trees with a small, medium and large DBH from each of 4 stockings (200, 400, 600 and 800 SPH) (see Table 1). The experimental plots were established in a naturally regenerated stand and thinned to their nominal stockings in 1972 when the tallest tree was 6.1 m . The trees were also pruned to 6.1 m. Hence all branches on the sample trees should have been formed when the plots were at their nominal stocking.

Table 1. Tree Number, DBH, and Nominal Stocking (SPH) for trees felled.

Tree Number	DBH (cm)	SPH
18	60.3	200
21	50.8	200
30	37.4	200
33	57.8	400
35	46.1	400
39	31.6	400
71	65.0	600
76	43.4	600
79	30.0	600
56	50.0	800
63	39.5	800
7	29.8	800

After the position of the branch clusters had been measured, the trees were cut into discs. Each branch cluster was in a separate disc, except where clusters overlapped. A sample branch was chosen from each disc containing branches. The sample branches were chosen by selecting a large, medium, small, medium, large diametered branch from consecutive clusters. While this appears to weight the sample towards larger branches, some clusters contain only small branches. The distribution of sample branches in terms of diameter at time of felling is actually weighted towards smaller branches (Table 2).

Table 2. Distribution of branch diameters at time of thinning for sample branches

Branch diameter (D) at time of thinning	Number of branches
missing	33
0 < D <= 1	46
1 < D <= 2	134
2 < D <= 3	83
3 < D <= 4	66
4 < D <= 5	57
5 < D <= 6	28
6 < D <= 7	13
7 < D <= 8	1
D > 8	3

A block, containing the sample branch, was cut from each disc using a chainsaw. These blocks were brought back to FRI and then cut in two, along a plane close to and approximately parallel to the branch pith, using a band saw (see Fig. 1a). Finally, an electric planer was used to expose the branch pith along the branch. The branch diameter, perpendicular to the branch pith was measured for each year that the stem had grown. (see Fig. 1b). The point where the measurement was taken was just above any obvious swelling).

Figure 1a. Diagram showing how sample branch was cut from the disc.

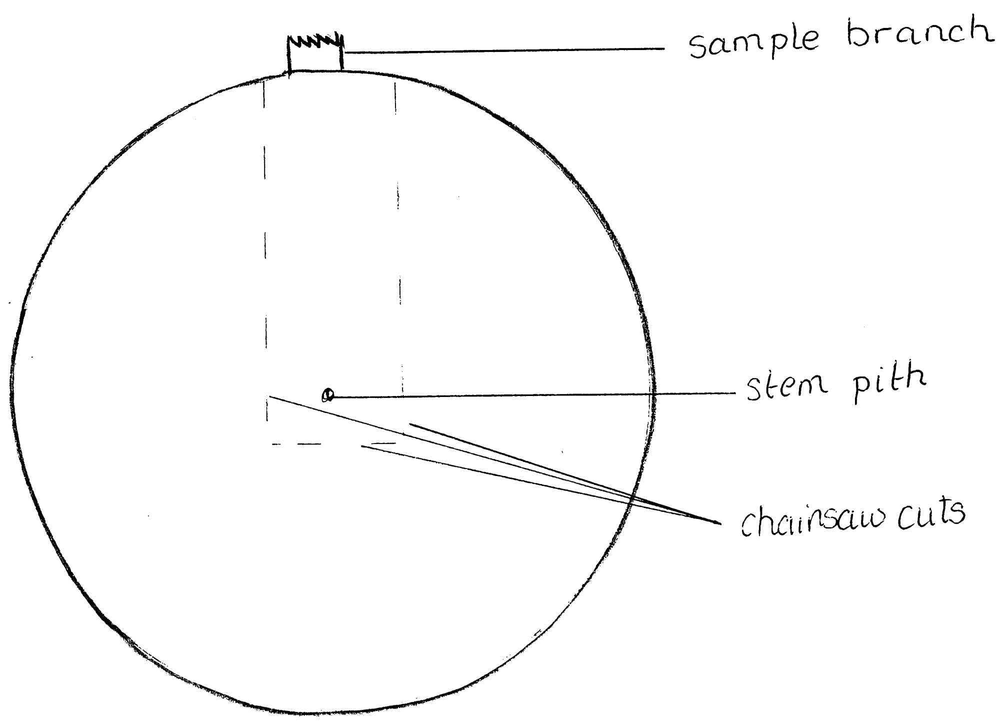
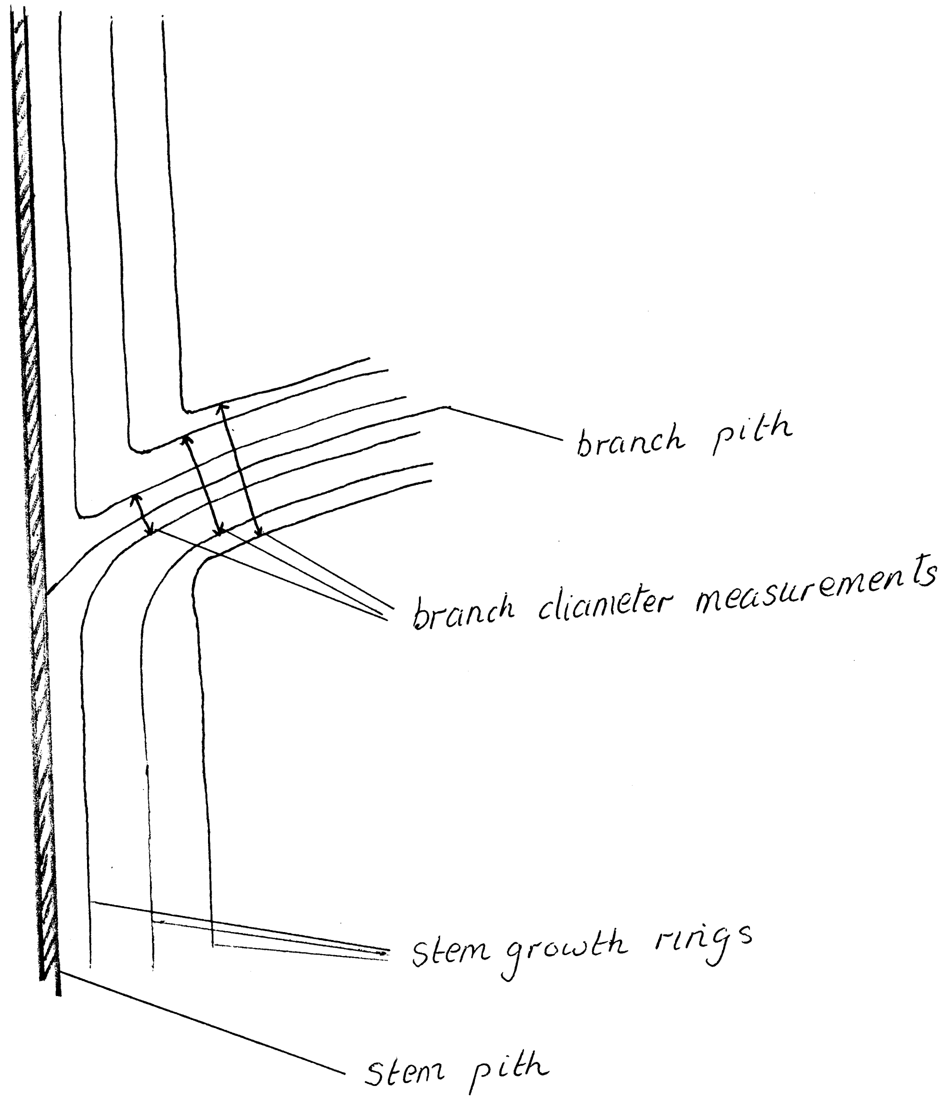


Figure 1b. Diagram showing how branch diameter was measured.



METHODS AND RESULTS

Predicting branch diameter increment

As a preliminary investigation, annual increments in branch diameter were calculated for one tree (Tree 71) and plotted against the initial age and the initial diameter. The variation in diameter increment with initial age was initially large but was clustered around zero by age 8 years. There was no obvious relationship between diameter increment and initial diameter.

Branch diameter increment was predicted as a function of initial branch diameter and initial age. However, for all the functions evaluated, the residuals were correlated with the measured branch diameter at the time of felling.

This result is not surprising, since branch diameter can be predicted as a function of maximum branch diameter and age (see previous study, above).

The approach of predicting branch diameter increment was therefore not pursued any further at this stage.

Predicting branch diameter as a function of age

An equation is needed which allows branch diameter to grow to a maximum and then decrease slightly with increasing age. One such equation is Eqn. 2. In the previous study (see above), a special case of Eqn. 2 was used to predict branch diameter from age.

$$D = A / (a2 + b2*A + c2*A^2) \qquad (2)$$

where:

D is the branch diameter
A is the branch age
a2,b2,c2 are model coefficients.

Another equation which would allow branch diameter to increase to a maximum with age and then decrease slightly in size is Eqn. 3

$$D = A / (a + b*A + c*A^{0.5}) \qquad (3)$$

where:

D is the branch diameter
A is the branch age
a, b, c are model coefficients.

Both equations are capable of having a maximum and at least one inflection point (Ratkowsky, 1990). From his diagrams, diameter appears to decline more slowly with increasing age once the maximum has been reached when using Eqn 3 compared to Eqn 2.

As a preliminary study, Eqns. 2 and 3 were fitted (using the SAS NLIN procedure) to data from individual branches with 5 or more growth rings from two of the trees in RO696. These were the largest tree sampled at 200 SPH and the average-sized tree at 800 SPH.

The residual mean sqares from fitting Eqns. 2 and 3 were compared. For both trees, Eqn 3, tended to provide the better fit to the older branches, while Eqn 2, provided the better fit for the younger branches.

In order to further compare these equations, a small set of branches covering the range of number of stem growth rings and final diameters was selected and Eqns 2 and 3 fitted. Also, the influence of the power A is raised to in Eqn 3. was investigated by fitting Eqn. 4 for each branch with p = 0.1, 0.3, 0.7, 0.9.

$$D = A / (a4 + b4*A + c4*A^p) \qquad (4)$$

where:

D is the branch diameter
A is the branch age
a4,b4,c4 are model coefficients
p = 0.1, 0.3, 0.7, 0.9

The residual sum of squares from fitting Eqns. 2 - 4 to the selected branches are shown in Table 3.

Table 3. Residual sum of squares from fitting Eqns. 2-4 to selcted sample branches.

Tree	Cluster Number (whorl)	Number of stem growth rings	final branch diameter (cm)	p =0.1 Eqn. 4	p = 0.3 Eqn. 4	p = 0.5 Eqn. 3	p =0.7 Eqn.4	p = 0.9 Eqn. 4	p =2.0 Eqn. 2
30	30	10	12	2.00	2.01	2.05	2.14	2.26	3.30
35	27	15	13	1.18	1.50	1.89	2.34	2.86	6.21
56	9	20	13	3.00	3.72	4.52	5.39	6.32	11.37
63	26	10	20	5.78	5.33	4.96	4.69	4.51	4.76
33	27	15	21	5.38	4.96	4.62	4.37	4.22	4.90
39	1	20	21	12.06	14.30	16.65	19.07	21.52	33.90
76	41	10	42	14.12	12.32	10.97	10.06	9.56	12.71
63	15	15	43	29.44	27.39	26.10	25.54	25.64	34.64
35	8	20	43	24.28	.	32.74	.	44.27	85.25
21	150	10	58	56.93	55.41	54.18	53.22	52.53	52.82
21	2	20	59	29.74	30.60	33.33	37.79	43.75	92.83

From this table it can be seen that, p=0.5 generally gives a better fit than p=2. For p< 1.0, in general, the smallest residual mean square occurs when p=0.1 for the smaller and the older branches and that the smallest residual mean square occurs when p=0.9 for the larger and younger branches.

The actual and predicted branch diameters were then plotted against age (see Appendix 1). From these figures it can be seen that Eqn 3. provides a better fit than Eqn. 2, however there was little practical difference Eqns. 3 and 4.

It was decided to investigate Eqn. 3 further as this provides a reasonable fit for all the above branches.

Eqn. 3 was then fitted to each of the 464 branches in the dataset using the SAS NLIN procedure.

The NLIN procedure failed to converge for 3 branches and 24 branches had large residual sum of squares.

Obviously we do not want to have separate model coefficients for each branch. We need to develop one equation where a, b, and c (Eqn. 3) are functions of variables related to the branch. The following analyses were aimed at achieving this.

The predicted values of a, b and c were plotted against
the number of stem growth rings below the branch,
the measured maximum diameter the branch reached (M).

(Tree age minus the number of stem growth rings below the branch plus one equals the age of the tree when the branch was formed).

The estimated maximum branch diameter was plotted against measured maximum branch diameter (M).

The estimated age at which the maximum branch diameter occurs was plotted against the age at which the observed maximum branch diameter first occurred (A_{min}).

These plots highlighted possible outliers which were examined in detail. Some combinations of a, b and c were found to be unrealistic, see Appendix 2.

The conclusions from these analyses are:

branch diameter growth is not related to the tree age when the branch was formed.

b is a linear function of M^{-1} (5)

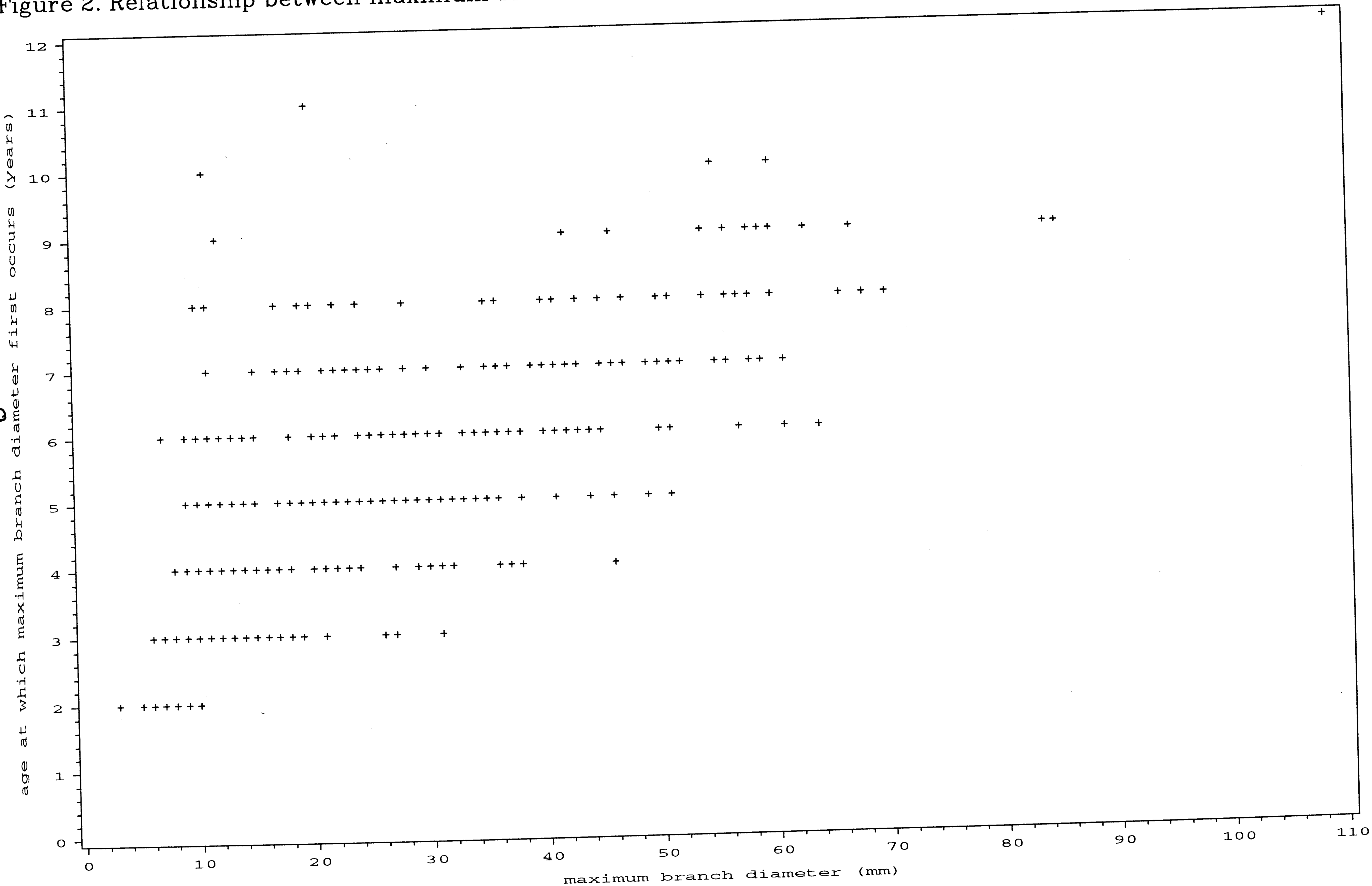
c^2/a is a linear function of M^{-1} (6)

a is a linear function of M^{x1} (7)

c is a linear function of M^{x2} (8)

The age at which the maximum branch diameter first occurred (A_{min}) was plotted against the observed maximum branch diameter (M) (see Fig. 2), A_{min} was predicted as a function of M. Only branches were A_{min} was less than the age of the branch at felling were considered.

