

NZ FRI PROJECT RECORD

NO. 3927

**PROJECTING INVENTORY DATA:
PREDICTING INDIVIDUAL TREE DIAMETER GROWTH**

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REPORT NO. 34

APRIL 1994

Note : This is an unpublished report and must not be cited as a literature reference.

FRI/INDUSTRY RESEARCH COOPERATIVES

EXECUTIVE SUMMARY

This report describes the development of tree diameter increment models that can be used to apportion stand basal area growth to tree diameter increment in stands aged at least 15 years, on which no further thinning and pruning will be carried out.

In a comparison of methods used to project individual *Pinus radiata* diameters through time it was shown that simply scaling diameter based on stand growth model basal area predictions under-estimates the variance of the diameter distribution of the projected diameters (Gordon and Lawrence 1992).

Of the methods compared the most promising was a distance-independent, individual tree growth model (Manley 1981), which accounted for the position of the tree within the stand diameter distribution when determining its increment.

Using a large set of stand, site, and climatic variables it was not possible to satisfactorily account for all effects in one formulation, so regionally based models were developed and fitted.

The models developed here can be used as adjuncts to stand growth models by adjusting the projected tree basal areas so that they sum to the value predicted at the stand level.

INTRODUCTION

This report describes the development of tree diameter increment models for apportioning stand basal area growth to tree diameter increment in stands aged at least 15, on which no further thinning and pruning will be carried out. The primary aim of this work is to produce a robust method for projecting inventory data for periods of one to a maximum of about fifteen years, ie. from mid-rotation inventory through until harvesting age.

In a comparison of methods used to project individual *Pinus radiata* diameters through time it was shown that simply scaling diameter based on stand growth model basal area predictions under-estimates the variance of the diameter distribution of the projected diameters (Gordon and Lawrence 1992). Of the methods compared the most promising was a distance-independent, individual tree growth model (Manley 1981), which accounted for the position of the tree within the stand diameter distribution when determining its increment.

It is feasible to use an individual tree model as an adjunct to a stand level growth model in such a way that the stand basal area increment is set by the stand level model but that the increment is apportioned between the trees based on the individual tree model.

NOTATION

G	stand basal area ($\text{m}^2 \text{ ha}^{-1}$)
T	stand age (years)
N	stand stocking (stems ha^{-1})
\bar{h}_{100}	mean top height (m)
S	site index (m)
d_i	breast height (1.4m) diameter over bark (Dbh) for tree i (mm)
\bar{d}_{100}	mean top diameter (mm)
Δd	Dbh annual increment (mm)
$Rain$	Total annual rainfall (for Sep-Aug year) (mm)

DATA

Plots were selected from the Permanent Sample Plot data base (McEwen, 1978; Pilaar and Dunlop, 1989) from seven growth modelling regions, to form a general data set. At least 15 trees measured for diameter were required in each plot to reduce the variability of estimates of stand parameters derived from each plot. This ruled out very small plots, particularly where the stocking was low. Plots were required to have had three or more consecutive measurements, with the first measurement somewhere between ages 15 and 25 inclusive. Only plots with

"normal" levels of mortality (i.e. excluding windthrow, poison thinnings etc.) were chosen, and all thinning operations were required to have been completed prior to the initial measurement. An estimate of \bar{h}_{100} was required.

A total of 291 plots were selected which met these criteria. They are listed in APPENDIX A.

Procedures were developed to derive the annual diameter increment of each tree after each plot measurement excluding the last. This involved some interpolation where re-measurement did not take place 12 months later. Trees which died were excluded from the data set from the last re-measurement where their increment was less than or equal to zero. At each measurement point, from which increments were derived, corresponding stand statistics were calculated.

Diameter increment in *Pinus radiata* has been clearly connected with moisture stress during the December to April period (Jackson *et al*, 1976). Hunter and Gibson (1984) derived a model for predicting site index which showed rainfall to be an important predictor. Studies with *Pinus sylvestris* have also shown that rainfall in the current growing season is one of the most important climatic factors in determining diameter increment (Reimer and Sloboda 1991).

For 30 meteorological stations a data base of monthly rainfall from 1968 to 1986 was created. As each plot was processed, when building the data set, this data base was searched for the closest station and the spring (September, October, November), summer (December, January, February) and total rainfall for that growth year, were calculated. Also calculated was a measure of effective rainfall, obtained by subtracting the precipitation intercepted by the canopy (a function of G) from the total rainfall (Myers and Talsma, 1991).

The average growing season temperature was not included because the data were not readily available and it was considered that the effects of temperature would be accounted for by latitude and altitude, both of which were contained in the data set.

The New Zealand radiata pine nutrition atlas (Hunter *et al* 1991) was developed by averaging nutrient concentrations based on foliage samples at a forest level. The forest means and standard deviations were related to the predominant soil type occurring in that forest in order to rank the fertility level for each nutrient into seven classes, from deficient through to a high probability of being satisfactory.

As soil fertility is related to productivity (Hunter and Gibson 1984) it appeared likely that these rankings could be related to diameter increment. To use the atlas, the latitude and longitude fields stored against each plot in the Permanent Sample Plot system were extracted and converted to New Zealand map grid eastings and northings. At each plot location the scores for the elements N , K , P , B and Mg were determined using the Terrasoft geographic information system, by overlaying the plot location data set and retrieving the matching score. This process revealed a number of incorrect plot locations within the sample plot system but eventually yielded scores for these five elements that could be matched to every plot.

Thus a large data set of diameter increments was built up. Each observation comprised measurements on the set of 24 variables listed in Table 1. A total of 65628 observations were compiled into the data set.

Table 1. Variables in Data Set

Variable Type	Variable Name
Tree	<i>Dbh</i> cm.
	<i>Dbh</i> increment cm.
Stand	<i>T</i>
	<i>G</i>
	<i>N</i>
	\bar{h}_{100}
	\bar{S}
	\bar{d}_{100}
Site	Year
	Latitude
	Longitude
	Altitude
	Distance from sea (km)
	Growth Modelling region
Nutrition Scores	Plot Identifier
	N
	K
	P
	B
Climate	Mg
	Spring rain (Sep,Oct,Nov) (mm)
	Summer rain (Dec,Jan,Feb) (mm)
	Growth year rain (Sep-Aug) (mm)

A subset of some 5000 observations were randomly selected using varying probabilities of selection in order to extract approximately 750 from each growth modelling region. This data set was used for the bulk of the analyses. By using a sample of the observations more reliable significance tests could be made because the residuals were less likely to be correlated. It was possible to draw further samples to assist with model validation by using an independent data set.

METHODS

Following the work of Manley (1981) using Kaingaroa data we focussed on simple diameter increment rather than its logarithm or tree basal area increment, as the dependent variable. A number of transformations and combinations of the independent variables were constructed, such as relative diameter $\frac{d_i}{\bar{d}_{100}}$ and relative spacing $\frac{1}{\bar{h}_{100}\sqrt{N}}$, which is the ratio of the average between-tree spacing to the stand mean top height. Interactions between rainfall and nutrient score were included as highly significant relationships between these variables and G increment have been reported by Benson *et al* (1992).

Several models that could be used to predict the increment were suggested by the data and by theoretical considerations. Initially three approaches were tried.

Diameter increment as a function of Time

The value of Δd through time resembles a unimodal, asymmetric frequency distribution (Prodan 1968) but the maximum value is usually reached early in a trees life. Most growth equations can be partitioned into two components representing expansion and growth decline. Zeide (1993) has shown that the decline component is invariably a function of T . By looking only at growth after age 15 a simple decline model can be assumed such as:

$$\Delta d = A e^{RT} \quad \dots (1)$$

where A represents growth potential and R the rate of decrease of the increment with time. A should be related to site variables (nutrient status, rainfall, etc.) and the rate R to stand density and the trees position within the diameter distribution.

Diameter increment as a function of Stand Basal Area

The decrease in diameter increment was clearly related to stand basal area in all regions suggesting a similar model but driven primarily by G .

$$\Delta d = A e^{RG} \quad \dots (2)$$

Again A represents growth potential and R the rate of decrease of the increment with time.

Diameter increment as a linear function of predictor variables

By initially assuming a linear model a large number of possible sets of predictors could be examined quickly. Although log-linear models have often been advocated (*e.g.* Wykoff 1990) this can lead to problems in correcting for the log bias especially if the residuals of the log-transformed models are not normally distributed (Zumrawi and Hann 1993, Manley 1981).

After determining which approach appeared most fruitful the variation in the set of predictor variables was examined in more depth using principal component analysis (PCA) and regression. PCA was used in an attempt to reduce the number of independent variables and remove as much multicollinearity as possible, as many of the dependent variables in the data set were highly correlated.

A model was then selected, fitted and analysed for regional effects.

RESULTS

Determining Models

The first two approaches to determining a useful model produced similar results. The rate of Δd decline, R , was related primarily to relative diameter with stocking and basal area mean annual increment also having significant coefficients. The growth potential A could be predicted by a wide variety of variables. Site variables such as latitude and rainfall were important but so were tree variables such as d and relative diameter and stand variables such as N , \bar{d}_{100} , \bar{h}_{100} and relative spacing.

Starting with a linear model approximately 60% of the variation in Δd was accounted for using five variables:

T , relative diameter ($\frac{d_i}{\bar{d}_{100}}$), relative spacing ($\frac{1}{\bar{h}_{100}\sqrt{N}}$), latitude and *Rain*.

Two comparisons were made between these three approaches. Firstly the mean and variation in the prediction errors on an independent subset of the data were compared. The linear model clearly showed less bias and greater precision.

