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# A COMPARISON OF THE REGIONAL HEIGHT MODELS USED IN NEW ZEALAND

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## FRI/INDUSTRY RESEARCH COOPERATIVES

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## EXECUTIVE SUMMARY

Since 1980 a series of regional stand growth models for *Pinus radiata* have been developed for use throughout New Zealand. Site index curves which form an integral part of each model have been produced which in some instances can predict similar results. Previous investigations into the apparent similarities have focussed on predictions from the models rather than the structure and precision of the individual models. This report describes a statistical comparison of the likelihood estimates of the individual regional models and a single 'national' model constructed from the pooled data. It also contrasts regional differences by graphing the confidence ellipses for the primary parameters of the models.

The results of these investigations suggest that the development of a single model for all New Zealand would negate much of the flexibility currently available through the regional models. Comparisons of the primary model parameters, which for statistical reasons can only be considered indicative, exhibit marked groupings of models and tend to support the results of earlier analyses based on predictions from the models.

## A COMPARISON OF THE REGIONAL HEIGHT MODELS USED IN NEW ZEALAND

### Introduction

Since 1980 a series of regional stand growth models for *Pinus radiata* in New Zealand have been developed based on the methodology described by Garcia (1979, 1983, 1984a). As part of the development of each model, site index curves are fitted to periodic height and age measurements collected from Permanent Sample Plots (McEwen 1979, Pilaar and Dunlop 1990) located in forests throughout the regions. These curves are used not only in the growth model itself, but also as important components of other stand modelling systems (eg. STANDPAK, Kininmonth 1987, Whiteside 1990, Gordon 1992).

Although the curves form an integral part of each regional model, there appears to be little difference between some predictions from the models. The significance of these differences has previously been addressed by Eggleston (1991), who concluded that there were essentially two main groups - the first consisting of those models representing the Central North Island pumice plateau (Dunningham & Lawrence 1987), Hawkes Bay, and Southland (Law 1988) and the second those models representing Nelson (Law 1990), Canterbury (Lawrence 1988) and the Auckland clay soils (Shula 1987, 1988).

Eggleston's results were based largely on differences in predictions from the various models and on sensitivity analyses. This report describes further investigation of the differences between the models by looking at statistical differences in the likelihood estimates and at the confidence limits surrounding the major model parameters.

## Background

### i) Methodology

Height growth in the regional growth models is described by (1), which assumes that the height increment of a stand at any time is a function of its current height.

$$\frac{dH^c}{dt} = b(a^c - H^c) \qquad \text{... (1)}$$

where

| | | |
|---|---|---|
| $H$ | = | top height (m) |
| $t$ | = | age (years) |

By integrating (1), an equation for top height can be derived which, given an height-age pair, enables top height to be calculated at any other age $t$.

$$H = a\left(1 - \left(1 - \frac{H_0^c}{a^c}\right)e^{-b(t - t_0)}\right)^{\frac{1}{c}} \qquad \text{... (2)}$$

where

| | | |
|---|---|---|
| $H$ | = | top height (m) |
| $H_0$ | = | top height (m) at time $t_0$. |
| $t$ | = | age (years) |
| $t_0$ | = | age (years) at $H$=0m |

Parameter $a$ is the asymptotic height as $t \rightarrow \infty$, $b$ is a 'time scale' factor which can be used to modify the rate of growth (see (3)) and $c$ determines the shape of the height-age curve.

By utilising $a$, $b$, or a linear combination of the two, the site index curves can be developed from (2) for any particular set of data. In all cases described here, best results were obtained by using $b$ such that:

$$b = \frac{-\ln\left(1 - \left(\frac{S}{a}\right)^c\right)}{20 - t_0} \qquad \text{... (3)}$$

assuming $H_0^c$ = 0

and where $S$ = site index (m) at age 20 years

Maximum likelihood estimators were used to obtain values for the model coefficients. A likelihood function can be created for a particular combination of parameters; the best estimates being the values for which the function has a maximum. In deriving the height models it was found convenient to use the logarithm of the likelihood function and sum the estimates over all pairs of successive measurements.

### ii) Height models

The six height models under consideration were all based on large amounts of data and, with one exception, have the same parameters in common (see Table 1). In the Hawkes Bay and Central North Island models, the $t_0$ term was omitted because it was practically insignificant. For the Auckland clays, an alternative term, $H_0^c$, was adopted in place of $t_0$. This term is not dissimilar to $t_0$, with the initial condition $H=H_0$ at $t=0$ years.

To facilitate the analysis, a common model form was fitted to each of the data sets, and to the pooled data from all the regions. This model is similar to those previously fitted, but it includes the $t_0$ term in all cases. The resulting parameter and log-likelihood estimates are detailed in Table 2. In some instances the parameter estimates differ substantially from those used in the models, suggesting a degree of over-parametrisation when $t_0$ is included.