Figure 2. Relationship between maximum branch diameter and the age at which it first occurs



The predicted relationship was:

$$A_{min} = 1.76\ M^{0..35} \qquad (9)$$

There were no obvious trends when the residuals were plotted against:

predicted values
number of stem growth rings below the branch
tree DBH
SPH

The error (actual -estimated) in predicting A_{min} generally varied between -3 and +3 years.

From Appendix 2:

$$A_{min} = 4\ a^2/c^2 \qquad (10)$$

Combining Eqns 5 -10 implies:

$$2(x1) - 2(x2) = 0.35$$
$$2(x2) - (x1) = -1.0$$

solving these simultaneous equations gives:

$x2 = -0.825$
$x1 = -0.65$

This solution depends on the observed relationship between A_{min} and M.

One equation (Eqn. 11) was then fitted to the sub-set of data used previously.

$$D = A/\ (a6\ M^{x1} + b6\ M^{-1}\ A + c6\ M^{x2}\ A^{0.5}) \qquad (11)$$

Eqn. 11 was fitted using various values for x1 with $2(x2) - x1 = -1$

The residual mean squares are shown in Table 4.

Table 4. Residual mean squares from fitting Eqn. 11 to a subset of the data.

x1	Residual mean square
-0.4	5.11
-0.5	4.81
-0.6	4.68
-0.7	4.76

The optimum value for x1 (obtained using the SAS NLIN procedure) was x1 =-0.61.

A similar analysis was repeated using the whole dataset (see Table 5).

Table 5. Residual mean squares from fitting Eqn. 11 to the whole dataset.

x1	Residual mean square
-0.3	4.97
-0.4	4.72
-0.5	4.76
-0.6	5.11

The optimum value for x1 (obtained using the SAS NLIN procedure) was x1 = -0.44.

It is of interest to note that the optimum value for x1 for the subset of data is close to the value predicted using Eqns. 5-10. However this is not the case for the whole dataset. A reason for this is that PROC NLIN is determining the best fit for all datapoints, not just the maximum values.

In all cases, the residuals were plotted against:

predicted values
age
maximum branch diameter (M)
tree DBH

The residual plots were similar for all values of x1. Those for x1=-0.44 are shown in Appendix 3.

From examining these plots, the following conclusions can be drawn.

There is no obvious trend in the residuals when plotted against predicted branch diameter or tree DBH. There is little trend when the resisuals are plotted against measured maximum diameter (M). However there is a trend when the residuals are plotted against age. Branch diameter is underpredicted at the later ages., i.e branch diameter is predicted to decline more rapidly than occurs in practice. This trend can be seen by examining the plots in Appendix 1.

Even though there is a trend in the residual plot, branch diameter is generally predicted to within 5 mm at the later ages. Hence the trend is not considered to be of practical importance.

This trend with age occurred with all values of x1.

The error in predicting A_{min} and M were plotted against their estimated values (see Appendix 3). In both cases there is a trend in the residual plot. This is likely to be due to the fact the optimum value for x1 (Eqn. 11) is not the value derived from solving Eqns 5-10.

SUMMARY

Branch diameter can be predicted as a function of age using:

$$D = A / (a6\, M^{x1} + b6\, M^{-1}\, A + c6\, M^{x2}\, A^{0.5})$$

where:

D is the branch diameter in mm
M is the maximum diameter of that branch in mm
A is the age of the branch in mm

a6 = 0.99
b6 = 1.92
c6 = -1.91
x1 = -0.44
x2 = -0.72
2(x2) - x1 = -1

A method for predicting the maximum branch diameter of a branch is given in Grace (1996).

FUTURE RESEARCH

The above analysis indicates that branch diameter can be predicted as a function of age with reasonable success. There are some trends in the residuals from fitting the chosen equation but these are unlikely to be of practical significance.

This equation is not general enough to predict branch diameter in all situations. For example, this approach is not capable of allowing branch diameter to change in response to silviculture treatment.

In implementing this equation in a computer graphics model, it was found that the age 1 diameter for branches at the top of annual shoots was rather large. This is considered to be due to the fact that the branches have not been growing for a full year (D. Pont pers. comm.). This problem should be overcome by relating branch growth to stem diameter growth (see below).

Predicting diameter increment is likely to be more flexible. When data were collected from a thinning trial, (RO905) in March 1996, stem diameter growth below the branch was measured as well as branch diameter growth. It is hoped that the additional data can be used to predict branch diameter increment from stem diameter increment.

REFERENCES

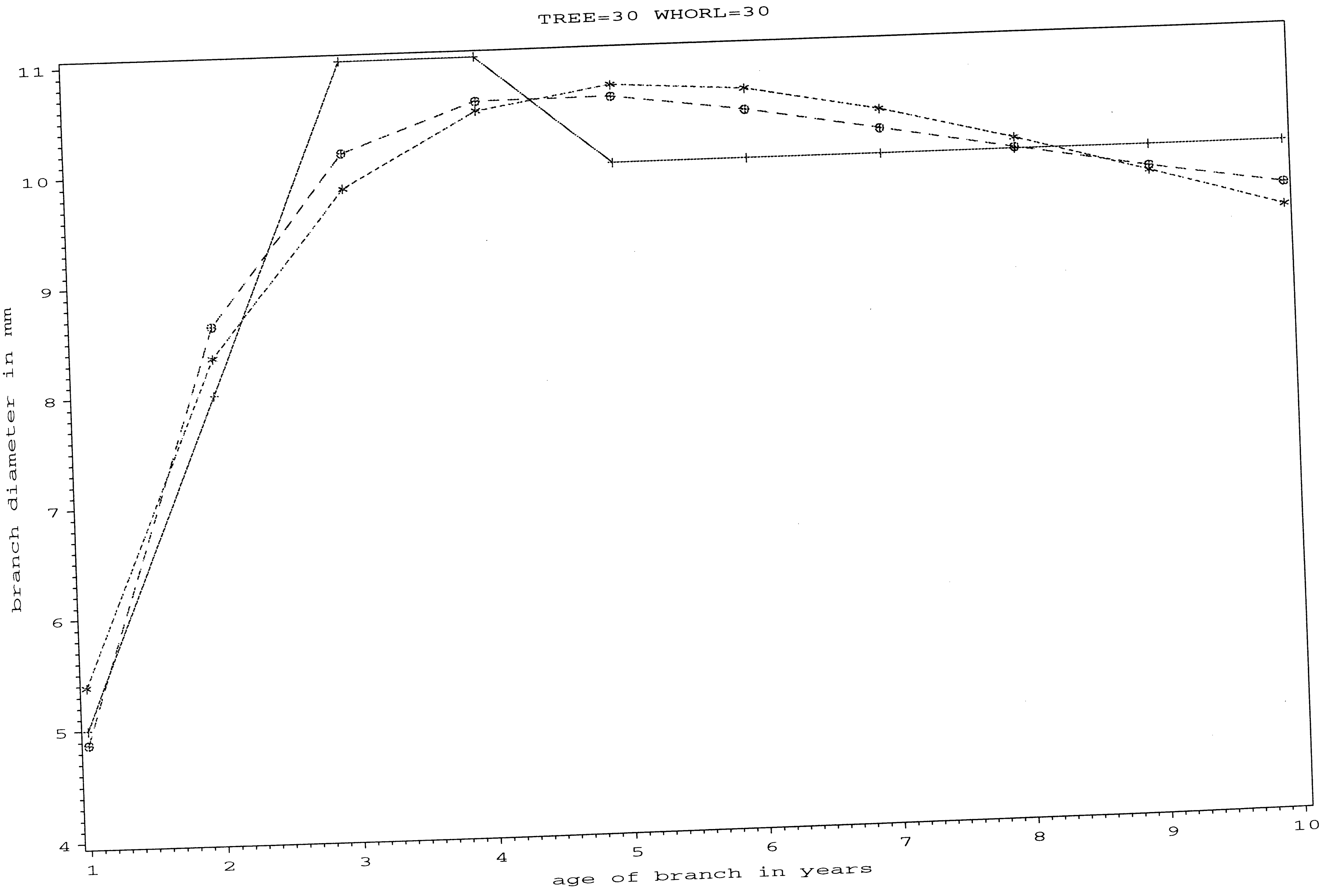
Grace, J.C. 1994 Methodology for collecting branch growth data. Stand Growth Modelling Co-operative Report No. 37.

Grace, J.C. 1996. Diameter of branches within a cluster in Experiment RO696. Stand Growth Modelling Co-operative Report No. 50.

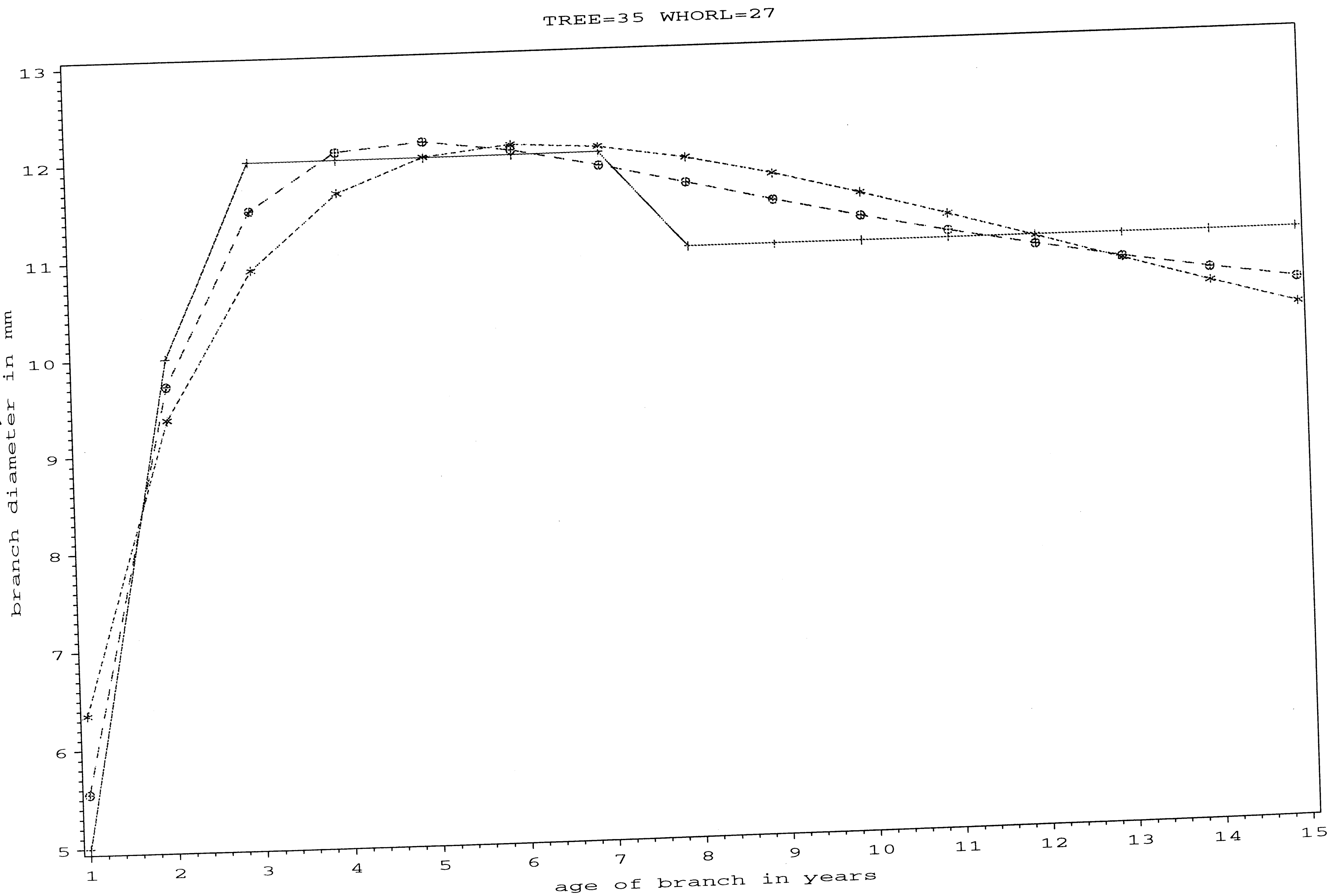
Ratkowsky, D.A. 1990. Handbook of nonlinear regression models. Marcel Dekker Inc. 241pp.

Appendix 1a. Actual and predicted branch diameter using Eqns. 2 and 3.

Key:		
	+ ———————+	actual branch diameter
	* - - - - - - - *	predicted branch diameter using Eqn. 2
	θ— — — —θ	predicted branch diameter using Eqn. 3


TREE=30 WHORL=30
branch diameter in mm
age of branch in years
4
5
6
7
8
9
10
11
1
2
3
4
5
6
7
8
9
10