Table 2. Comparison of Errors

Model	Mean prediction error (mm)	Standard Deviation (mm).
$\Delta d = A e^{RT}$	0.105	5.070
$\Delta d = A e^{RG}$	0.268	5.358
$\Delta d = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ..$	0.021	4.892

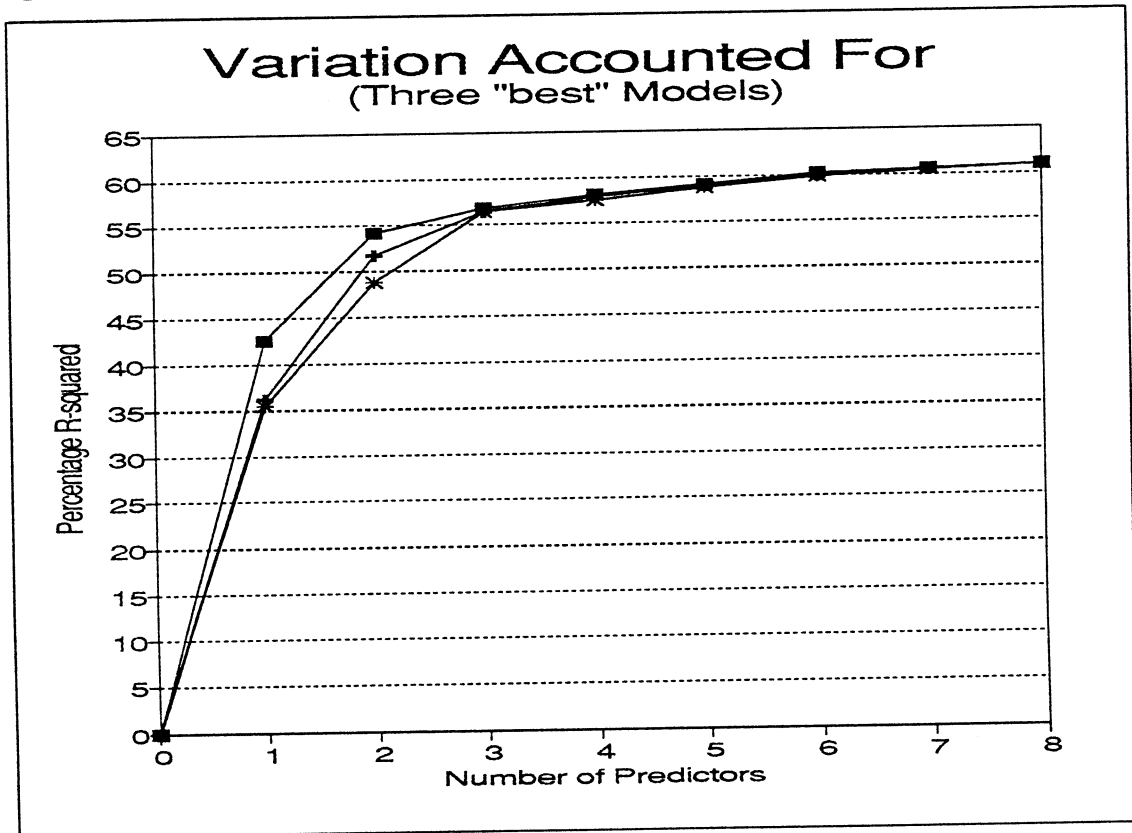
Secondly the errors in the predictions were plotted over the predicted values, the independent variables and other variables in the data set by region for the time-based and the linear model. Trends in these errors were clearly shown in the time-based model especially with relative diameter. Diameter increment was over-estimated for small relative diameters and *vice versa*.

In contrast the error plots from the linear model showed no trends in most regions. However this model generally over-estimated diameter increment in the Sands region, especially for trees with large relative diameters. Some trends were also visible for the Canterbury and Southland regions. Although clearly imperfect this linear model was selected as the most promising approach.

Variable Selection

To initially select the "best" linear model, all-subset regressions were calculated. For the complete set of predictor variables (raw, transformed and combined), the models giving the three highest R^2 values with 1, 2, .. predictors were selected and the results tabulated. Figure 1 shows the percentage variation accounted for as the number of predictors were increased.

Figure 1.



Relative spacing and relative diameter were the best predictors of Δd and these two variables consistently featured in models with more than two terms. Models with more than three predictors showed only small gains in R^2 . The three best three-predictor models with R^2 around 0.57 were, in order:

$$\frac{d_i}{\bar{d}_{100}}, \frac{1}{\bar{h}_{100}\sqrt{N}}, T$$

$$\frac{d_i}{\bar{d}_{100}}, \frac{1}{\bar{h}_{100}\sqrt{N}}, \frac{1}{T}$$

$$\frac{d_i}{\bar{d}_{100}}, \frac{1}{\bar{h}_{100}\sqrt{N}}, \frac{\bar{h}_{100}}{T}$$

Principal Components

In an attempt to describe the interrelationships among the predictor variables, and to check the validity of the model, principal components were calculated and used as independent variables in a regression analysis on Δd .

The eigenvectors are the linear transformation coefficients of the variables from which the components are determined. The eigenvalues, or component variances, of the first eight principal components differed considerably in magnitude indicating a large degree of multicollinearity as was expected. Six had relatively large values which implies there are at least that many under-lying factors required to fully describe the behaviour of the predictor variables.

Table 3. Size of Main Eigenvectors

Eigenvalue	Cumulative Proportion of total Variation
9.22	0.26
6.45	0.45
4.81	0.59
4.09	0.70
2.51	0.77
2.09	0.83
1.40	0.87
1.07	0.90

The regression analysis on Δd showed four or five of the components to be good predictors. The eigenvectors of these components were examined in an attempt to determine which underlying variables should be included in the model. Table 4 shows the components in order of importance as predictors and the variables which contribute most to each one.

Table 4. Composition of Principal Components.

Principal Component	Important Variables
2	G MAI, N , G , $\frac{1}{\bar{h}_{100}\sqrt{N}}$
8	$\frac{d_i}{\bar{d}_{100}}$, Dbh
3	Altitude, Mg , Distance from sea, B . ¹
5	K , $N \times P$, Summer Rainfall. ¹
4	$\frac{1}{T}$, \bar{d}_{100} , T

¹ - No clear interpretation was possible as several variables had similar coefficients. The variables with larger coefficients are listed.

Combined Model

The principal component analysis indicated that the linear model including $\frac{1}{\bar{h}_{100}\sqrt{N}}$, $\frac{d_i}{\bar{d}_{100}}$ and T had incorporated many of the underlying factors in the set of predictor variables, but that there were still complex local or regional effects. As Figure 1 indicates, extending this model by incorporating more variables produced very little return in terms of proportion of variation accounted for. Also the difference in R^2 and $C(p)$ ¹ (Mallows 1973) between the "best" n -variable models became very small when more than 3 predictors were used.

For this reason attention was focussed on the 3 predictor model and it was extended to allow for non-linear relationships and fitted with a boundary condition so that predictions of negative increments were set to zero. Both $\frac{1}{\bar{h}_{100}\sqrt{N}}$ and $\frac{d_i}{\bar{d}_{100}}$ had significant exponents while T did not. To simplify analysis new transformed variables were generated using the estimated exponents to produce a linear model.

¹ The $C(p)$ statistic is a measure of the error variance plus the bias introduced by not including important variables in the model.

The combined linear model was :

$$\hat{\Delta d} = \beta_0 + \beta_1 T + \beta_2 \left(\frac{d_i}{\bar{d}_{100}} \right)^{0.6} + \beta_3 \left(\frac{1000}{\bar{h}_{100} \sqrt{N}} \right)^{1.5} \quad \dots (3)$$

Residuals were tabulated over the range of the estimates and predictor variables and examined for trends by region. Three regions showed up as having clear trends in the Δd residuals with relative diameter. These were Canterbury, Sands and Southland, but other regions also illustrated some bias. Model 3 was then fitted separately for each region and the hypothesis that a single equation was adequate was tested. Table 5 shows the analysis of variance of the residual sums of squares.

Table 5. ANOVAR of Residual SS.

Source	DF	Residual SS	Residual MS	F
Hypothesis Model				
Single Equation	5307	126241.1	23.788	
Maximum Model				
Regional Equations				
Canterbury	760	12818.9		
Clays	754	18615.4		
GoldenDowns	741	16486.5		
HawkesBay	707	12911.5		
Kaingaroa	802	13324.8		
Sands	776	21117.0		
Southland	743	18768.2		
	5283	114042.3	21.587	
Difference	24	12198.8	508.28	23.5 **

There is a significant decrease in the residual sum of squares between the hypothesis and the maximum models and the hypothesis that a single equation model fits the data adequately was rejected.

Examination of the prediction errors of the by-region model on an independent data set showed Δd in three of the smallest relative diameter classes from Hawkes Bay was under-estimated. It appeared that the exponent of $\frac{d_i}{\bar{d}_{100}}$ was affecting the fit. This parameter

has the effect of exaggerating or under-stating the effect of relative diameter. Rather than retain one model formulation for all regions it was decided to fit separate linear models to each region. The linear variable $\left(\frac{d_i}{\bar{d}_{100}}\right)^{1.6}$ was used as it resulted in the best predictions of the variance of the diameter distributions when a range of exponent values were tried.

Regional Models

An all-regressions model selection procedure was followed on a regional basis. Table 7 shows the variables selected and approximate R^2 values. Parameter estimates were made with a boundary condition so that predictions of negative increments were set to zero. The model fitted was:

$$x = f(\text{predictor variables})$$

$$\Delta d = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Plots of residuals over predicted values and tree, stand and site variables showed no error trends. Table 6 summarizes the residuals by region. Estimated coefficients (with standard errors) are given in table 8. Plots of residuals over T by site index classes are given for each region in appendix C.