14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branch diameter in mm
age of branch in years
5
6
7
8
9
10
11
12
13
1
2
3
4
5
6
7
8
9
10
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12
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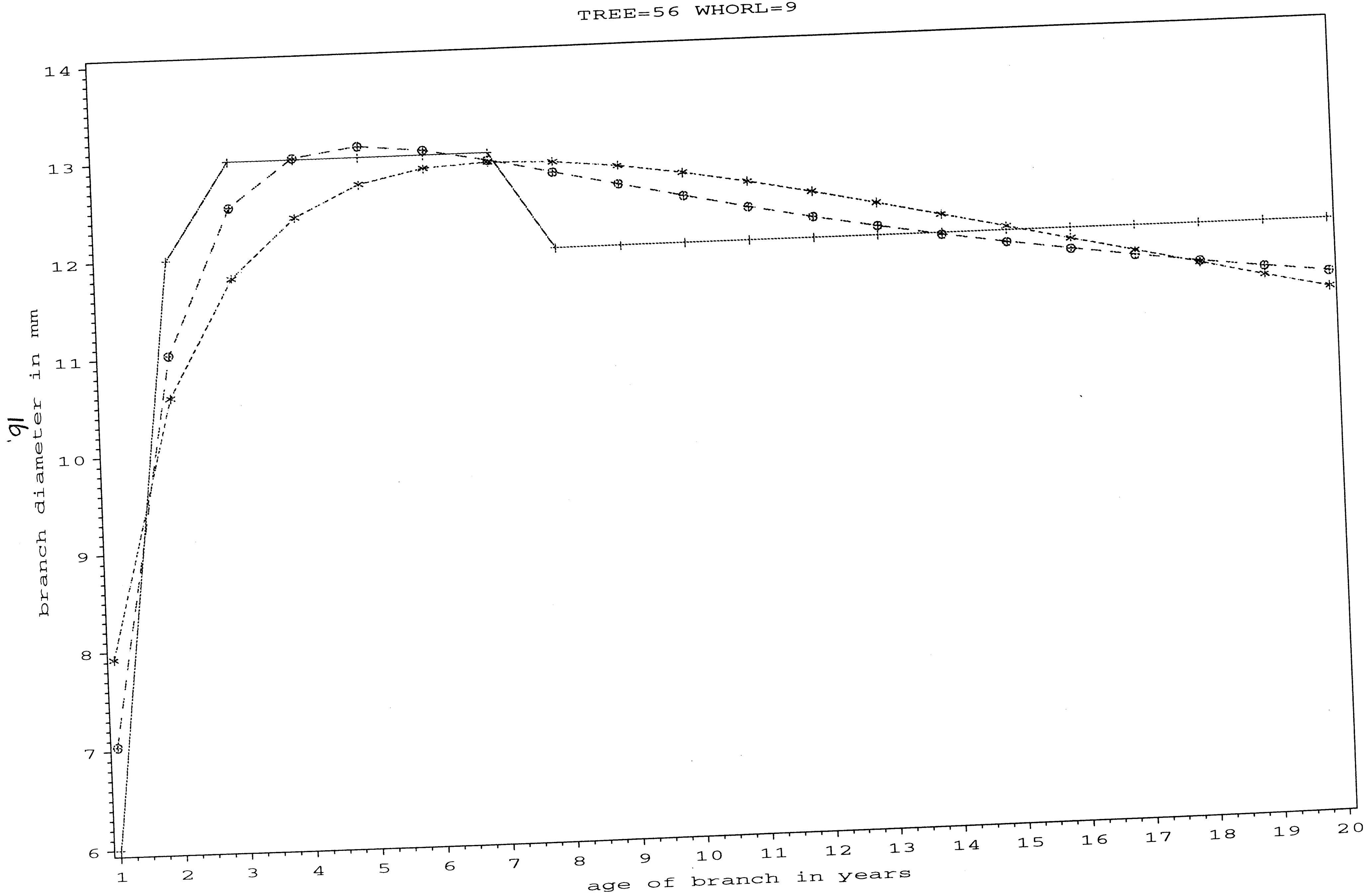
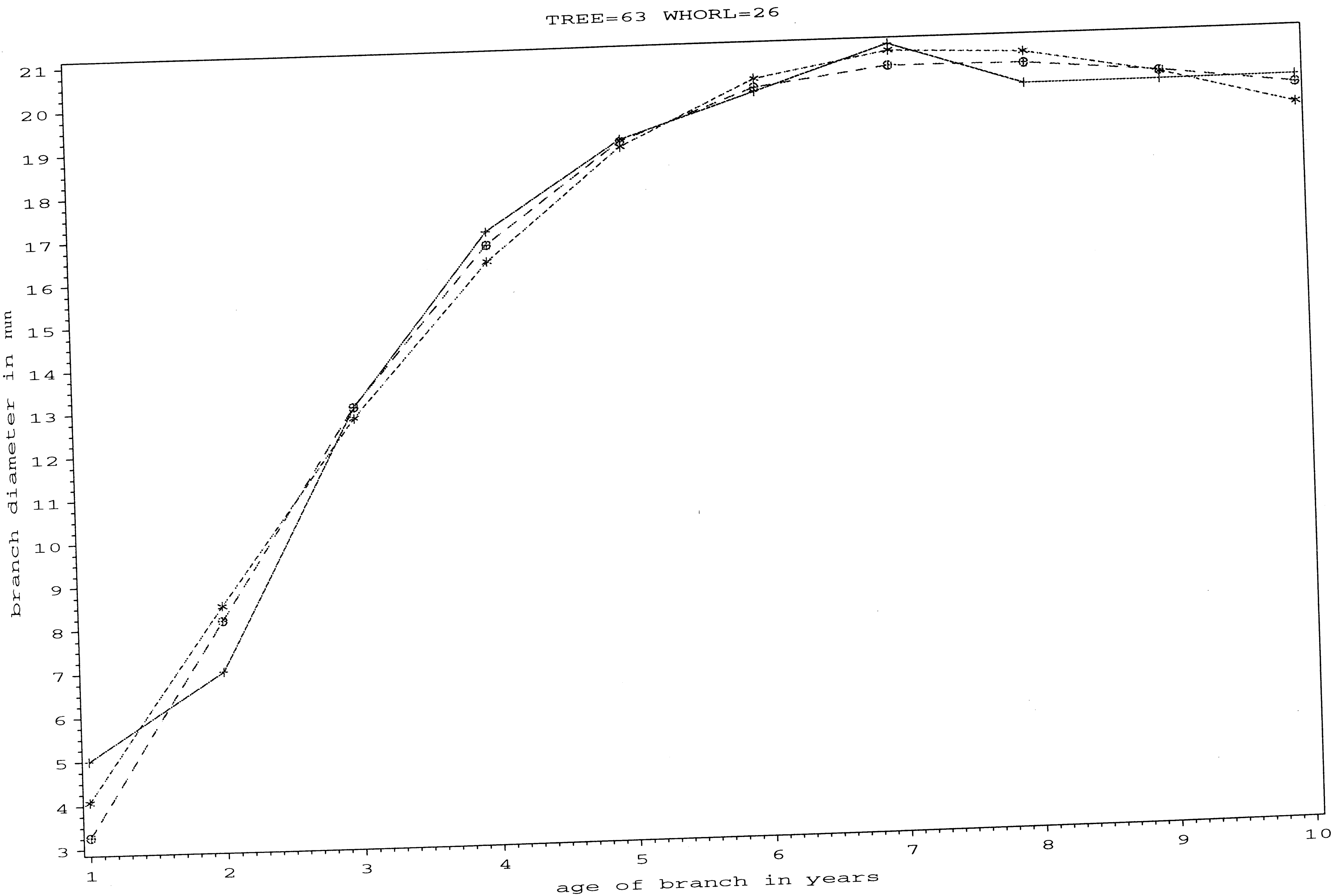
14
15

TREE=56 WHORL=9

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branch diameter in mm
age of branch in years
3
4
5
6
7
8
9
10
11
12
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14
15
16
17
18
19
20
21
1
2
3
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5
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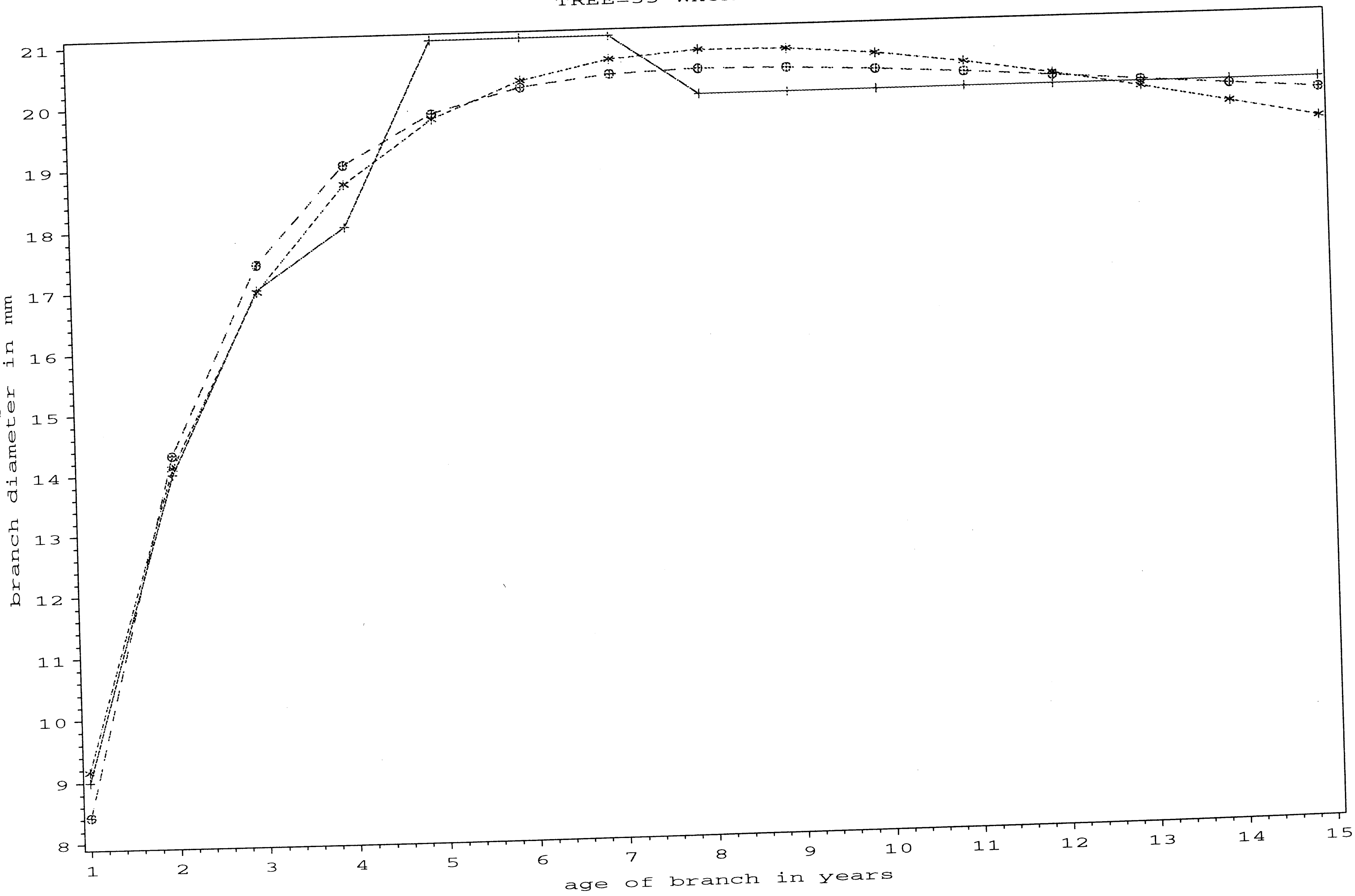
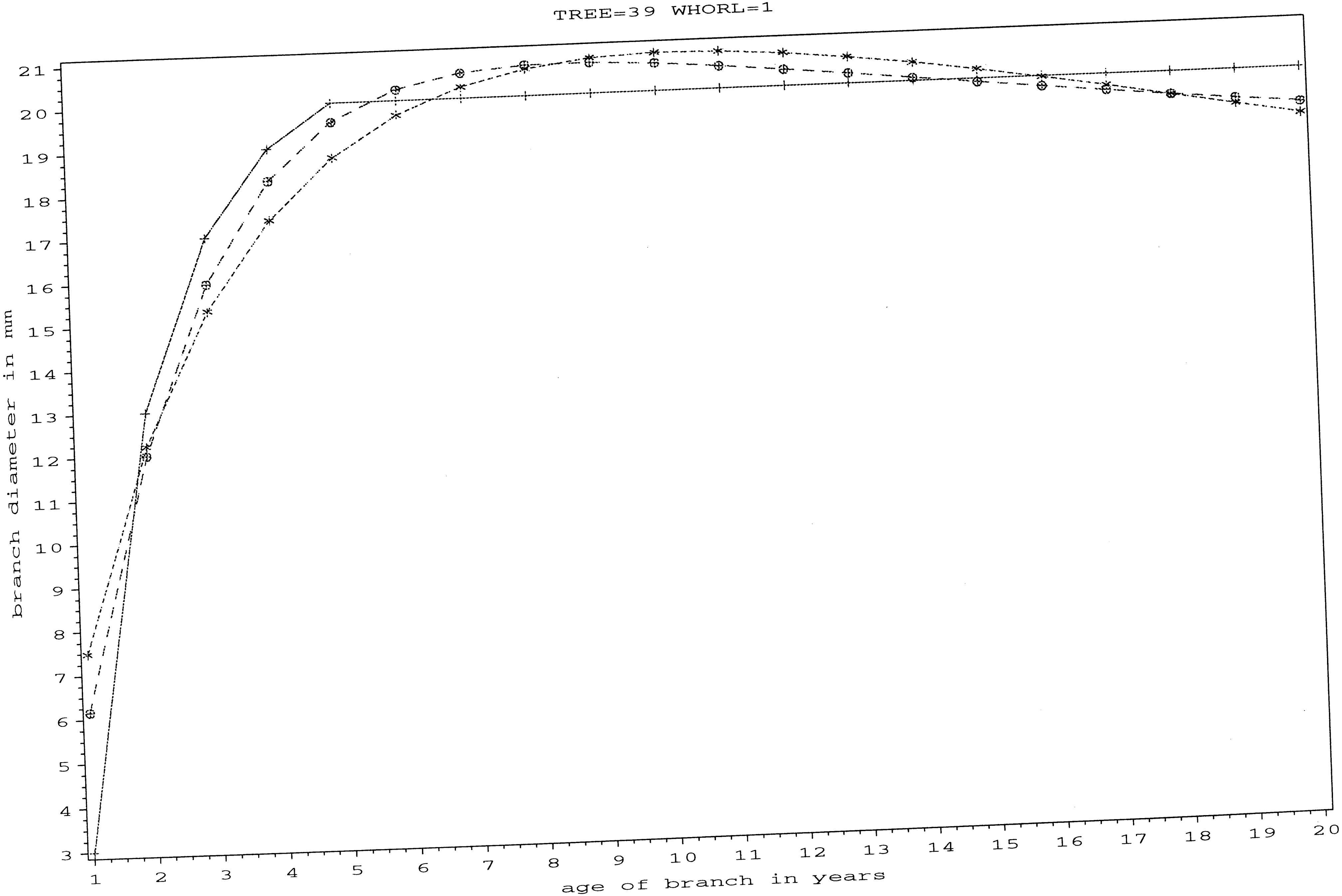
9
10

TREE=33 WHORL=27

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branch diameter in mm
age of branch in years
21
20
19
18
17
16
15
14
13
12
11
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9
8
7
6
5
4
3
1
2
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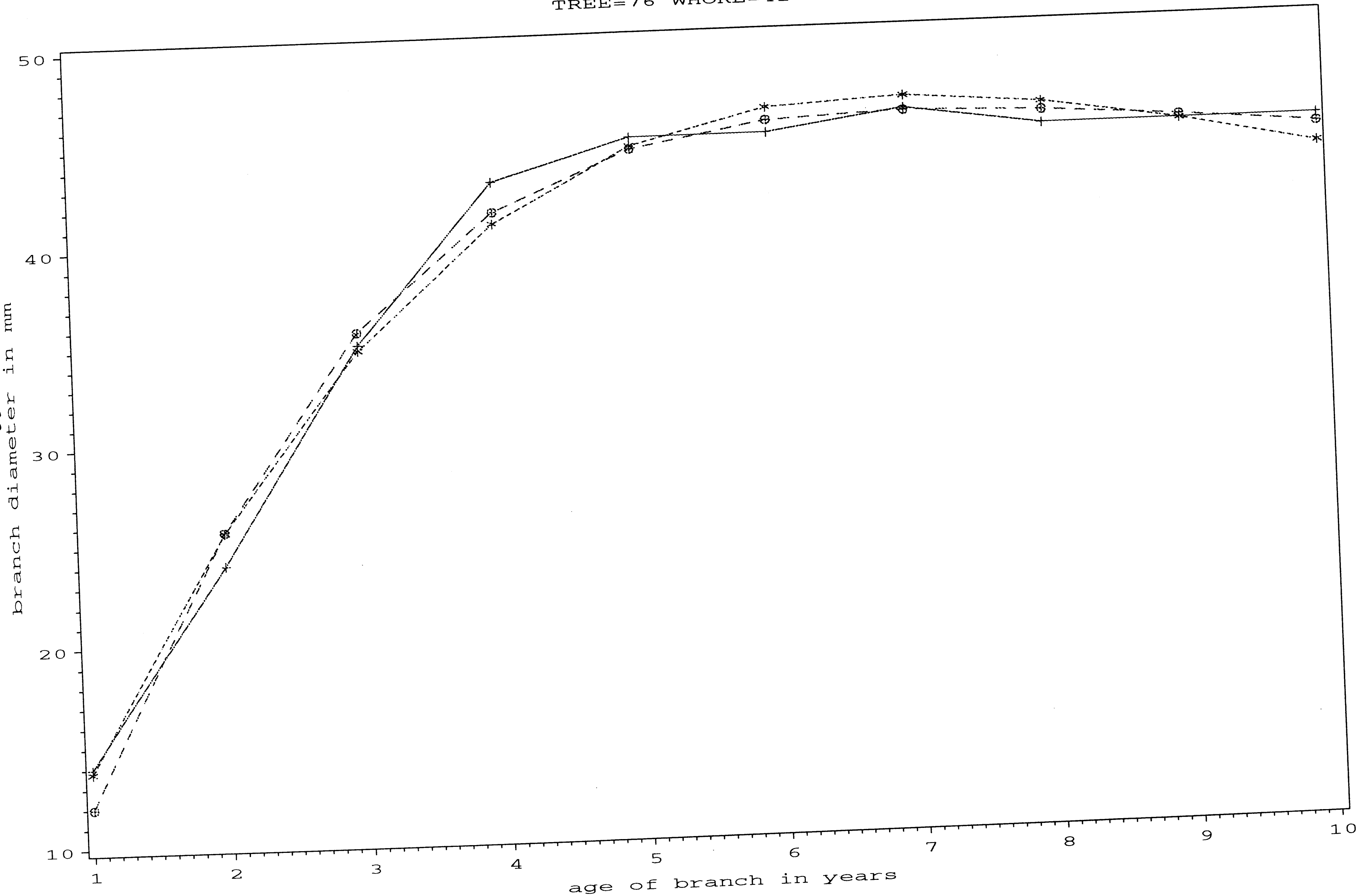
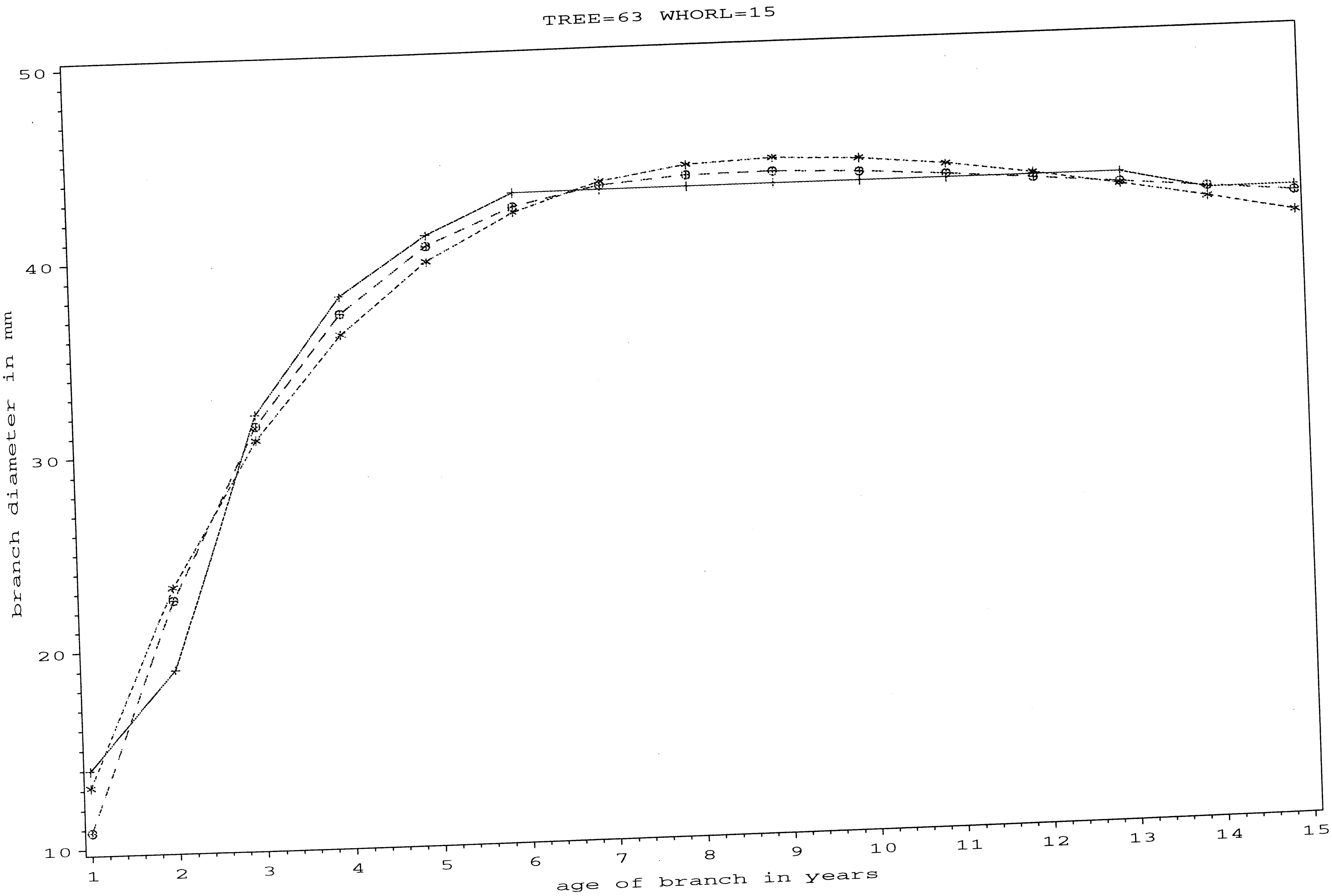
TREE=76 WHORL=41

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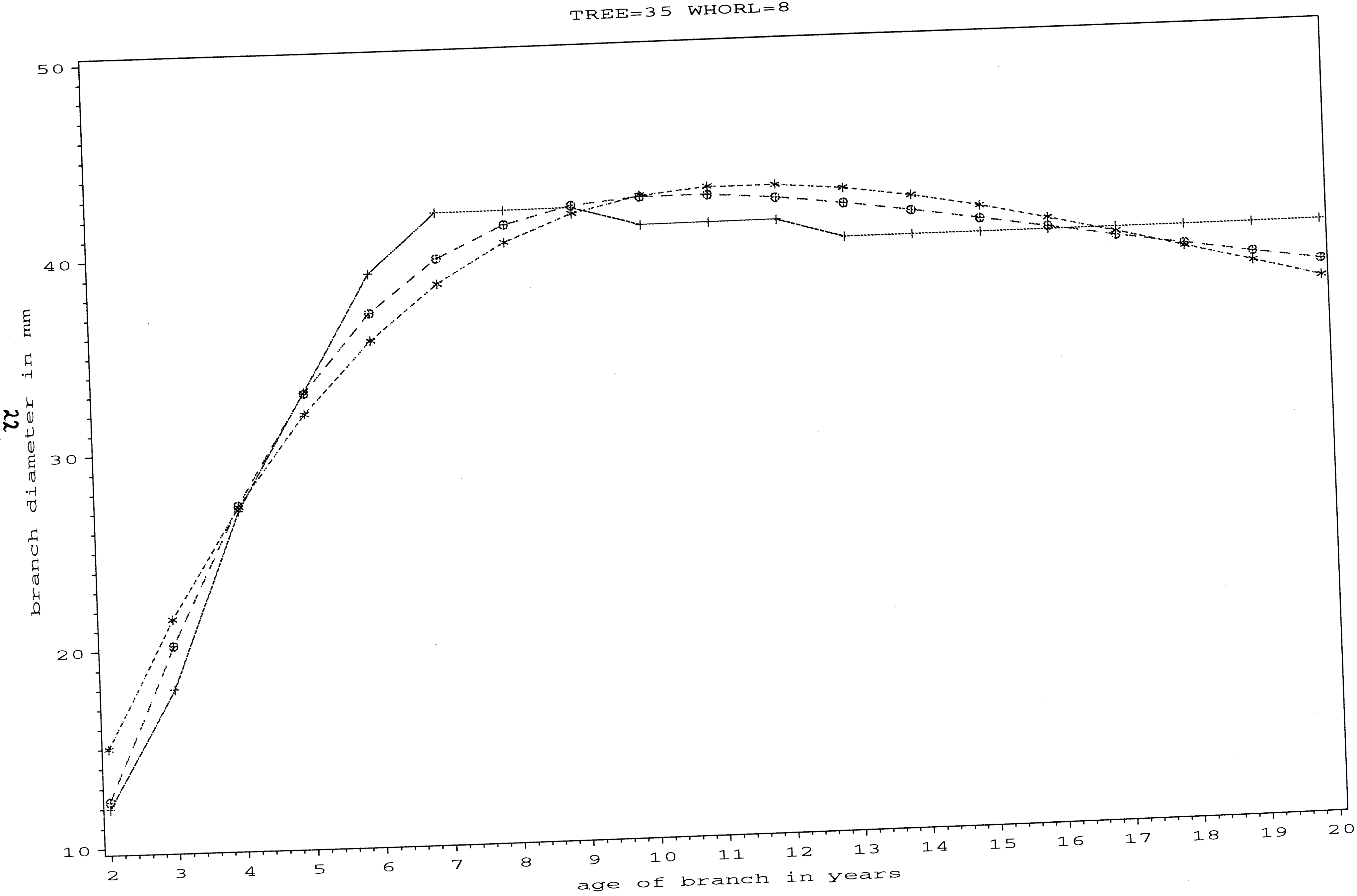
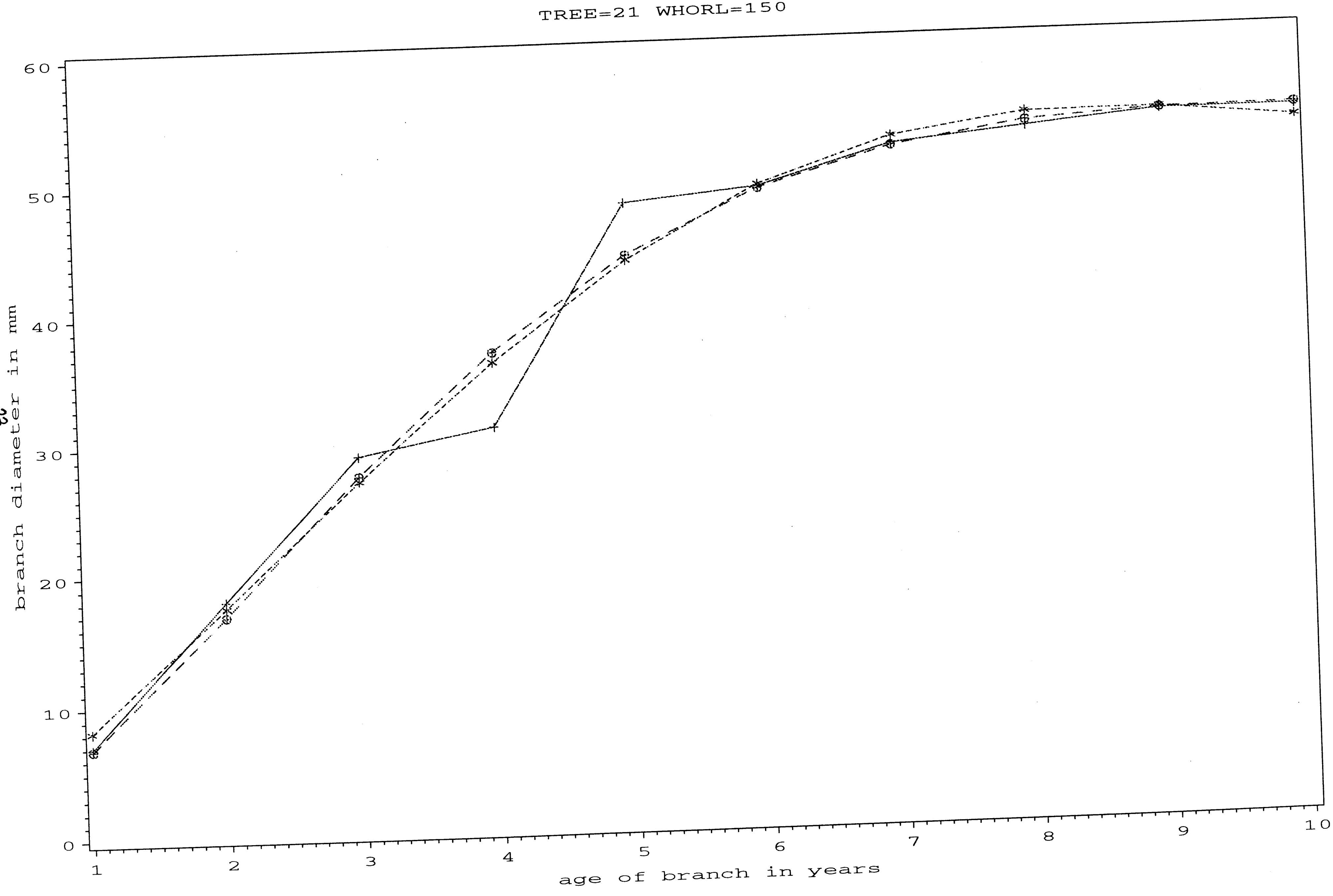
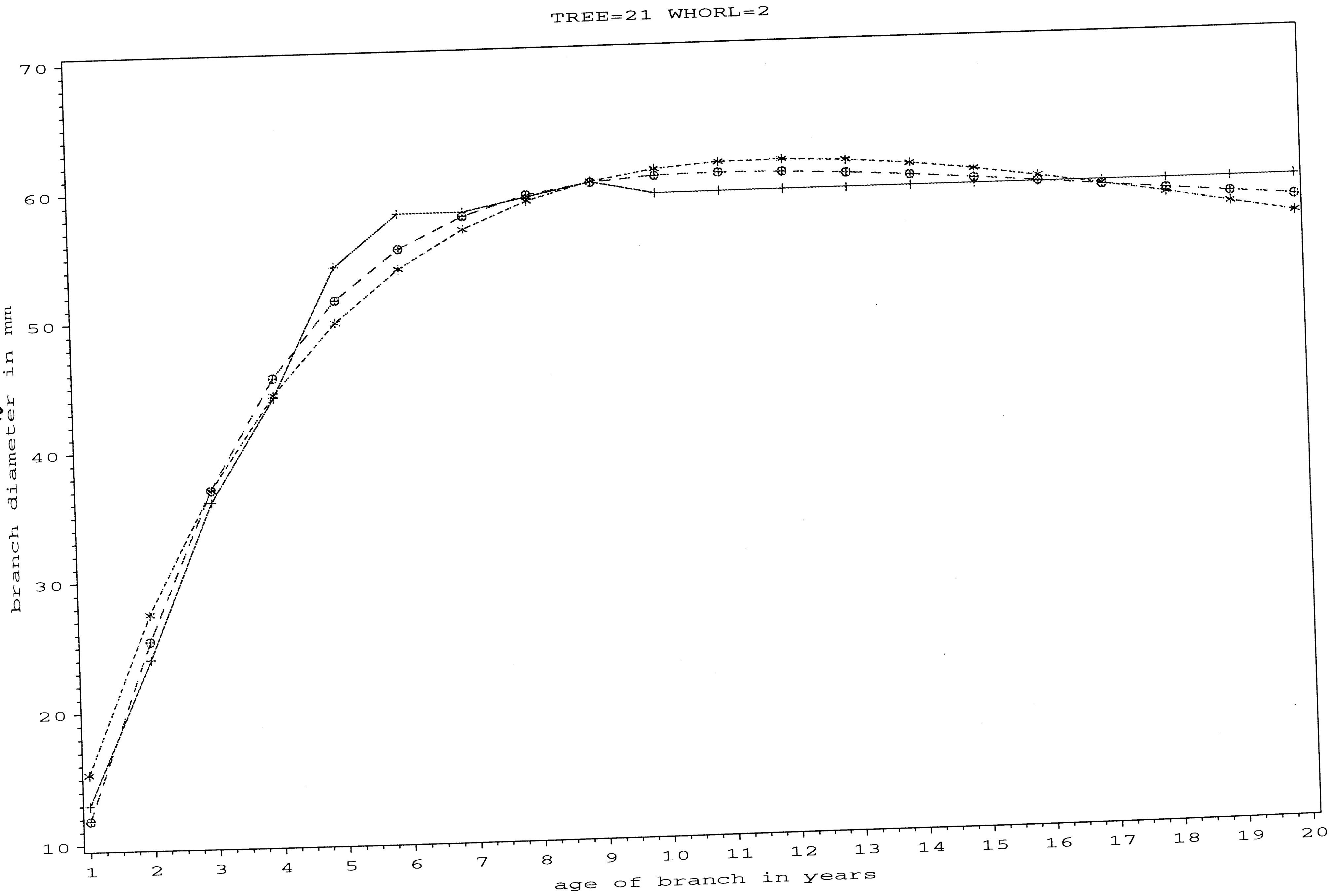
branch diameter in mm
age of branch in years
10
20
30
40
50
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