Table 6. Residuals

Region	n	Residual mean (mm)	Residual Std Dev. (mm)
CANTY	763	0.09	3.77
CLAYS	757	-0.04	4.72
GDNS	745	0.00	4.65
HBAY	711	0.04	4.25
KANG	806	0.04	4.01
SANDS	779	-0.03	4.79
SOUTH	747	0.05	4.45

Table 7. Variables used in Models.

	Approx. R^2	$\left(\frac{d_i}{\bar{d}_{100}}\right)^{1.6}$	$\frac{1000}{\bar{h}_{100}\sqrt{N}}$	$\left(\frac{1000}{\bar{h}_{100}\sqrt{N}}\right)^2$	$2000\sqrt{\frac{G}{\pi N}}$	$\frac{\bar{h}_{100}}{T}$	$\frac{1}{T}$	$Rain$	N	G^2	$\frac{1}{G}$	\bar{h}_{100}	$Altitude^2$
Canterbury	0.58	•	•	•				•	•			•	
Clays	0.48	•	•	•	•								
GoldenDowns	0.55	•	•		•	•							
HawkesBay	0.75	•	•				•			•			
Kaingaroa	0.63	•	•		•	•	•						
Sands	0.52	•	•		•	•							•
Southland	0.76	•						•			•		
<i>Coefficient Labels</i>		β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}

• - variable included in the model.

Table 8. Model Coefficients and Standard Errors.

Canterbury	β_0	β_1	β_2	β_3	β_7	β_8	β_{11}
<i>coefficient</i>	262.6	15.88	-134.6	26.97	0.006571	-0.06036	-3.367
<i>std. error</i>	31.2	0.67	15.5	3.06	0.000713	0.00757	0.383

Clays	β_0	β_1	β_2	β_3	β_4
<i>coefficient</i>	8.051	11.99	-3.756	2.953	-0.02209
<i>std. error</i>	2.854	0.82	2.669	0.661	0.00316

Golden Downs	β_0	β_1	β_2	β_4	β_5
<i>coefficient</i>	-13.63	11.19	5.478	-0.01677	9.206
<i>std. error</i>	2.04	0.83	0.296	0.00303	1.456

Hawkes Bay	β_0	β_1	β_2	β_6	β_9
<i>coefficient</i>	-25.39	17.70	7.244	176.1	0.0006145
<i>std. error</i>	1.68	0.86	0.488	22.9	0.0001197

Kaingaroa	β_0	β_1	β_2	β_4	β_5	β_6
<i>coefficient</i>	-4.238	15.27	9.491	-0.03545	8.738	-248.9
<i>std. error</i>	2.109	0.703	0.803	0.00460	1.418	42.3

Sands	β_0	β_1	β_2	β_4	β_5	β_{12}
<i>coefficient</i>	-20.49	9.460	6.685	-0.01056	10.34	0.0004823
<i>std. error</i>	1.79	0.951	0.293	0.00216	0.99	0.0000634

Southland	β_0	β_1	β_7	β_{10}
<i>coefficient</i>	-19.31	15.77	0.008928	514.4
<i>std. error</i>	0.82	0.88	0.000649	19.1

Model Evaluation - Kaingaroa

The set of plot data from Kaingaroa used by Gordon and Lawrence (1992) to compare projection methods was used to evaluate the regional model for Kaingaroa derived here. Following the same methods as Gordon and Lawrence (1992) the ratio of the projected diameter distribution variance to the actual was plotted over projection period, initial stocking

and relative spacing. Quadratic trend lines with confidence limits on the mean were overlaid. The results shown in Figures 2, 3 and 4 are promising. No significant bias is shown in any of these figures.

Figure 2.

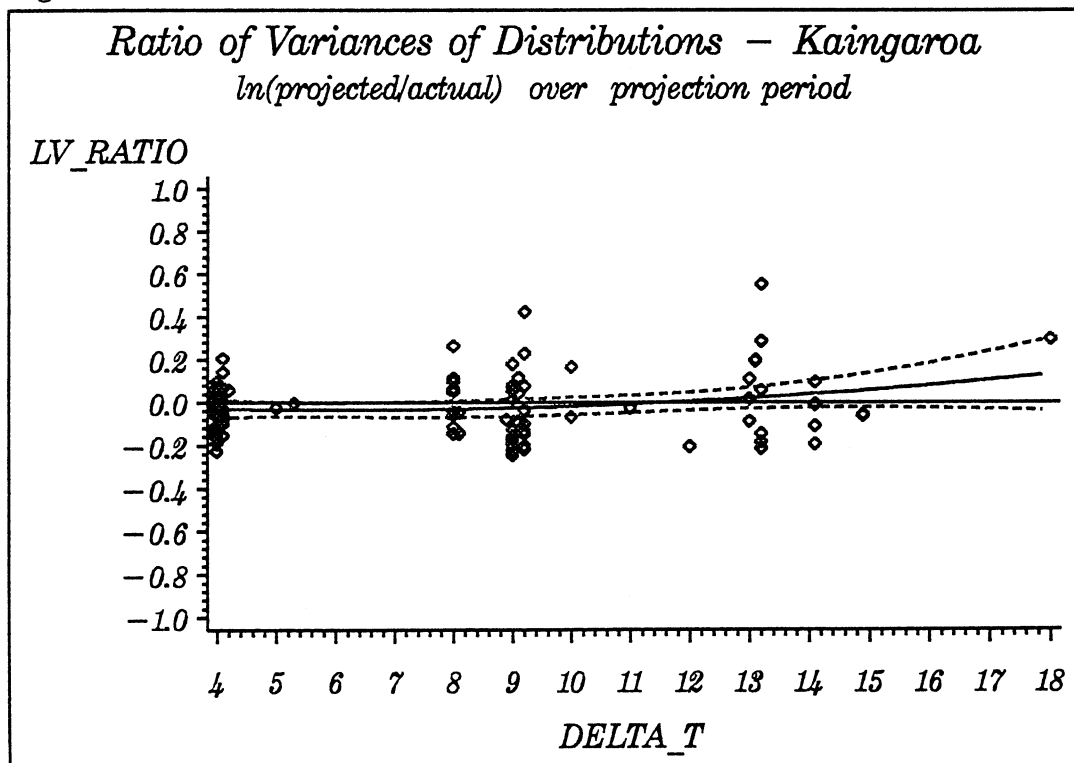


Figure 3.

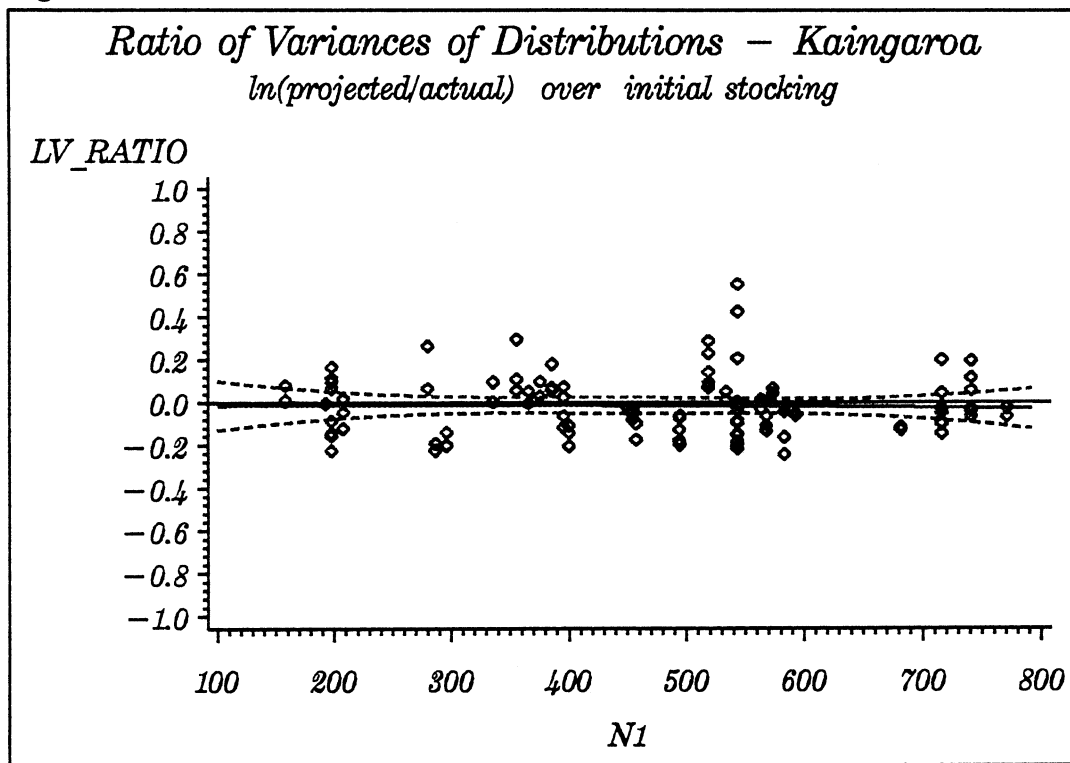
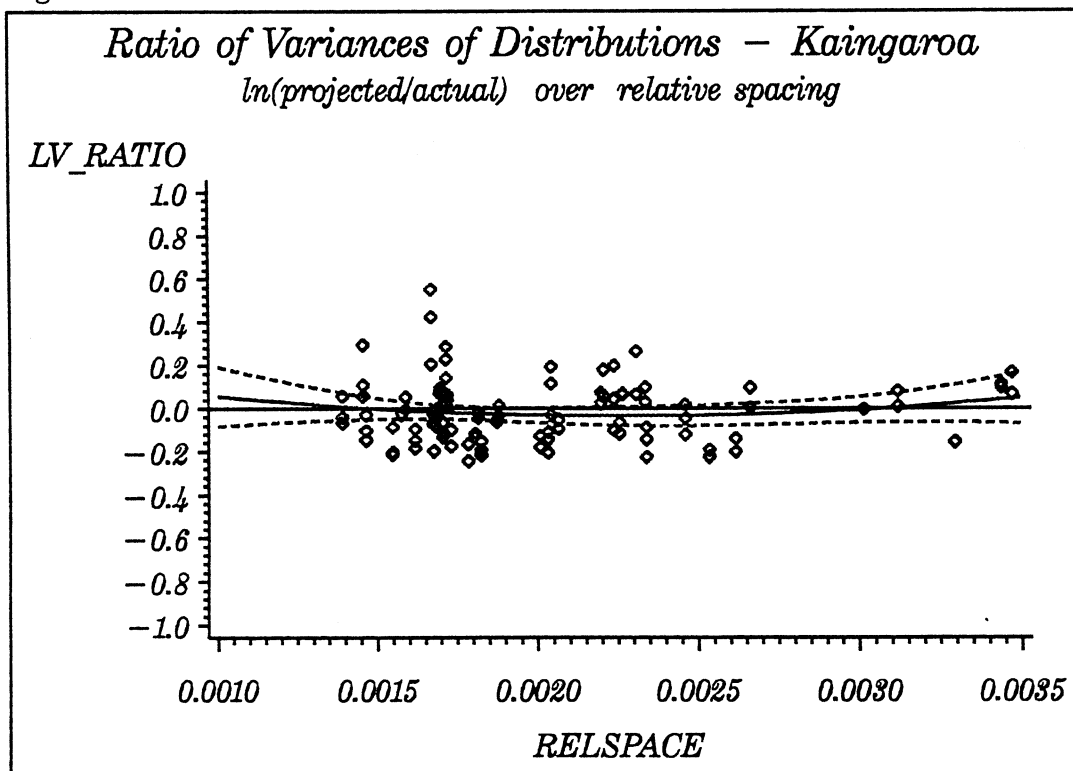