TREE=35 WHORL=8



TREE=21 WHORL=150




TREE=21 WHORL=2
branch diameter in mm
10
20
30
40
50
60
70
age of branch in years
1
2
3
4
5
6
7
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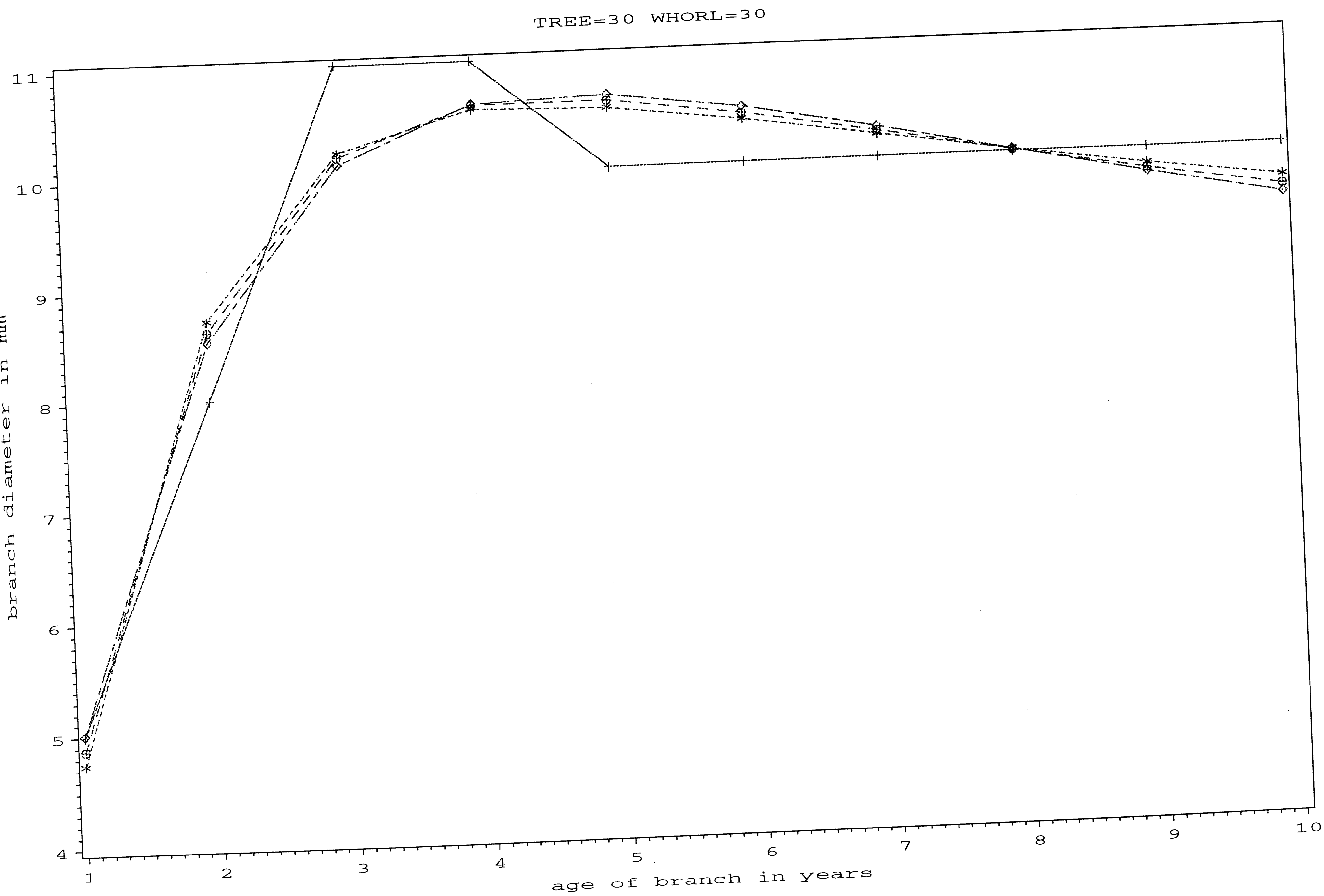
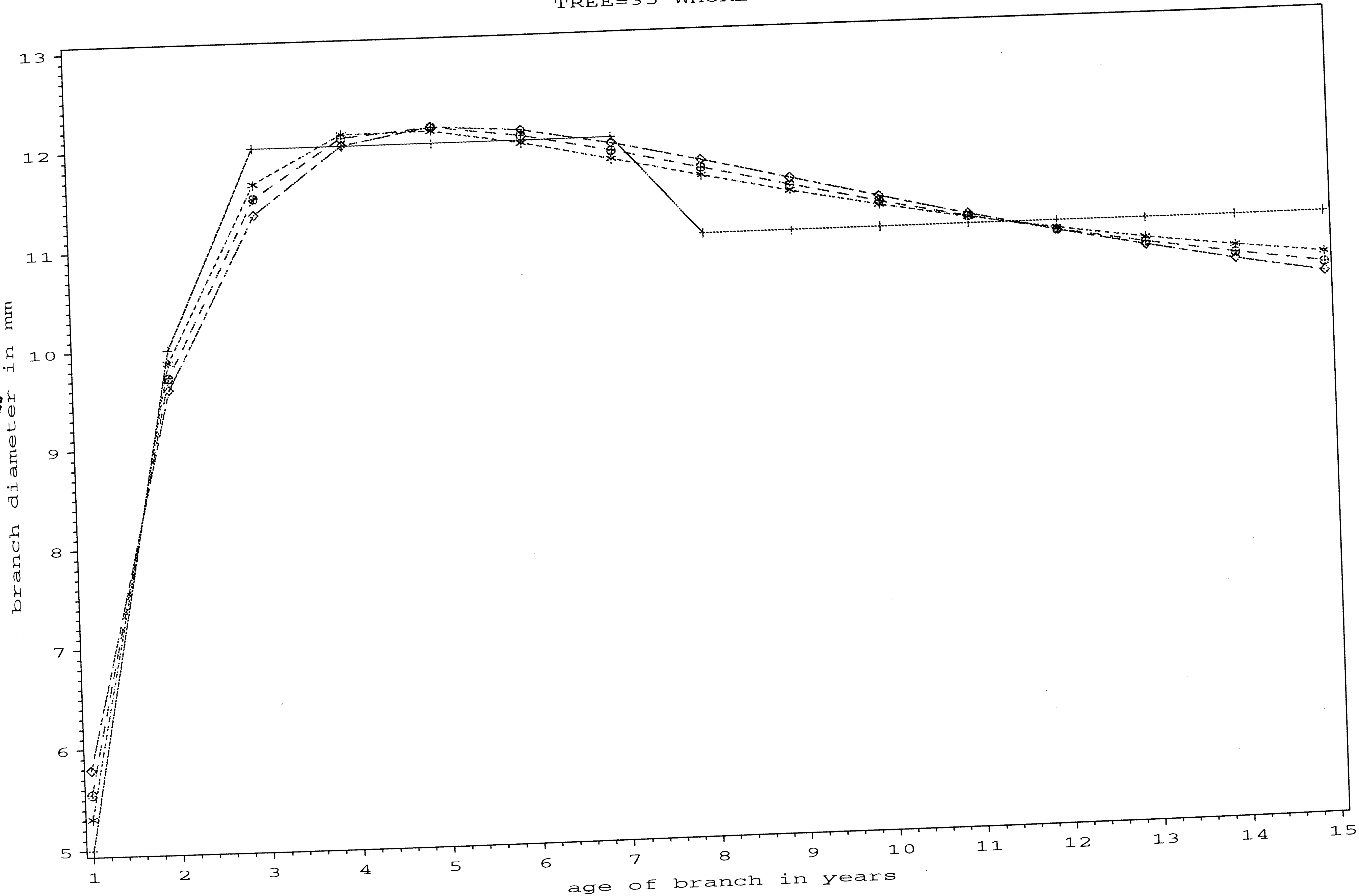
Appendix 1b. Actual and predicted branch diameter using Eqns. 3 and 4.

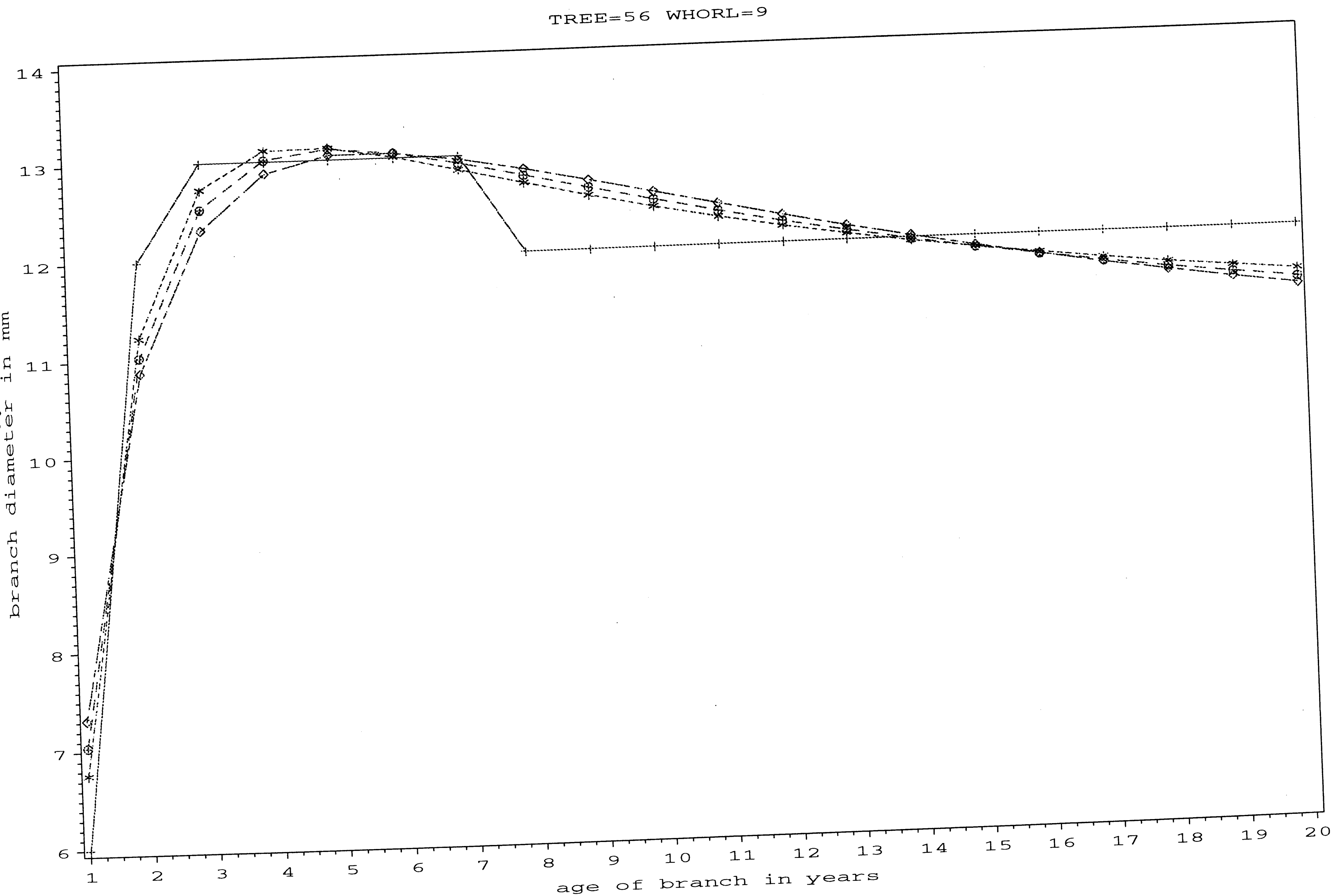
Key:	
+————————+	Actual branch diameter
* - - - - - - - *	predicted branch diameter with p=0.1 in Eqn. 4
⊕— — — —⊕	predicted branch diameter using Eqn. 3 (i.e. p=0.5 in Eqn. 4)
◇——- —— ◇	predicted branch diameter with p=0.9 in Eqn. 4



TREE=30 WHORL=30
branch diameter in mm
age of branch in years

TREE=35 WHORL=27





TREE=56 WHORL=9
branch diameter in mm
age of branch in years
6
7
8
9
10
11
12
13
14
1
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18
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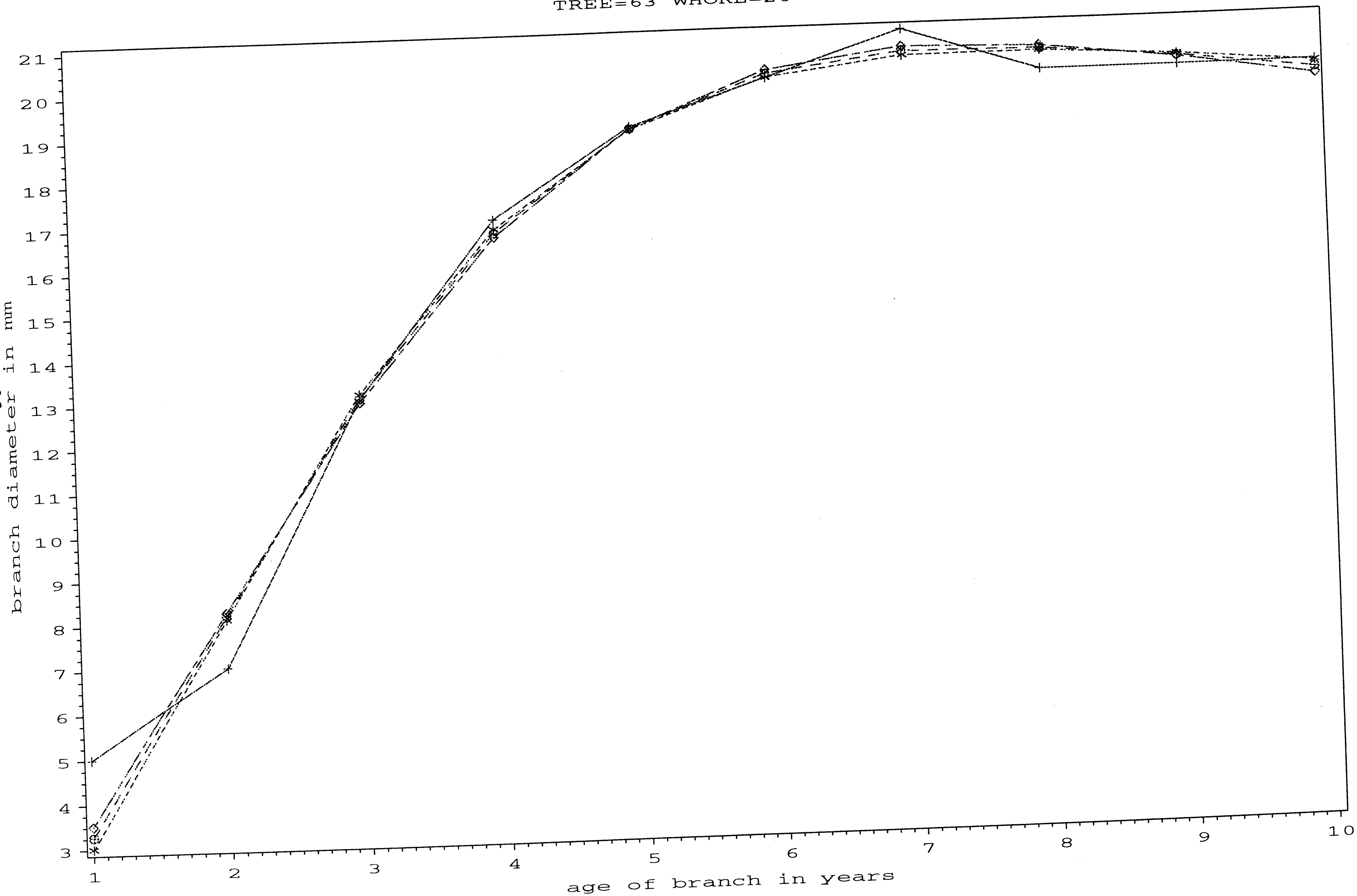
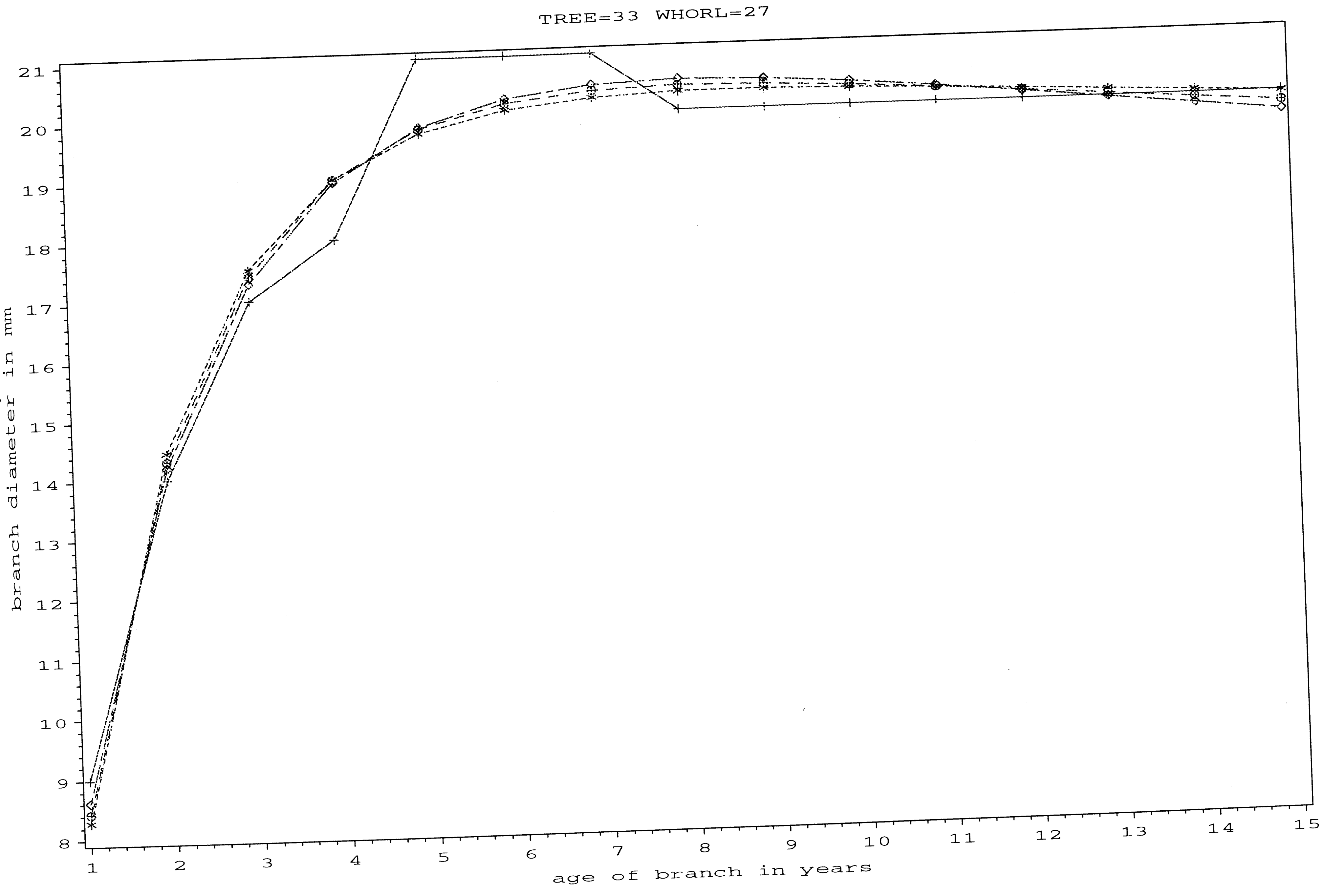
20

TREE=63 WHORL=26

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branch diameter in mm
8
9
10
11
12
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age of branch in years
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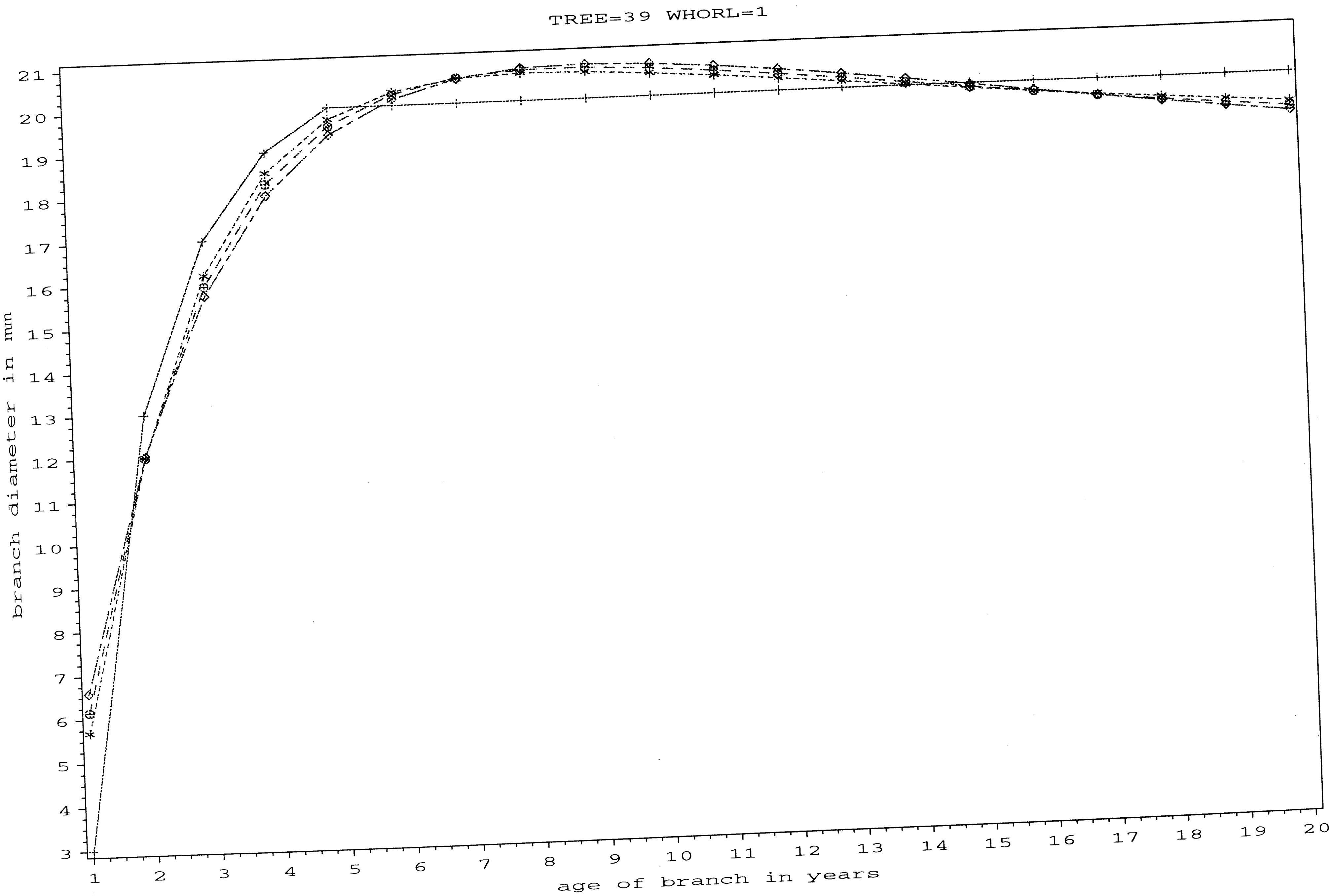
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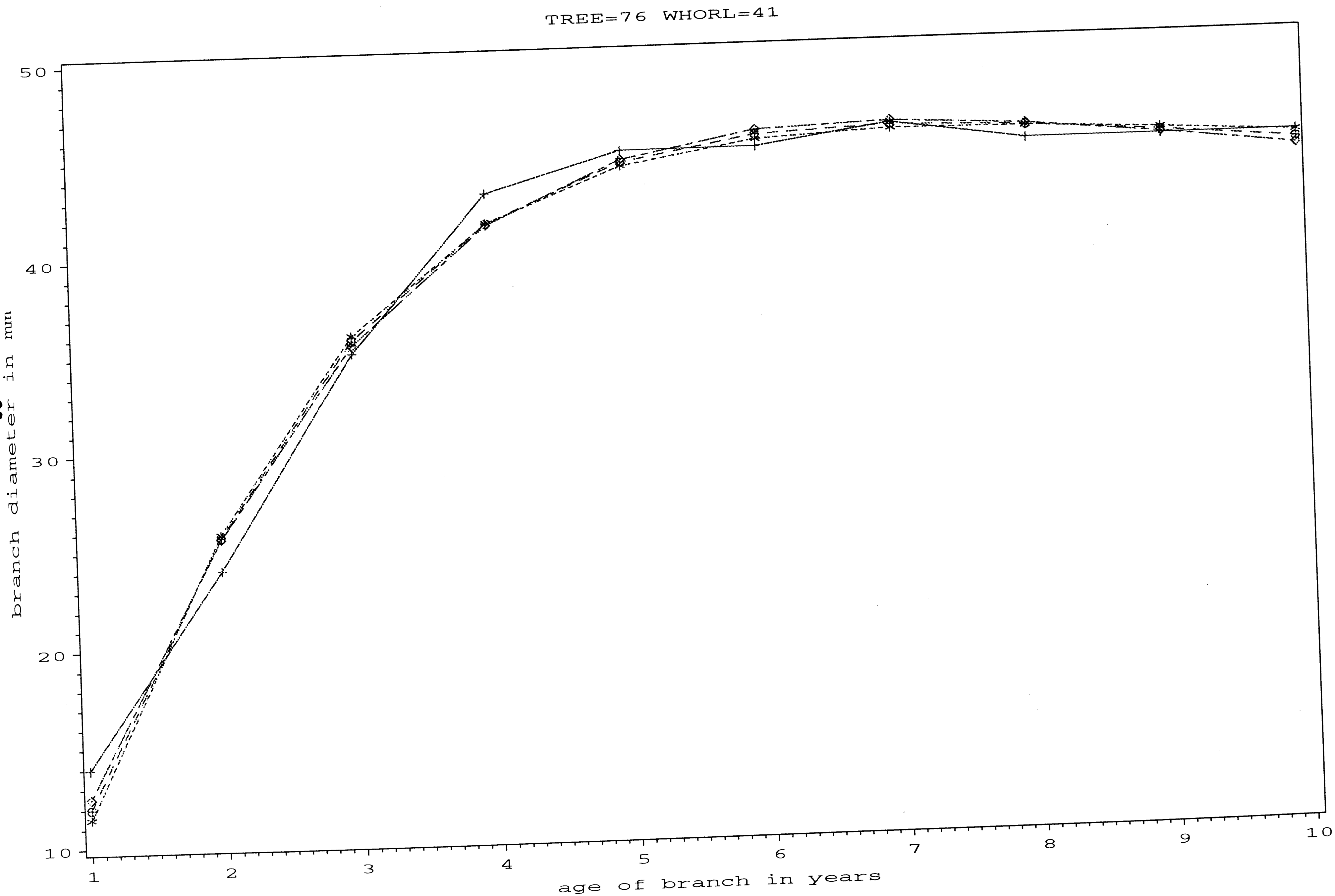
branch diameter in mm
age of branch in years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branch diameter in mm
age of branch in years
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20
30
40
50
1
2
3
4
5
6
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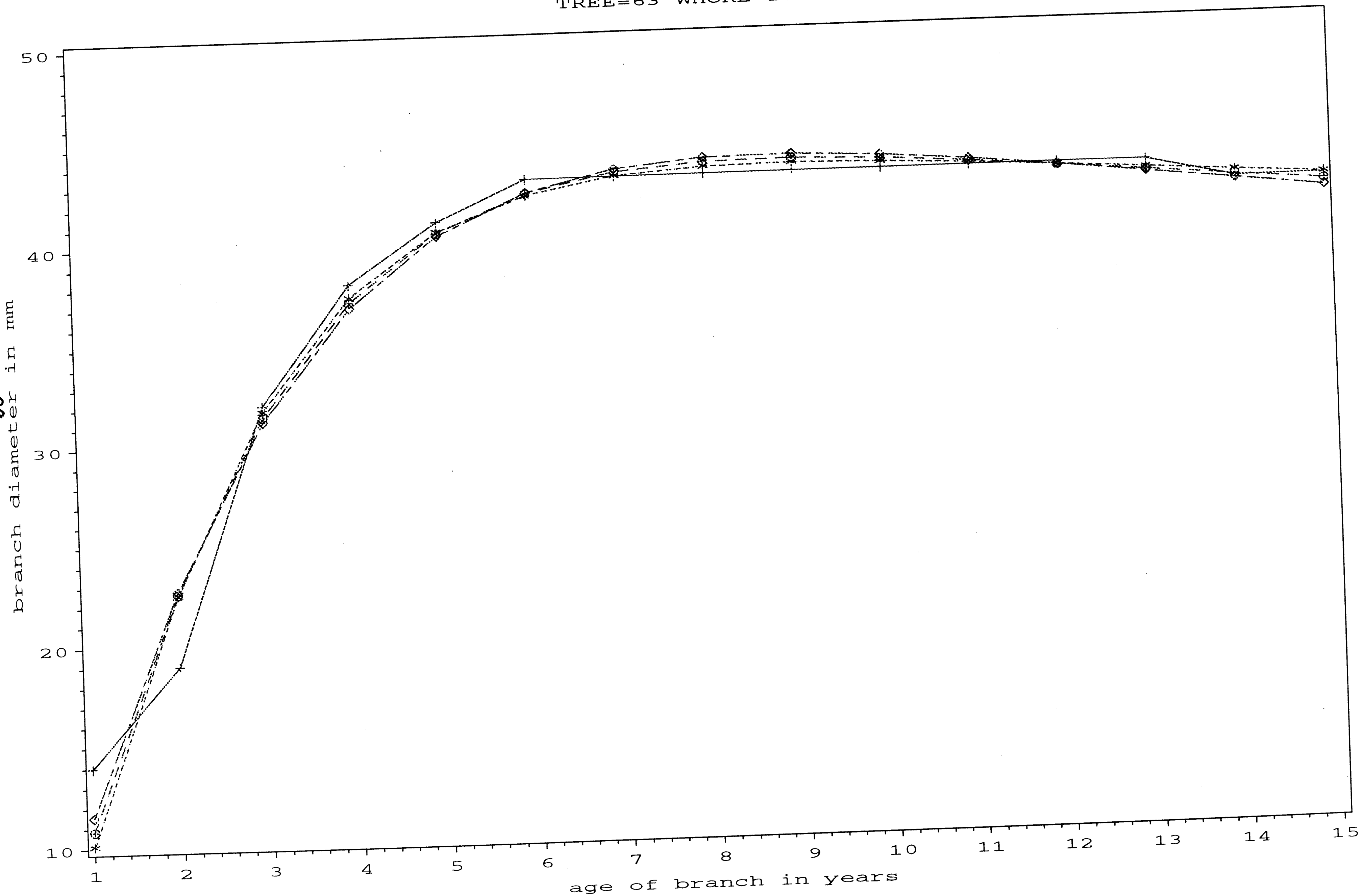
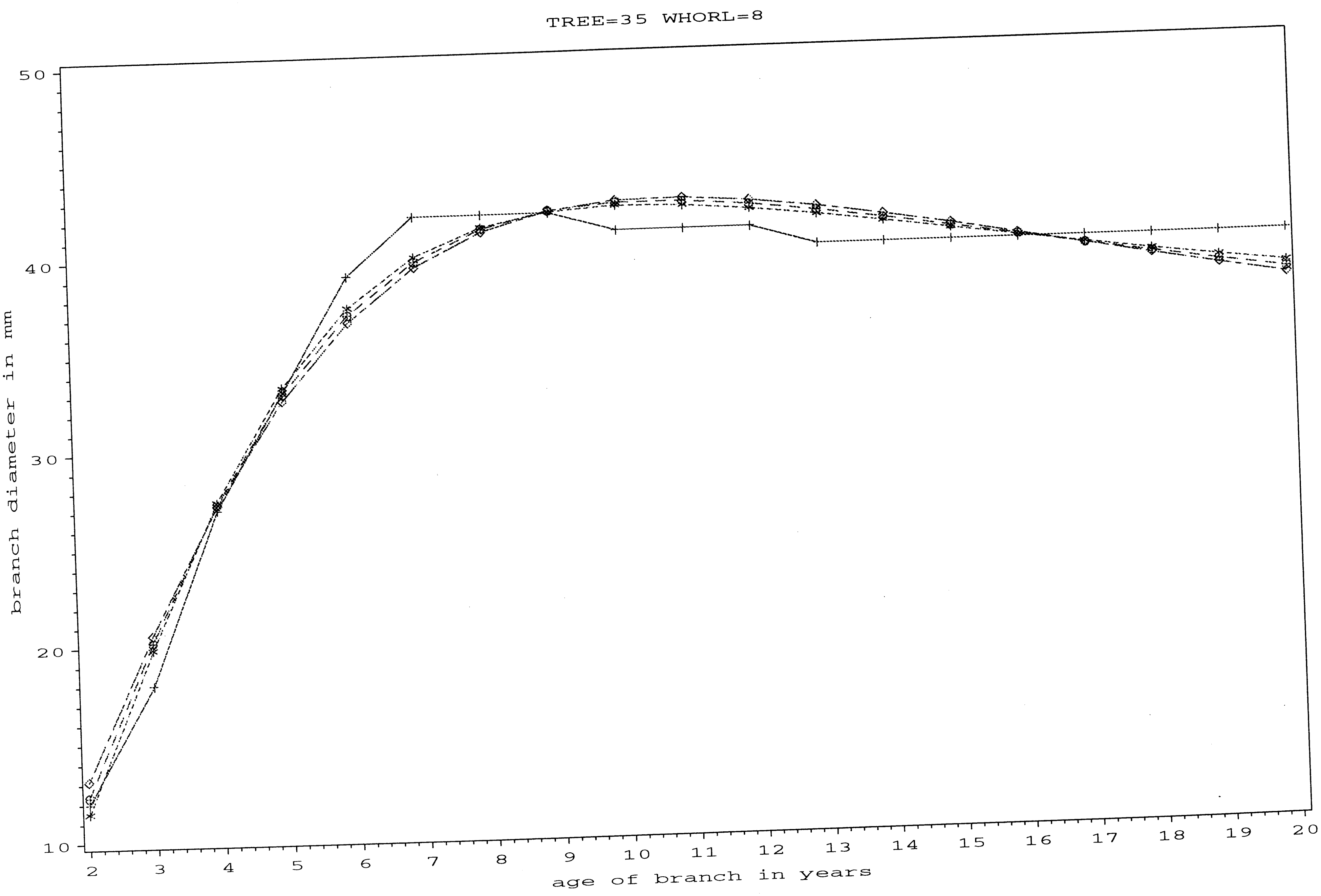
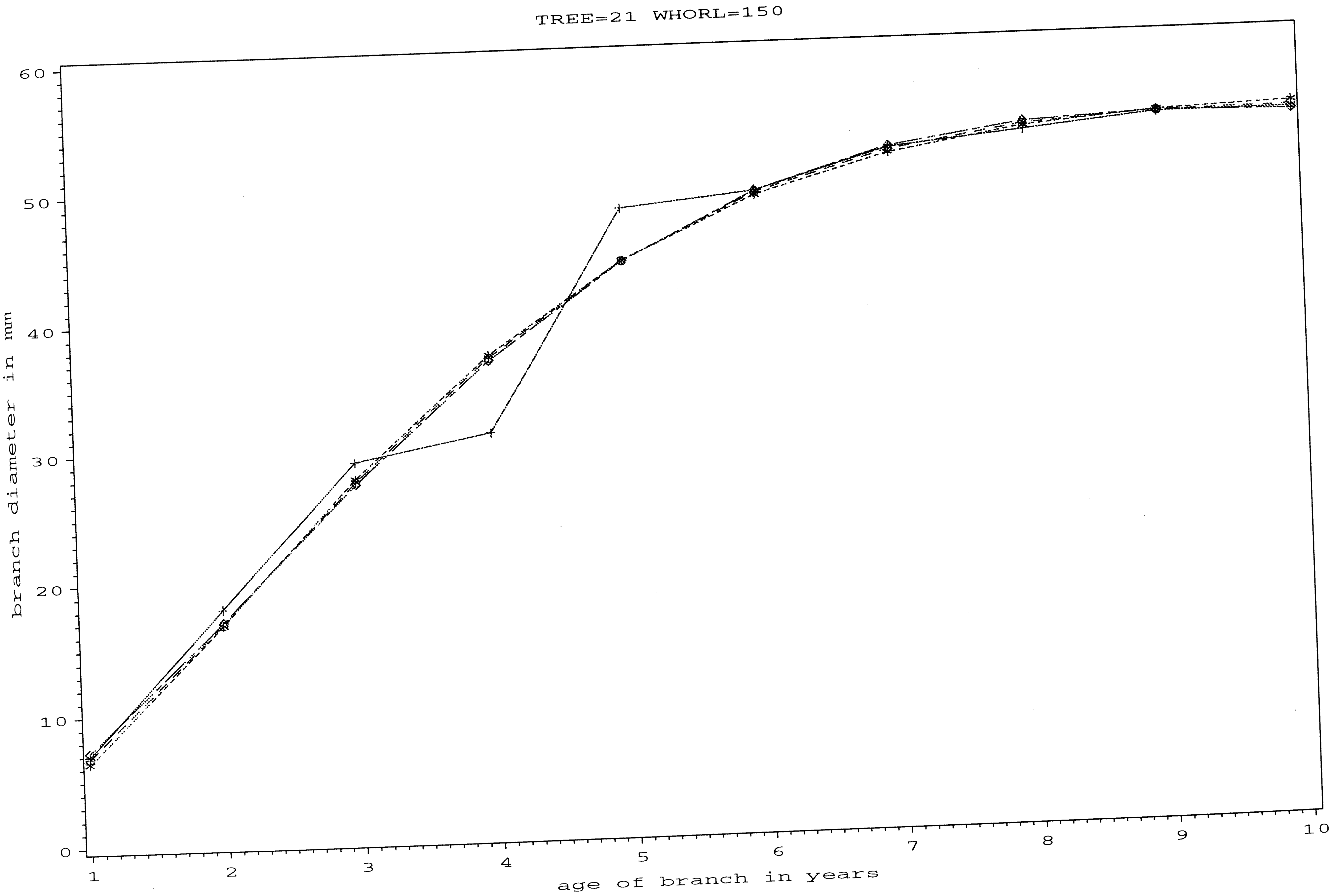
8
9
10