Figure 4.



DISCUSSION

The lack of success in accounting for regional differences using site, climatic and nutrition variables was not wholly unexpected. The measures of nutrition used are highly aggregated values which, although appropriate at the national level, may not be accurate enough at the plot and tree level. Soil descriptions could have been gathered from soil maps but would not have provided the detail that is needed to predict growth potential (Hunter and Gibson 1984). Most validated soil- site quality models have relatively low precision (Verbyla and Fisher 1989). Rainfall was calculated for the actual growth periods but figures had to be taken from the closest meteorological station to the plot. It appears likely that unless a major increase is made in the number of local site variables connected with permanent sample plots, and their resolution of measurement, regional models may be the most practical method of accounting for the observed differences in growth.

The data set developed for this study may be of use in examining the boundaries of the current growth modelling regions. This is one of the aims of the Growth Variation among Sites project.

The Canterbury and Southland regional models both include rainfall as a significant predictor of increment. This could be used to evaluate drought effects on increment when analysing risk, but here serves to reduce one source of variation in the data which should result in better estimates of the parameters. It seems likely that growth is rarely limited by rainfall in any other region. The Southland model was unusual in that stand relative spacing, which is formed with \bar{h}_{100} , did not appear in the best model as a significant predictor, whereas G was significant.

The Clays model presented here has a quadratic in relative spacing but the linear term is not significant ($\alpha = 0.05$). It is retained, as the best 3-variable model included the linear term not the quadratic. The R^2 value increased from 0.467 to 0.481 on adding the quadratic term.

The P nutrition scores did not prove to be significant predictors of diameter increment in this region. The Clays-fertilizer stand-level model can be used in conjunction with predictions of diameter increment to model the effect of P availability on the Auckland clays. The low R^2 of the increment model for Clays (Table 7) may reflect the lack of accurate tree-level nutrition information.

In some cases site index and the closely related variable $\frac{\bar{h}_{100}}{T}$ proved equally good as predictors.

As $\frac{\bar{h}_{100}}{T}$ does not require any assumptions about the subsequent growth of stand height (ie. a height model), it was used instead of site index if there was no clear difference between the two.

The Hawkes Bay model includes a G^2 term. Clearly increment decreases with age but the data set contained some plots from Hawkes Bay with high basal area values which showed very little reduction in increment.

Close proximity to the coast reduces stand height growth in sand dune forests. Estimates of the distance from the sea in the Sands region that were included in the data set were too coarse to be of any use. Altitude proved a significant variable in the model for this region, but an accurate measure of distance from the sea may have been a better predictor of diameter increment. Thirteen plots with a range of altitude from 8 to 120 m above sea level were included in the data set. APPENDIX B contains a list of all variable ranges used in estimating the model coefficients. Care must be taken to ensure models are not applied outside their data ranges.

APPLICATION

Stand inventory provides estimates of stand parameters at a known age. Using stand-level growth models these parameters can be "grown" through time to a target age. Using the model presented here, increments can be calculated for each tree and aggregated to give an estimate of stand basal area at the target age. What is required is a method of scaling the diameter increments so these two independent estimates are equal. In this way the diameter increment model becomes an adjunct to the stand-level model.

For bounded plots the target basal area for a plot (G_{P2}) is calculated as

$$G_{P2} = G_{P1} \frac{G_{S2}}{G_{S1}}$$

where subscripts P and S refer to Plot and Stand and 1 and 2 refer to the time of inventory and the target age respectively.

As stocking at the target age may differ from measured stocking (mortality predicted by the stand growth model), the bounded plot area (A) must be altered to allow for this. ie.

$$A_2 = \frac{n}{N_2}$$

where n is the number of trees in the bounded plot.

What is then required is a scaling factor f so that

$$G_{P2} = \frac{k}{A_2} \sum_{i=1}^n (d_i + f\Delta d_i)^2$$

where $k = \frac{\pi}{4000000}$

The *increments* should be scaled, rather than the projected diameters, so that there is no possibility of the diameters of suppressed trees decreasing with time. By rearranging this, a quadratic equation for f can be found which has a solution:

$$f = \frac{2 \sum d_i \Delta d_i + \sqrt{4(\sum d_i \Delta d_i)^2 - 4 \sum \Delta d_i^2 \left(\sum d_i^2 - \frac{A_2 G_{P2}}{k} \right)}}{2 \sum \Delta d_i^2}$$

By adding the scaled increment $f\Delta d$ to each tree in the bounded plot, estimates of N_2 and G_{P2} equivalent to the stand level model will be produced.

When point sampling is used, plot basal area can easily be changed by altering the basal area factor (BAF). That is:

$$BAF_2 = \frac{G_{P2}}{n}$$

However to ensure the correct stocking is calculated from the plot, the diameter increments must be scaled. Each tree in a point sample plot contributes to the stocking estimate in inverse proportion to its tree basal area. A scaling factor f is required so that

$$N_2 = \frac{BAF_2}{k} \sum_{i=1}^n \frac{1}{(d_i + f\Delta d_i)^2}$$

In this case f must be calculated numerically.

CONCLUSION

Relative diameter, as a measure of the trees position in the dominance hierarchy, is a useful predictor of the individual trees diameter increment. By using the size of each tree in relation to the stand in which it belongs, these regional diameter increment models should provide a better means of projecting *Pinus radiata* inventory data than is currently possible. Until methods for predicting tree height growth and mortality have been added, the models cannot be used for inventory growth projection on their own. However they can be used as adjuncts to stand growth models by adjusting the projected tree basal areas so that they sum to the value predicted by the stand model.

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APPENDIX A. Plots Used.

Plot Identifier	GM Region	Plot Identifier	GM Region
AK_149_0_2_0	SANDS	CY_432_0_4_0	CANTY
AK_149_0_3_0	SANDS	CY_432_0_5_0	CANTY
AK_242_0_3_0	SANDS	CY_447_0_15_0	CANTY
AK_286_1_5_0	CLAYS	CY_447_0_17_0	CANTY
AK_286_1_6_0	CLAYS	CY_447_0_24_0	CANTY
AK_286_2_3_0	CLAYS	CY_447_0_25_0	CANTY
AK_286_3_5_0	CLAYS	CY_447_0_2_0	CANTY
AK_286_3_6_0	CLAYS	CY_447_0_31_0	CANTY
AK_286_4_2_0	CLAYS	CY_447_0_6_0	CANTY
AK_286_4_6_0	CLAYS	CY_447_0_8_0	CANTY
AK_286_4_7_0	CLAYS	CY_560_2_1_0	CANTY
AK_35_0_14_0	SANDS	NN_183_0_1_0	GDNS
AK_35_0_16_0	SANDS	NN_184_0_1_0	GDNS
AK_35_0_1_0	SANDS	NN_234_0_3_0	GDNS
AK_35_0_5_0	SANDS	NN_278_1_13_0	GDNS
AK_35_0_8_0	SANDS	NN_278_1_9_0	GDNS
AK_401_0_11_0	CLAYS	NN_376_0_1_0	GDNS
AK_401_0_13_0	CLAYS	NN_376_0_2_0	GDNS
AK_401_0_1_0	CLAYS	NN_379_1_10_0	GDNS
AK_401_0_3_0	CLAYS	NN_379_1_11_0	GDNS
AK_401_0_7_0	CLAYS	NN_379_1_12_0	GDNS
AK_401_0_8_0	CLAYS	NN_379_1_2_0	GDNS
AK_401_0_9_0	CLAYS	NN_379_1_6_0	GDNS
AK_427_0_2_0	SANDS	NN_379_1_7_0	GDNS
AK_427_0_3_0	SANDS	NN_379_1_8_0	GDNS
AK_434_0_11_0	SANDS	NN_379_1_9_0	GDNS
AK_434_0_15_0	SANDS	NN_421_0_10_0	GDNS
AK_434_0_17_0	SANDS	NN_421_0_12_0	GDNS
AK_434_0_19_0	SANDS	NN_421_0_16_0	GDNS
AK_434_0_25_0	SANDS	NN_421_0_9_0	GDNS
AK_434_0_2_0	SANDS	NN_446_1_68_3	GDNS
AK_434_0_31_0	SANDS	NN_446_1_68_4	GDNS
AK_434_0_36_0	SANDS	NN_446_1_68_6	GDNS
AK_434_0_4_0	SANDS	NN_446_1_75_1	GDNS
AK_434_0_6_0	SANDS	NN_446_1_75_3	GDNS
AK_434_0_7_0	SANDS	NN_446_1_75_4	GDNS
AK_434_0_8_0	SANDS	NN_446_1_75_5	GDNS
AK_439_0_2_0	CLAYS	NN_446_1_75_6	GDNS
AK_439_0_6_0	CLAYS	NN_446_1_75_7	GDNS
AK_458_1_3_0	CLAYS	NN_446_1_76_10	GDNS
AK_458_2_2_0	CLAYS	NN_446_1_76_11	GDNS
AK_501_3_13_0	CLAYS	NN_446_1_76_13	GDNS
AK_501_4_5_0	CLAYS	NN_446_1_76_14	GDNS
AK_501_4_9_0	CLAYS	NN_446_1_76_2	GDNS
AK_501_6_9_0	CLAYS	NN_446_1_76_3	GDNS
AK_569_1_7_0	CLAYS	NN_446_1_76_5	GDNS
AK_570_2_10_0	CLAYS	NN_446_1_76_6	GDNS
AK_570_2_13_0	CLAYS	NN_446_1_76_9	GDNS
AK_570_2_15_0	CLAYS	NN_446_1_77_1	GDNS
AK_570_2_22_0	CLAYS	NN_446_1_77_11	GDNS
AK_570_2_2_0	CLAYS	NN_446_1_77_12	GDNS
AK_656_0_11_0	CLAYS	NN_446_1_77_2	GDNS
AK_656_0_13_0	CLAYS	NN_446_1_78_26	GDNS
AK_656_0_15_0	CLAYS	NN_446_1_78_51	GDNS
AK_656_0_16_0	CLAYS	NN_446_2_76_12	GDNS
AK_656_0_20_0	CLAYS	NN_446_2_76_15	GDNS
AK_656_0_21_0	CLAYS	NN_446_2_76_4	GDNS
AK_656_0_22_0	CLAYS	NN_446_2_76_7	GDNS
AK_656_0_8_0	CLAYS	NN_446_2_76_8	GDNS
AK_862_0_15_0	SANDS	NN_446_2_77_5	GDNS
AK_862_0_18_0	SANDS	NN_446_2_77_8	GDNS
AK_862_0_20_0	SANDS	NN_462_0_69_10	GDNS
AK_862_0_21_0	SANDS	NN_462_0_69_4	GDNS
AK_862_0_27_0	SANDS	NN_462_0_69_5	GDNS
AK_862_0_31_0	SANDS	NN_462_0_69_6	GDNS
AK_862_0_32_0	SANDS	NN_462_0_69_9	GDNS
AK_862_0_52_0	SANDS	NN_462_0_72_1	GDNS
AK_918_1_13_0	SANDS	NN_462_0_78_2	GDNS
AK_918_1_7_0	SANDS	NN_462_0_78_4	GDNS
AK_964_0_7_0	SANDS	NN_462_0_78_5	GDNS
CY_189_0_1_0	CANTY	NN_514_1_4_0	GDNS
CY_432_0_1_0	CANTY	NN_514_3_1_0	GDNS
CY_432_0_2_0	CANTY	NN_514_3_2_0	GDNS
CY_432_0_3_0	CANTY	NN_514_3_7_0	GDNS

Plot Identifier	GM Region	Plot Identifier	GM Region
RO_1_0_5_0	KANG	SD_188_0_22_0	SOUTH
RO_1_0_6_0	KANG	SD_54_0_10_0	SOUTH
RO_1_0_8_0	KANG	SD_54_0_11_0	SOUTH
RO_416_0_0_0	KANG	SD_54_0_12_0	SOUTH
RO_421_0_0_0	KANG	SD_54_0_13_0	SOUTH
RO_464_0_0_0	KANG	SD_54_0_15_0	SOUTH
RO_488_0_3_0	KANG	SD_54_0_1_0	SOUTH
RO_681_0_21_0	KANG	SD_54_0_2_0	SOUTH
RO_681_0_22_0	KANG	SD_54_0_3_0	SOUTH
RO_681_0_23_0	KANG	SD_54_0_4_0	SOUTH
RO_681_0_24_0	KANG	SD_54_0_6_0	SOUTH
RO_681_0_37_0	KANG	SD_54_0_7_0	SOUTH
RO_681_0_39_0	KANG	SD_54_0_8_0	SOUTH
RO_681_0_41_0	KANG	SD_588_0_2_0	SOUTH
RO_681_0_42_0	KANG	SD_588_0_3_0	SOUTH
RO_681_0_43_0	KANG	SD_588_0_4_0	SOUTH
RO_681_0_44_0	KANG	SD_588_0_5_0	SOUTH
RO_685_2_7_0	KANG	SD_588_0_6_0	SOUTH
RO_685_2_8_0	KANG	SD_588_0_7_0	SOUTH
RO_685_4_16_0	KANG	SD_715_0_7_0	SOUTH
RO_690_0_3_0	KANG	SD_715_0_8_0	SOUTH
RO_690_0_4_0	KANG	SD_715_0_9_0	SOUTH
RO_693_0_0_0	KANG	SD_90_0_28_0	SOUTH
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RO_695_2_23_0	KANG	WN_1100_1_29_1	HBAY
RO_695_2_4_0	KANG	WN_1100_1_29_2	HBAY
RO_695_3_11_0	KANG	WN_1100_1_2_11	HBAY
RO_695_3_14_0	KANG	WN_1100_1_35_2	HBAY
RO_695_4_10_0	KANG	WN_1100_1_42_3	HBAY
RO_695_5_18_0	KANG	WN_1100_1_45_1	HBAY
RO_695_5_21_0	KANG	WN_1100_1_45_2	HBAY
RO_695_6_17_0	KANG	WN_1100_1_47_1	HBAY
RO_696_1_15_0	KANG	WN_1100_1_63_1	HBAY
RO_696_1_5_0	KANG	WN_1150_1_27_1	HBAY
RO_696_2_2_0	KANG	WN_1150_1_28_1	HBAY
RO_696_3_10_0	KANG	WN_1150_1_28_2	HBAY
RO_696_4_16_0	KANG	WN_1150_1_29_1	HBAY
RO_696_4_24_0	KANG	WN_1150_1_29_2	HBAY
RO_696_6_11_0	KANG	WN_1150_1_36_7	HBAY
RO_696_6_21_0	KANG	WN_1150_1_39_1	HBAY
RO_696_8_6_0	KANG	WN_1150_1_39_3	HBAY
RO_746_0_0_0	KANG	WN_1150_1_50_1	HBAY
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RO_911_1_1_0	KANG	WN_1280_1_39_5	SANDS
RO_911_1_3_0	KANG	WN_1280_1_45_1	SANDS
RO_911_1_4_0	KANG	WN_1280_1_46_1	SANDS
RO_955_4_12_0	KANG	WN_1280_1_47_3	SANDS
RO_955_4_13_0	KANG	WN_1300_1_107	SANDS
RO_955_4_15_0	KANG	WN_1300_1_2_2	SANDS
RO_955_4_5_0	KANG	WN_1320_1_220	HBAY
RO_955_6_11_0	KANG	WN_1320_1_25_1	HBAY
RO_955_6_12_0	KANG	WN_1320_1_25_2	HBAY
RO_955_6_15_0	KANG	WN_1320_1_25_3	HBAY
RO_955_6_20_0	KANG	WN_1320_1_31_1	HBAY
RO_955_6_5_0	KANG	WN_1320_1_41_1	HBAY
RO_955_6_9_0	KANG	WN_1320_1_65_1	HBAY
RO_955_7_12_0	KANG	WN_1320_1_65_2	HBAY
RO_955_7_15_0	KANG	WN_154_0_1_0	SANDS
RO_955_7_18_0	KANG	WN_295_1_1_0	HBAY
RO_955_7_25_0	KANG	WN_295_2_4_0	HBAY
RO_955_7_30_0	KANG	WN_296_0_1_0	HBAY
RO_955_9_13_0	KANG	WN_296_0_2_0	HBAY
RO_955_9_15_0	KANG	WN_296_0_9_0	HBAY
RO_955_9_17_0	KANG	WN_313_1_27_4	HBAY
RO_955_9_1_0	KANG	WN_354_0_10_0	HBAY
RO_955_9_4_0	KANG	WN_354_0_12_0	HBAY
RO_955_9_8_0	KANG		
SD_130_0_14_0	SOUTH		
SD_170_0_1_0	SOUTH		
SD_170_0_22_0	SOUTH		
SD_180_0_13_0	SOUTH		
SD_180_0_14_0	SOUTH		
SD_180_0_22_0	SOUTH		
SD_180_0_23_0	SOUTH		
SD_180_0_24_0	SOUTH		
SD_188_0_1_0	SOUTH		
SD_188_0_21_0	SOUTH		

APPENDIX B. Variable Ranges.