TREE=63 WHORL=15

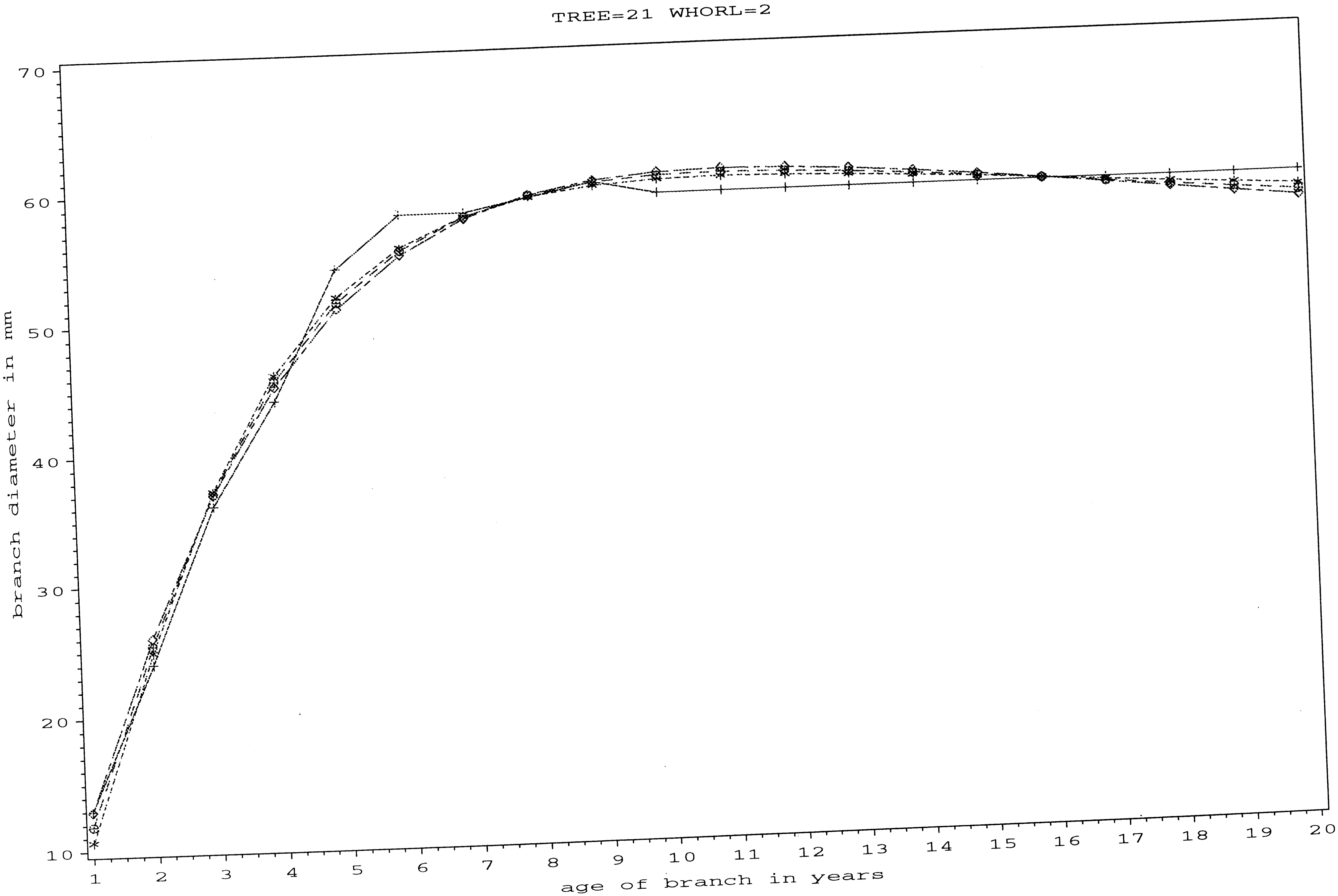




TREE=35 WHORL=8
branch diameter in mm
10
20
30
40
50
age of branch in years
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20



TREE=21 WHORL=150
branch diameter in mm
0
10
20
30
40
50
60
age of branch in years
1
2
3
4
5
6
7
8
9
10


TREE=21 WHORL=2
branch diameter in mm
10
20
30
40
50
60
70
age of branch in years
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Appendix 2. Equation for branch diameter growth.

The equation being used to predict branch diameter from age is:

$$D = \frac{A}{a + b\,A + c\,A^{0.5}}$$

When branch diameter is maximum dy /dx = 0.

$$\frac{dy}{dx} = \frac{(a + bA + cA^{0.5}) - A\,(b + 0.5cA^{-0.5})}{(a + bA + cA^{0.5})^2}$$

dy/dx = 0 implies:

$$a + 0.5\,c\,A^{0.5} = 0$$

$$A^{0.5} = \frac{-a}{0.5\,c}$$

$$A^{0.5} = -2a/c$$

This is the age at which the maximum branch diameter occurs. However for the solution to be valid one of a or c must be negative.

The maximum diameter, M, occurs when:

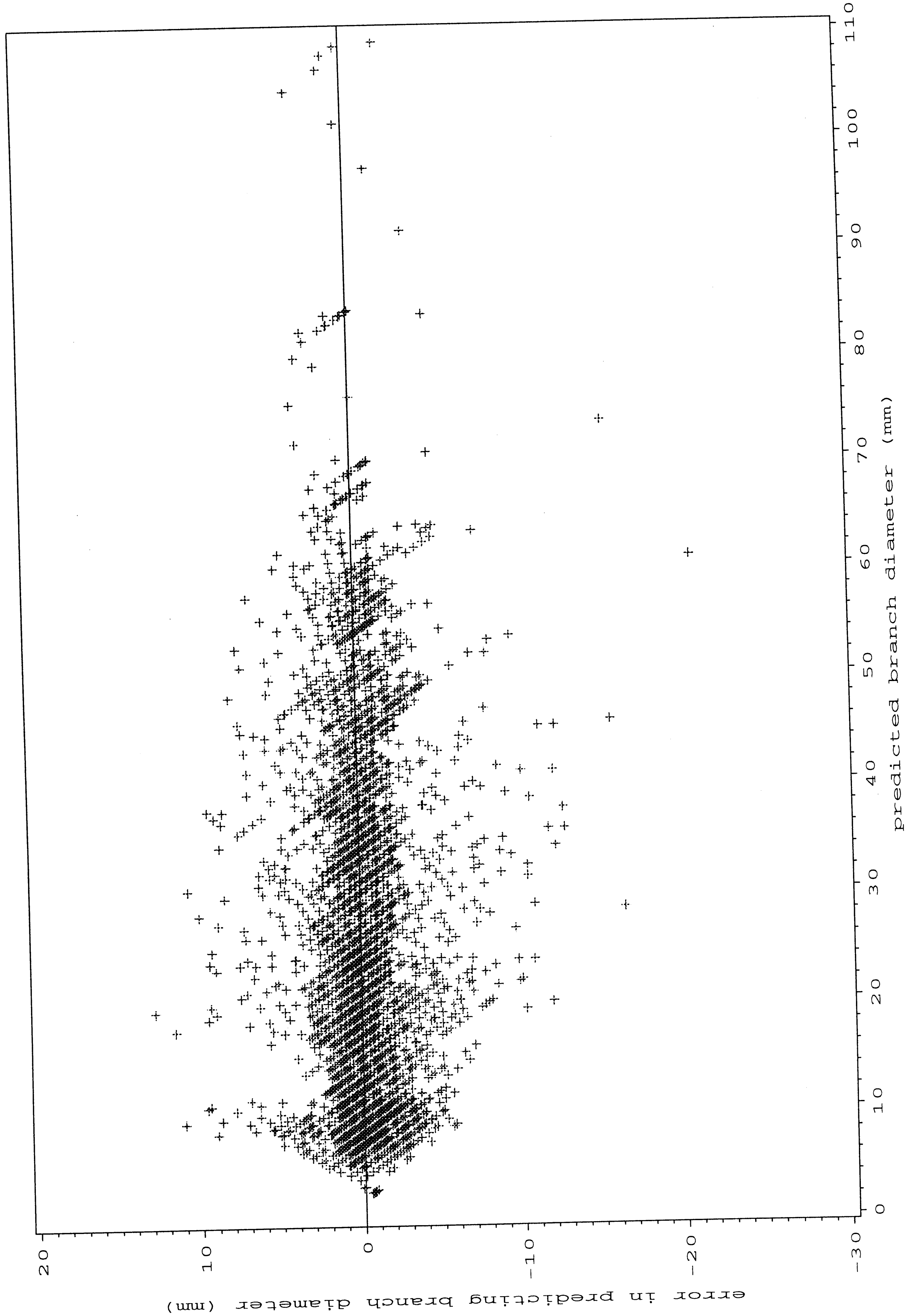
$$D = \frac{4a^2/c^2}{a + b\,4a^2/c^2 - c2a/c}$$

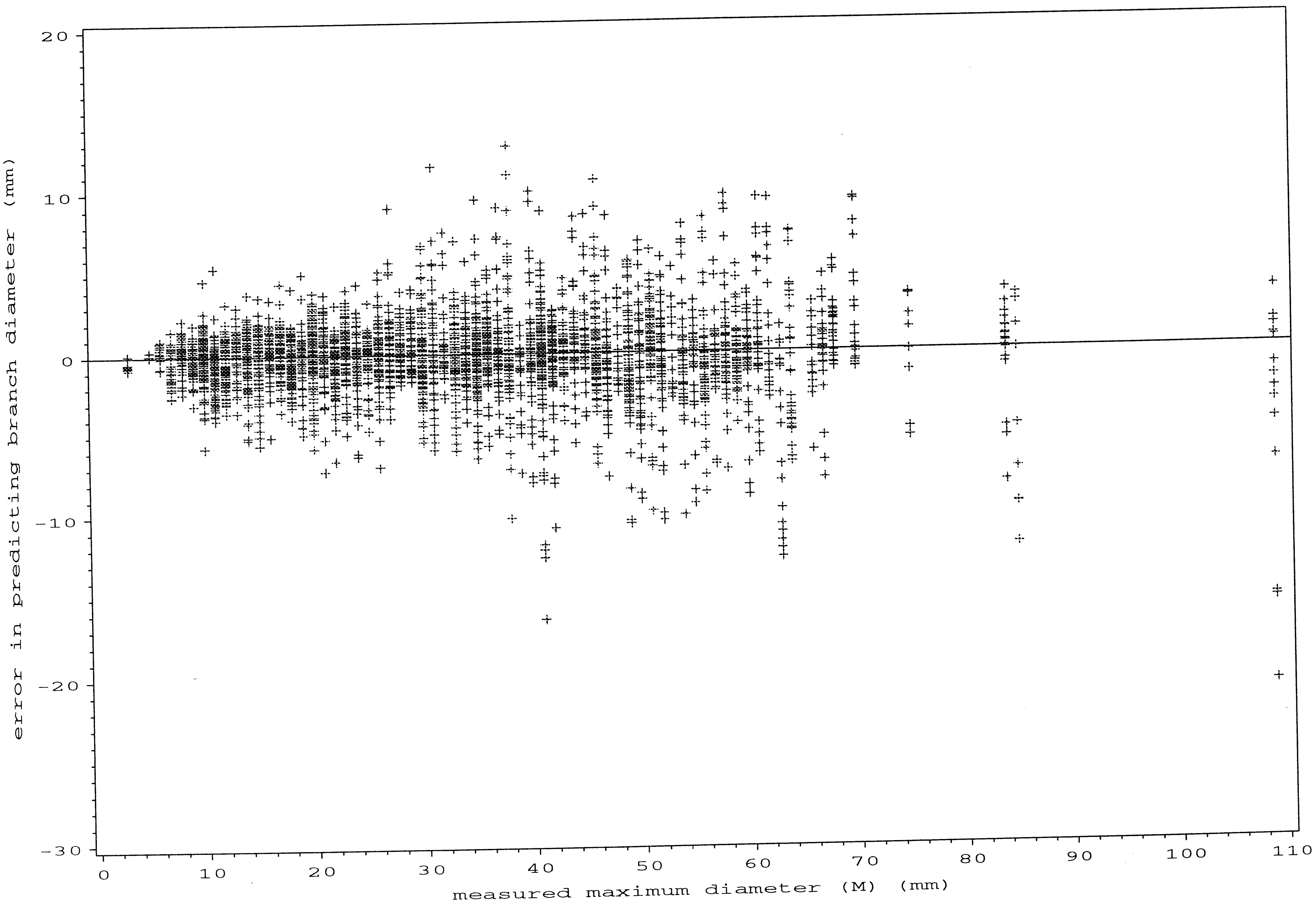
$$D = \frac{4a^2/c^2}{b\,4a^2/c^2 - a}$$