Canterbury

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	9.2264398	33.0000000
T (years)	15.1000000	19.6769634	29.8000000
G (m ² ha ⁻¹)	28.6700000	45.7713613	65.7400000
\bar{h}_{100} (m)	17.1000000	25.0345550	31.4000000
N (stems ha ⁻¹)	350.0000000	602.6780105	950.0000000
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	232.3475894	315.1325266	395.6131447
S (m)	21.1000000	25.3586387	29.0000000
$\frac{d_i}{\bar{d}_{100}}$	0.2553191	0.7731759	1.2953020
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.0012231	0.0016971	0.0024132
<i>Rain</i> (mm)	359.0000000	693.6164921	1369.00
<i>Altitude</i> (m)	155.0000000	252.1204188	445.0000000

Clays

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	11.9858839	45.8000000
T (years)	15.0000000	20.7907652	29.1000000
G (m ² ha ⁻¹)	14.4800000	40.8156728	65.0900000
\bar{h}_{100} (m)	17.0000000	30.1525066	44.7000000
N (stems ha ⁻¹)	170.0000000	399.2387863	1252.00
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	210.7630104	366.3076677	484.4810712
S (m)	23.4000000	29.3910290	34.5000000
$\frac{d_i}{\bar{d}_{100}}$	0.2760085	0.8292537	1.2328767
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.0010852	0.0017593	0.0034833
<i>Rain</i> (mm)	851.0000000	1542.82	2542.00
<i>Altitude</i> (m)	30.0000000	104.2559367	335.0000000

GoldenDowns

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	12.2350336	38.4000000
T (years)	14.8000000	19.7332886	31.0000000
G (m ² ha ⁻¹)	8.4400000	37.3282685	86.7900000
\bar{h}_{100} (m)	15.0000000	26.2430872	36.9000000
N (stems ha ⁻¹)	180.0000000	544.6483221	1630.00
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	156.3528698	316.0607309	489.4902807
S (m)	20.0000000	26.8414765	31.1000000
$\frac{d_i}{\bar{d}_{100}}$	0.1596244	0.8114002	1.2347188
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.000887040	0.0019449	0.0039644
<i>Rain</i> (mm)	859.0000000	1364.92	2756.00
<i>Altitude</i> (m)	68.0000000	345.6093960	571.0000000

HawkesBay

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	11.0874824	42.1000000
T (years)	14.7000000	19.7586498	33.0000000
G (m ² ha ⁻¹)	12.5800000	56.9703657	99.5800000
\bar{h}_{100} (m)	18.2000000	27.7288326	43.2000000
N (stems ha ⁻¹)	119.0000000	793.4978903	2045.00
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	223.0954220	369.0591273	584.2546250
S (m)	23.2000000	28.1167370	33.5000000
$\frac{d_i}{\bar{d}_{100}}$	0.1453202	0.7634758	1.2570755
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.000838316	0.0017748	0.0047010
<i>Rain</i> (mm)	717.0000000	1433.26	2461.00
<i>Altitude</i> (m)	280.0000000	444.8298172	754.0000000

Kaingaroa

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	8.2194789	30.0000000
T (years)	15.0000000	18.8699752	37.0000000
G (m ² ha ⁻¹)	16.9900000	50.2568734	76.3400000
\bar{h}_{100} (m)	16.7000000	31.8611663	50.6000000
N (stems ha ⁻¹)	158.0000000	875.7320099	2702.00
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	166.8381218	314.4506617	564.8163515
S (m)	22.9000000	33.8619107	38.1000000
$\frac{d_i}{\bar{d}_{100}}$	0.2738462	0.7270992	1.2592593
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.000639950	0.0013449	0.0034332
<i>Rain</i> (mm)	781.0000000	1612.33	2267.00
<i>Altitude</i> (m)	87.0000000	301.6836228	652.0000000

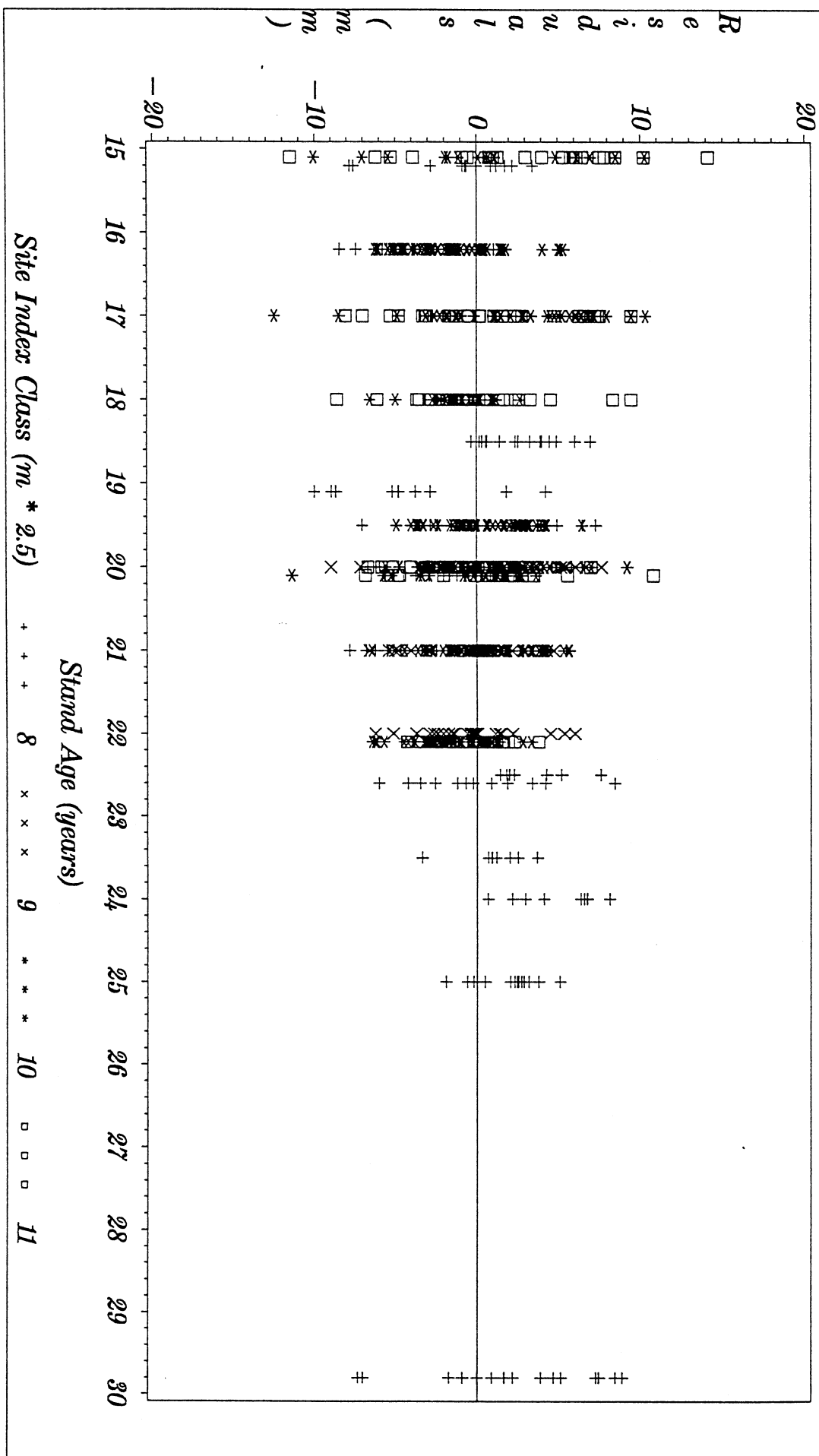
Sands

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	9.6798718	60.4000000
T (years)	14.6000000	24.7524359	46.0000000
G (m ² ha ⁻¹)	9.3600000	38.3340769	66.7900000
\bar{h}_{100} (m)	16.5000000	26.9834615	37.8000000
N (stems ha ⁻¹)	109.0000000	610.1012821	2404.00
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	160.2633353	342.3866746	664.2304196
S (m)	19.5000000	24.1046154	32.7000000
$\frac{d_i}{\bar{d}_{100}}$	0.3258786	0.8417937	1.3442623
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.000865233	0.0019772	0.0037819
<i>Rain</i> (mm)	690.0000000	1230.06	1616.00
<i>Altitude</i> (m)	8.0000000	42.7256410	120.0000000

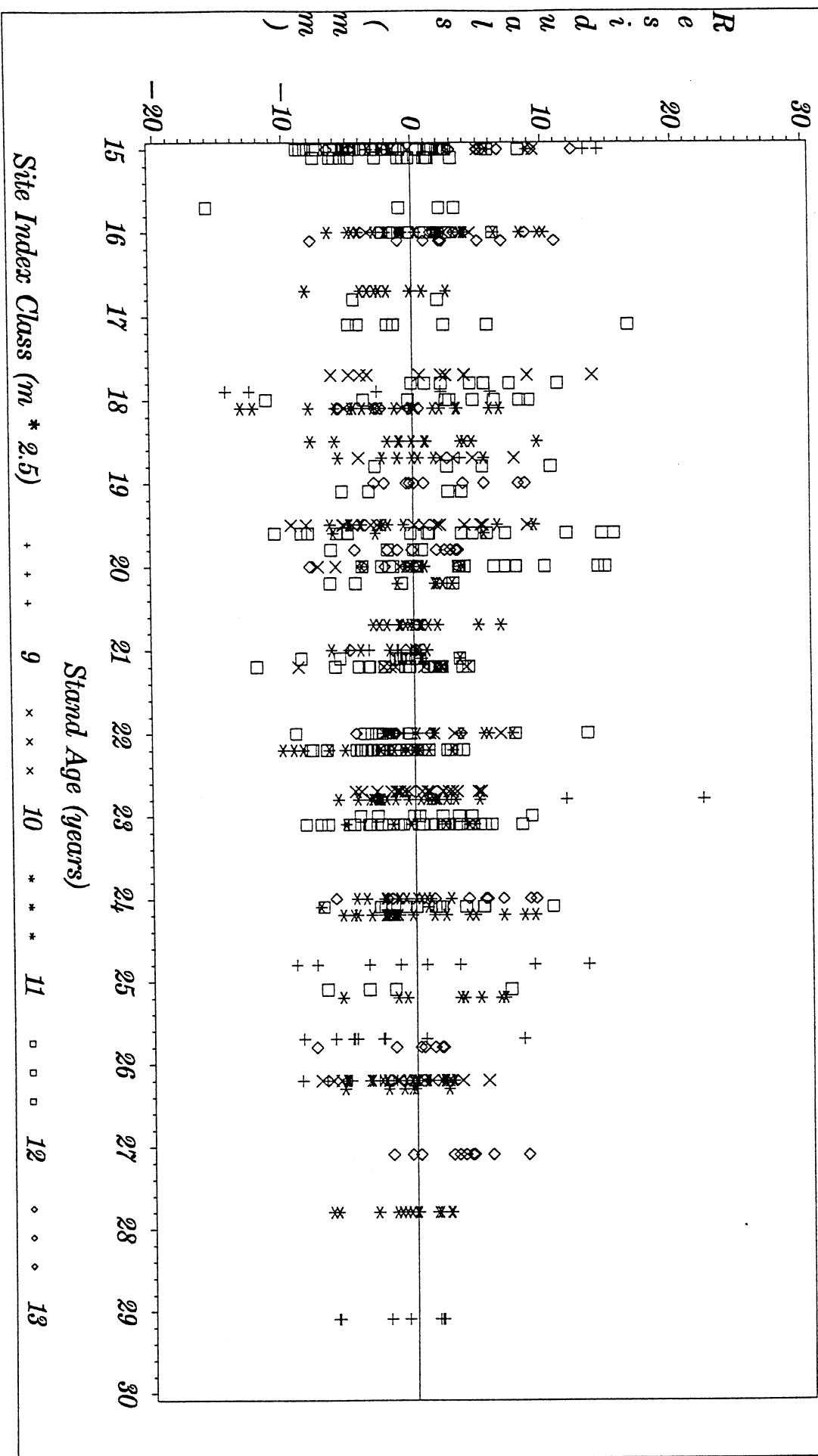
Southland

Variable	Minimum	Mean	Maximum
Δd (mm)	0.1000000	11.4566265	41.8000000
T (years)	14.6000000	21.7163320	33.1000000
G (m ² ha ⁻¹)	18.3200000	53.6477644	88.8400000
\bar{h}_{100} (m)	16.5000000	25.0748327	40.3000000
N (stems ha ⁻¹)	138.0000000	1078.34	2876.00
$2000\sqrt{\frac{G}{\pi N}}$ (mm)	173.2254045	317.9065358	684.2123848
S (m)	20.6000000	23.5161981	27.9000000
$\frac{d_i}{\bar{d}_{100}}$	0.0982533	0.7913889	1.2151899
$\frac{1}{\bar{h}_{100}\sqrt{N}}$	0.000689000	0.0017105	0.0035264
$Rain$ (mm)	523.0000000	919.9585007	1498.00
$Altitude$ (m)	50.0000000	133.3962517	270.0000000

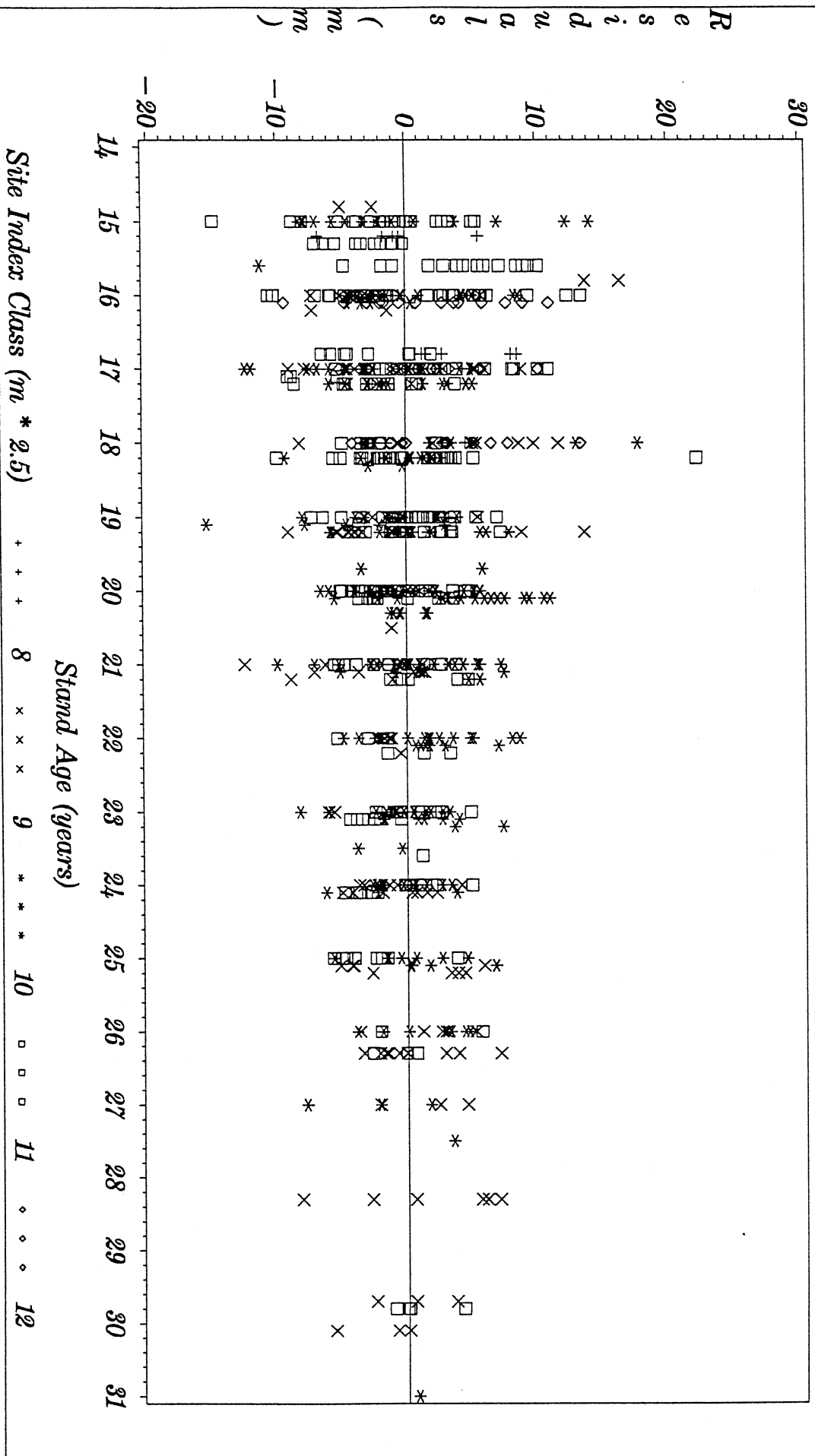
*Fig. 5 Canterbury Region.
Diameter Increment Residuals vs. Age*



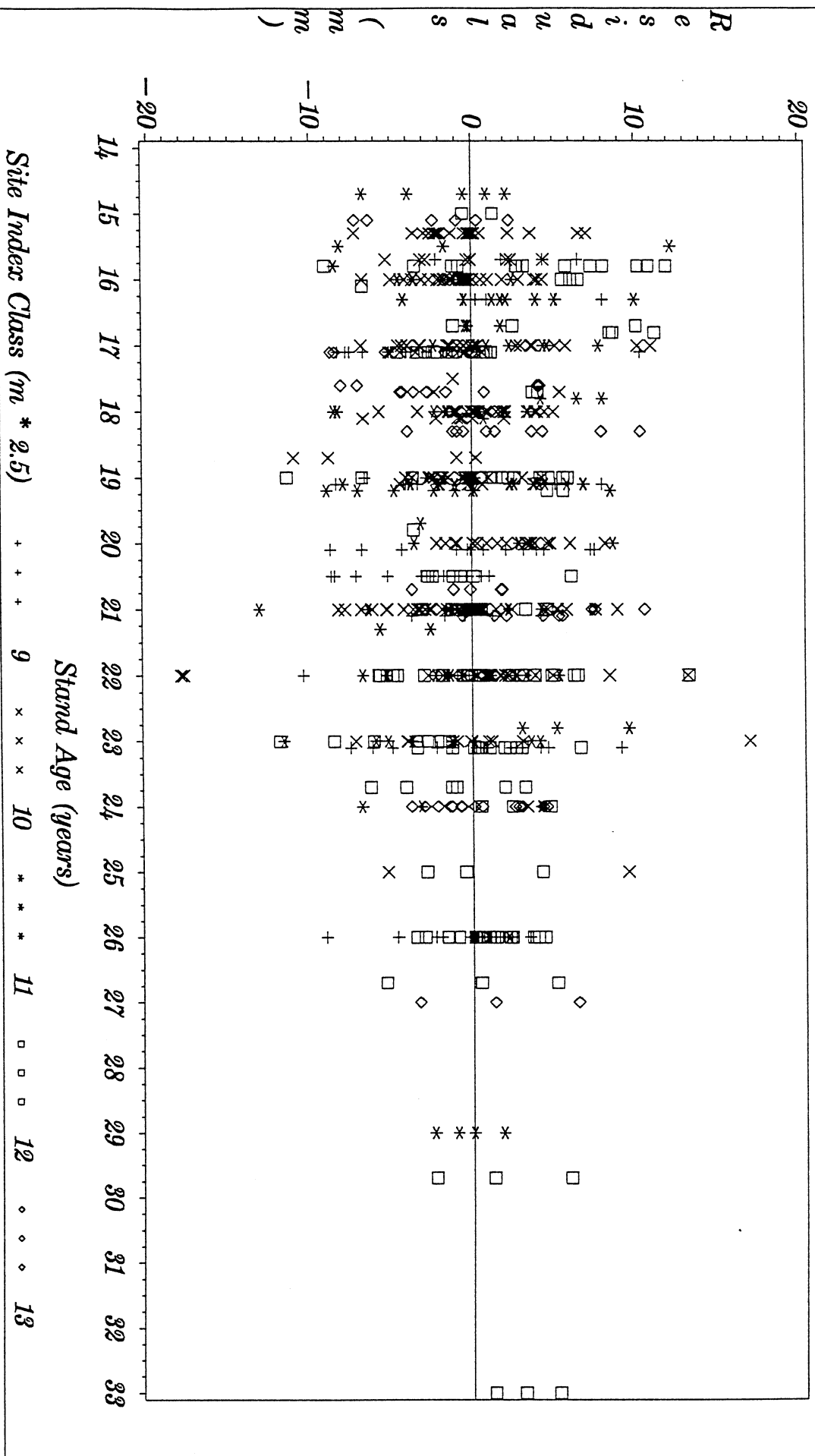
*Fig. 6 Clays Region
Diameter Increment Residuals vs. Age*