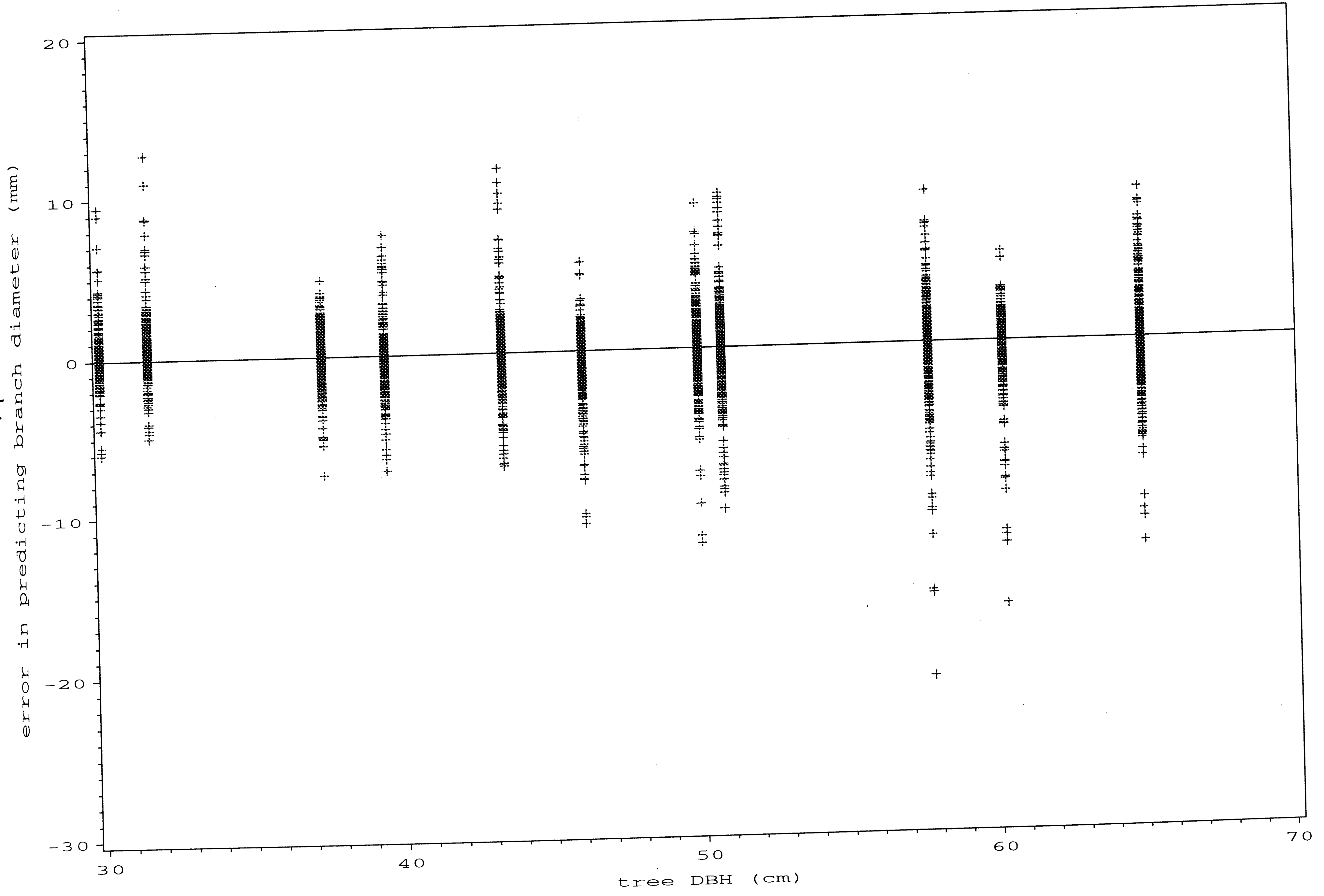
$$D = \frac{1}{b - c^2/4a}$$

The analyses (see main text) indicate that all three parameters are functions of the maximum diameter, M. The above equation indicates that b is a function of M^{-1} and c^2/a is also a function of M^{-1}.

Appendix 3. Residual plots from fitting Eqn. 11

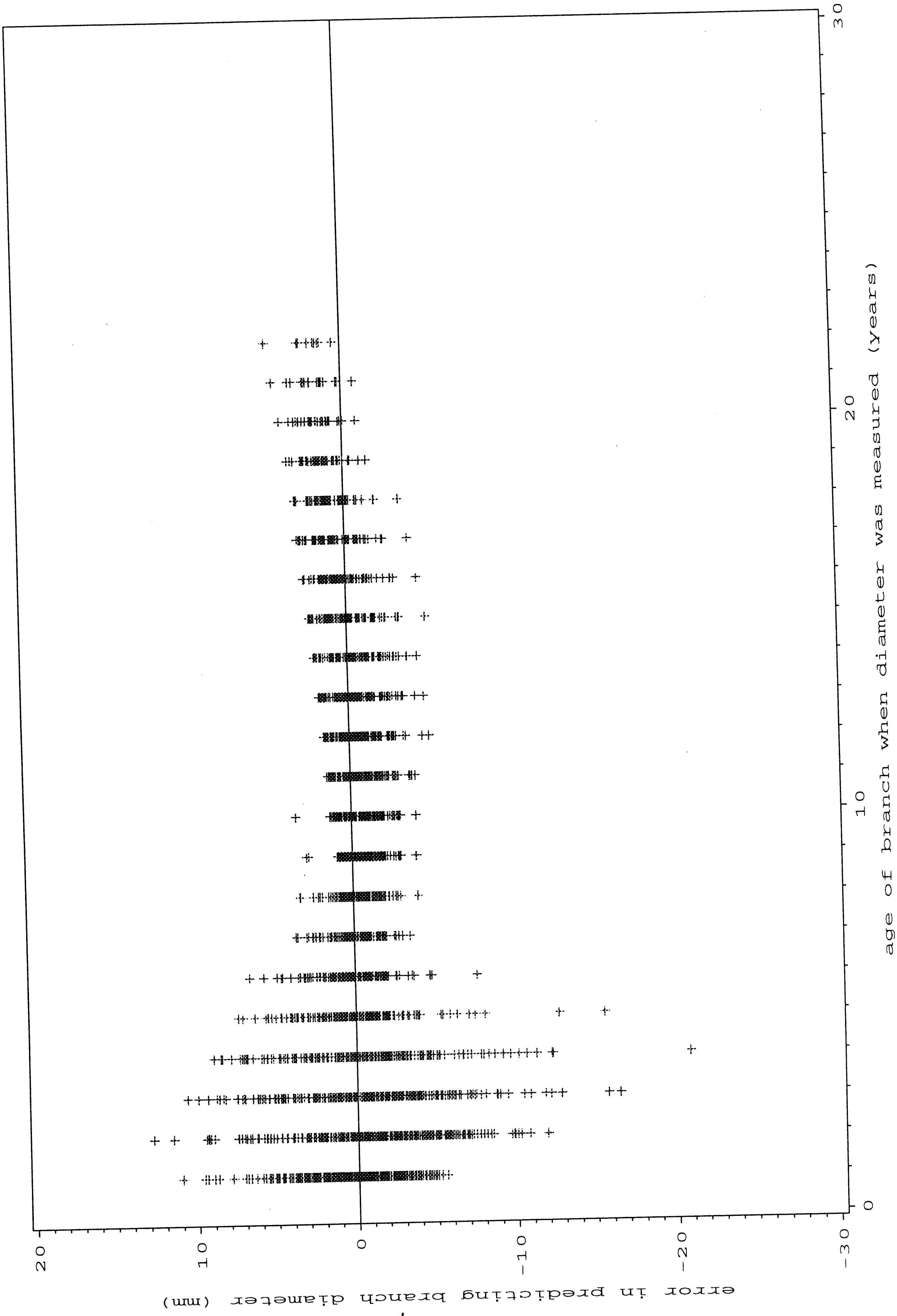

error in predicting branch diameter (mm)
20
10
0
-10
-20
-30
predicted branch diameter (mm)
0
10
20
30
40
50
60
70
80
90
100
110

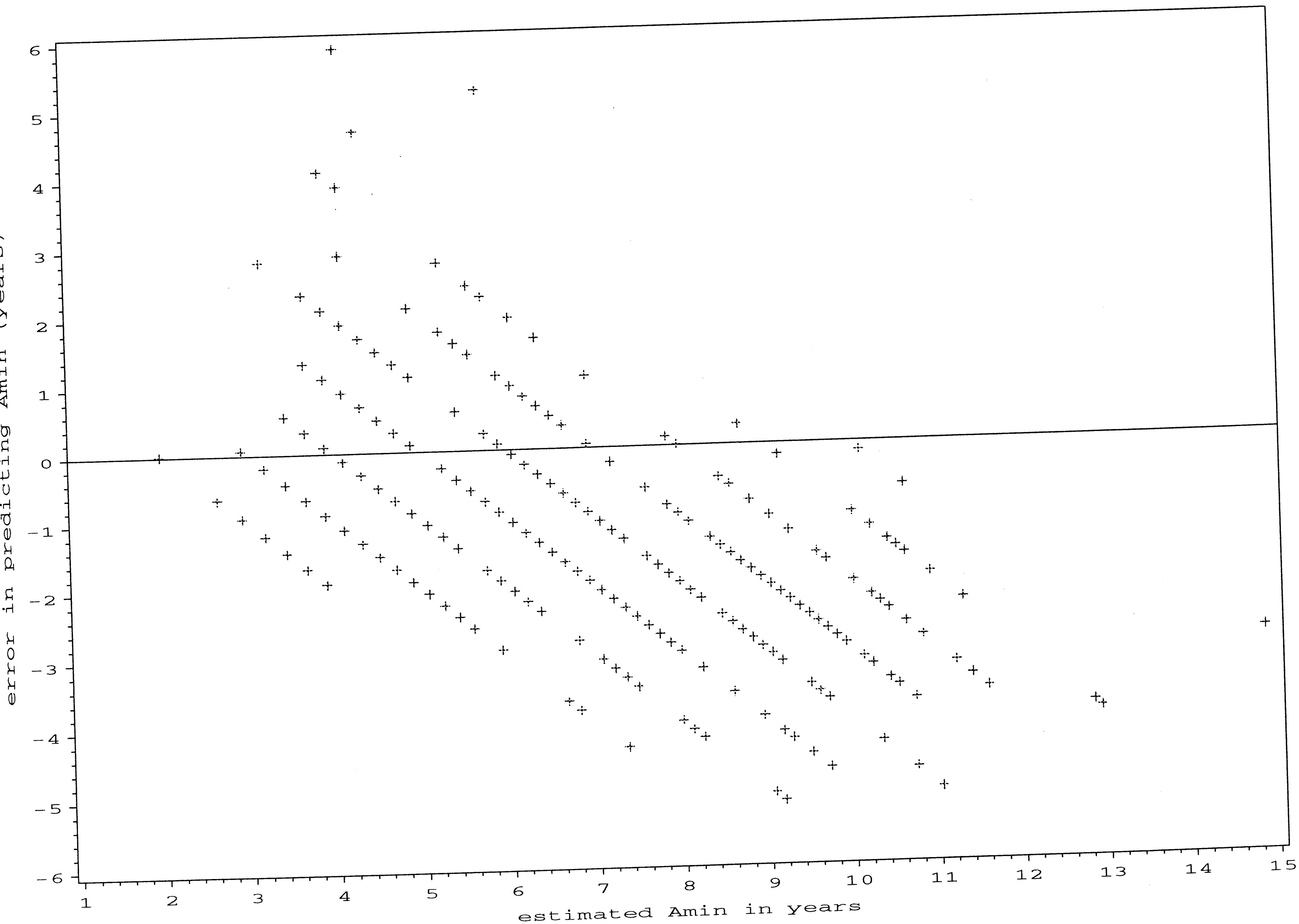

20
10
0
−10
−20
−30
error in predicting branch diameter (mm)
0
10
20
30
40
50
60
70
80
90
100
110
measured maximum diameter (M) (mm)

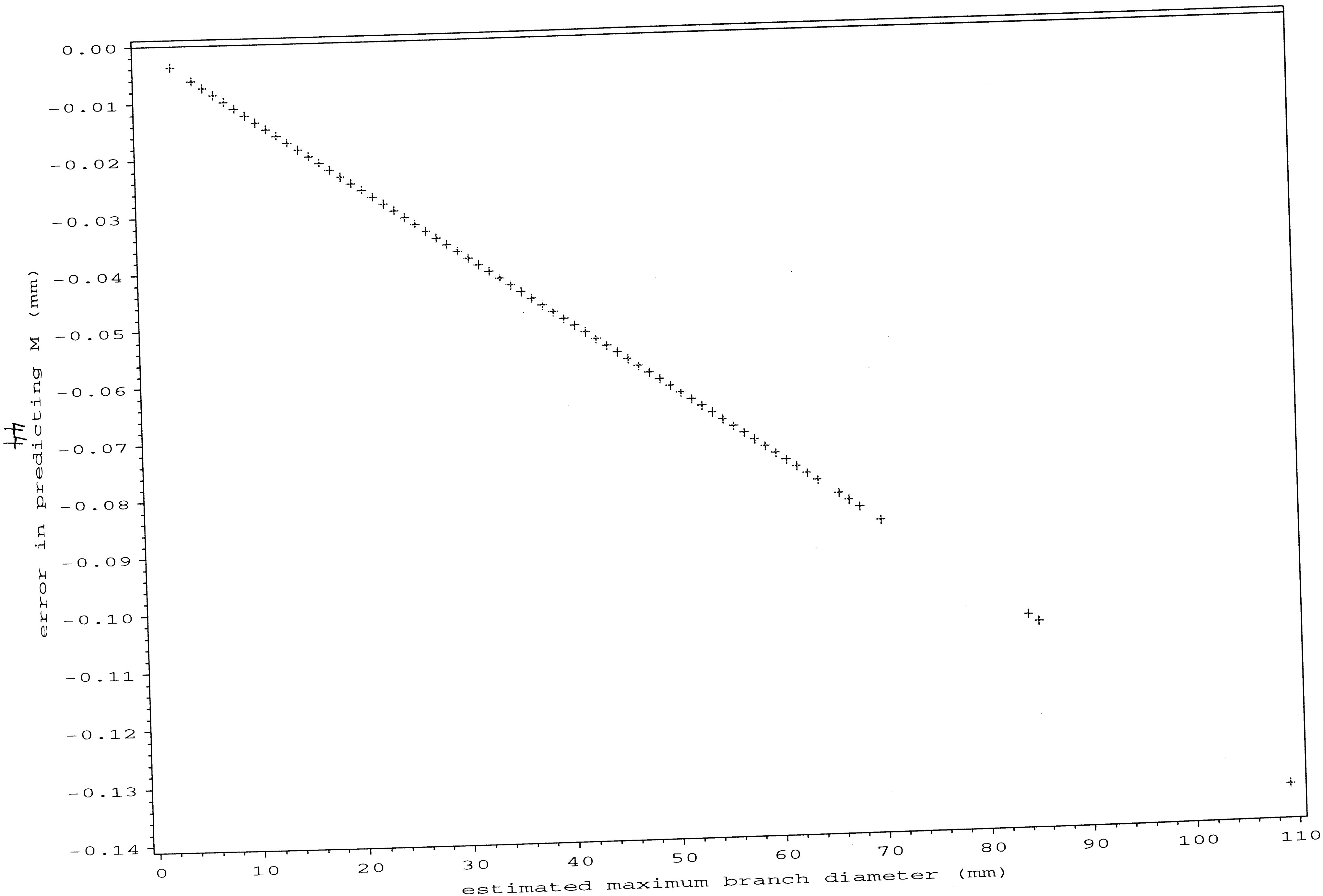


error in predicting branch diameter (mm)
20
10
0
-10
-20
-30
30
40
50
60
70
tree DBH (cm)

41


age of branch when diameter was measured (years)
error in predicting branch diameter (mm)
0
10
20
30
20
10
0
-10
-20
-30


error in predicting Amin (years)
estimated Amin in years
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


0.00
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.10
-0.11
-0.12
-0.13
-0.14
error in predicting M (mm)
0
10
20
30
40
50
60
70
80
90
100
110
estimated maximum branch diameter (mm)