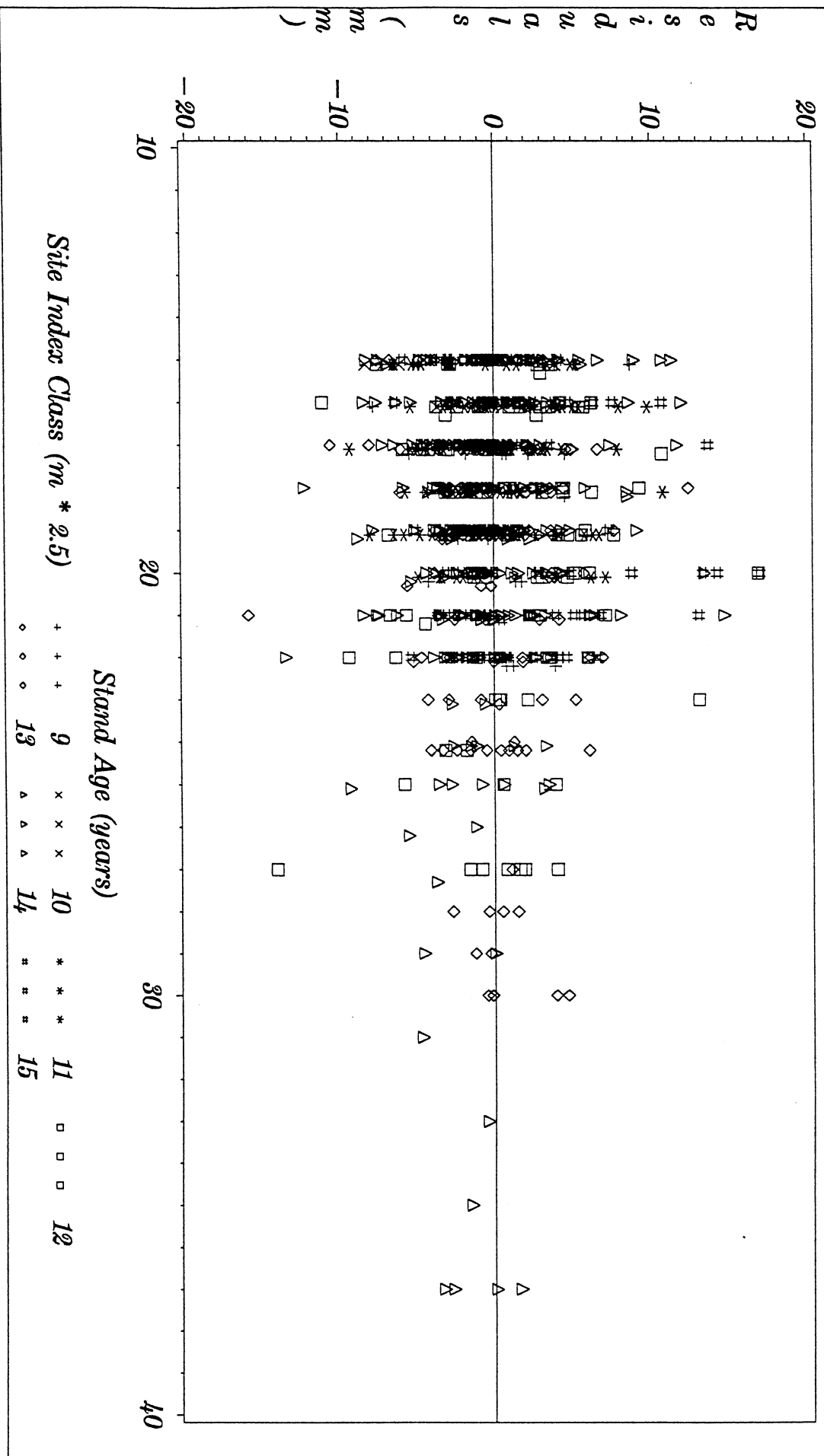
*Fig. 7 Golden Downs Region.
Diameter Increment Residuals vs. Age*



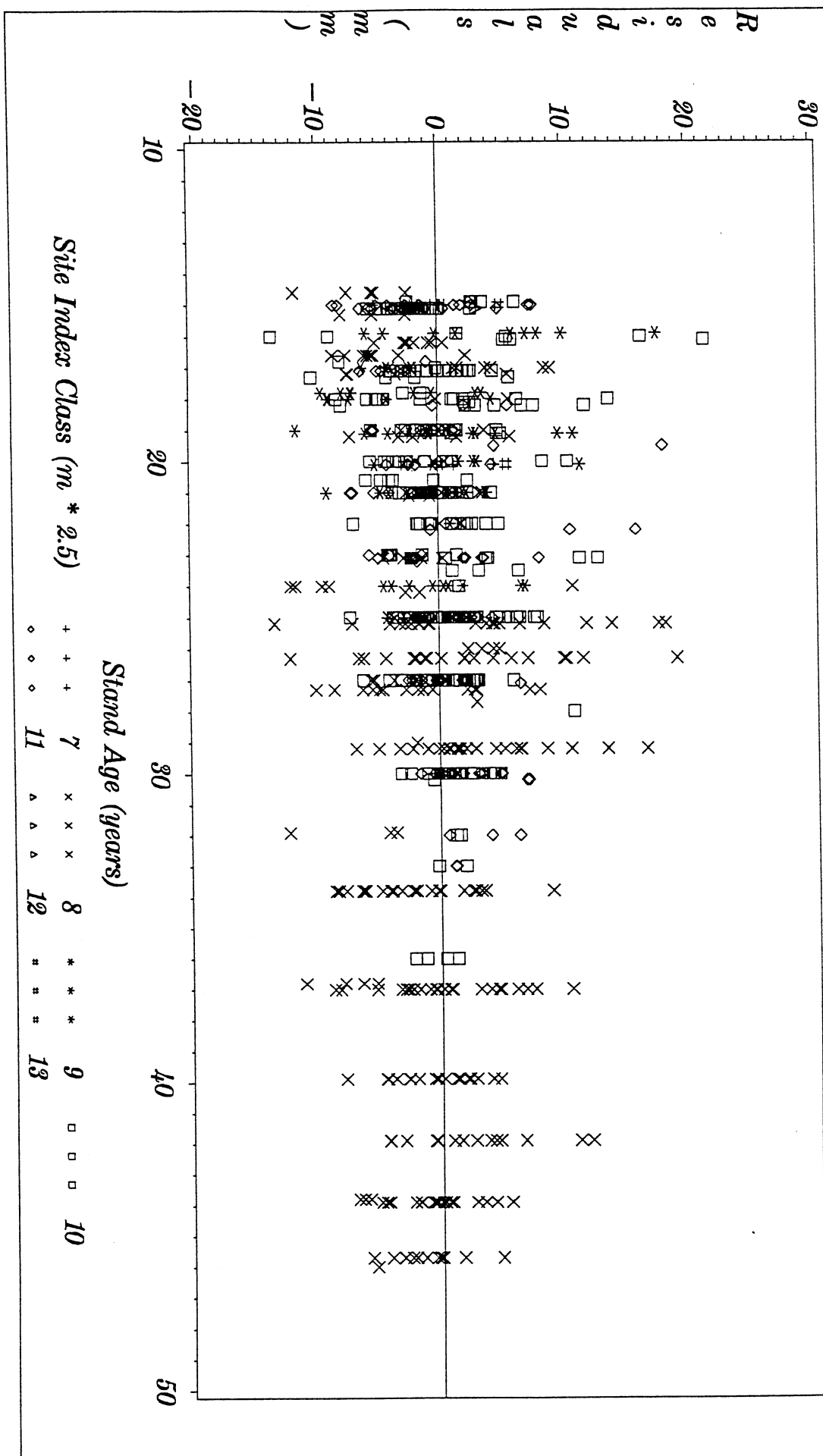
*Fig. 8 Hawkes Bay Region.
Diameter Increment Residuals vs. Age*



*Fig. 9 Kaingaroa Region.
Diameter Increment Residuals vs. Age*



*Fig. 10 Sands Region.
Diameter Increment Residuals vs. Age*



*Fig. 11 Southland Region
Diameter Increment Residuals vs. Age*