## Table 1: Height model parameters

| Model | # plots | # meas | a | c | $t_0$ | $H_0^c$ |
|---|---|---|---|---|---|---|
| Hawkes Bay | 167 | 723 | Yes | Yes | No | No |
| Southland | 258 | 1199 | Yes | Yes | Yes | No |
| C.N. * | 284 | 2551 | Yes | Yes | No | No |
| Nelson | 385 | 2023 | Yes | Yes | Yes | No |
| Canterbury | 120 | 590 | Yes | Yes | Yes | No |
| Auckland clays | 114 | 535 | Yes | Yes | No | Yes |

* Central North Island

## Table 2: Parameter estimates

| Model | $a$ | $c$ | $t_0$ | $\lambda^*$ |
|---|---|---|---|---|
| Hawkes Bay | 70.343 | 0.795 | 0.682 | 1028.3 |
| Southland | 60.376 | 0.653 | 0.562 | 1760.0 |
| C.N. | 64.626 | 0.712 | 0.009 | 3908.3 |
| Nelson | 60.386 | 0.781 | 1.225 | 2738.2 |
| Canterbury | 53.704 | 0.740 | 1.275 | 921.4 |
| Auckland clays | 56.065 | 0.800 | 0.857 | 524.7 |
| Pooled data | 62.190 | 0.742 | 0.634 | 10235.68 |

* Log-likelihood estimate

## Model comparisons

### i) Comparison of likelihood estimates

The hypothesis of a common model for all regions can be tested with a likelihood ratio test. The $\chi^2$ statistic can be calculated from the likelihood estimates to test the significance of the difference between the individual models and the combined model (Wilks 1962, Seber and Wild 1989):

$$\chi^2 = -2*(\Sigma\lambda_i - \lambda_c) \qquad \text{... (4)}$$

$$= 1290.352$$

where $\lambda_i$ = log-likelihood of individual model $i$

$\lambda_c$ = log-likelihood of the combined model

Taking into account the number of parameters in each of the models, plus the variances, the calculated result can then be compared with the equivalent $\chi^2$ tabular value for 25 degrees of freedom and desired level of confidence. In this case, the large calculated value implies a highly significant difference between the individual and combined models and the hypothesis of a single common model must be rejected. A similar result would have been obtained using the models as published.

### ii) Comparison of model parameters

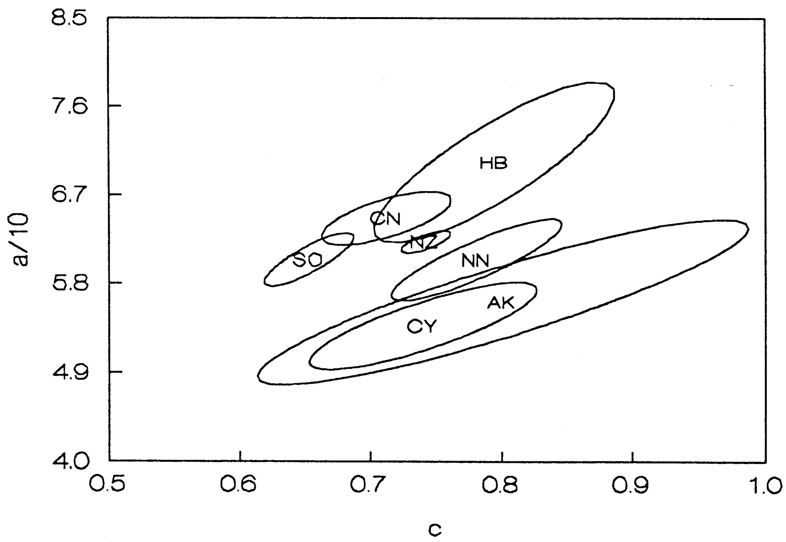
An alternative method of comparison is to investigate the parameter estimates for each region and their standard errors. In (2), $a$ is the asymptote for that model, $c$ determines the shape of the curve and $t_0$ is the point where it crosses the age axis at $H$=0m. Obviously $a$ and $c$ are the major parameters, $t_0$ having only a minor effect on the overall performance. If the coordinates of $a$ and $c$ $(c,a)$ are plotted, an ellipse representing the 95% confidence limits for the parameters can be drawn around each point with its size determined by the standard errors of the parameters and orientation by the correlation coefficient. The equation for each ellipsoid (5) can be solved by equating to $\chi^2_{2df} = 5.99$ and plotted for each model by substituting the polar equivalents of *c* and *a*. The result is shown in Figure 1.

$$\frac{1}{1-r^2}\left(\frac{c^2}{s_c^2}+\frac{a^2}{s_a^2}-2r\frac{ca}{s_c s_a}\right) \qquad \text{... (5)}$$

where

| | | |
|---|---|---|
| $c$ | = | *c* parameter estimate |
| $s_c$ | = | standard error of *c* |
| $a$ | = | *a* parameter estimate |
| $s_a$ | = | standard error of *a* |
| $r$ | = | correlation coefficient of *a* and *c* |

**Figure 1: Parameter confidence limits**




Key :

| | | | | | |
|---|---|---|---|---|---|
| HB | = | Hawkes Bay | CY | = | Canterbury |
| SO | = | Southland | AK | = | Auckland |
| CN | = | Central North Island | NZ | = | Pooled data |
| NN | = | Nelson | | | |

## Discussion

The initial comparison of likelihood estimates highlights likely differences between models without identifying which models are significantly different. The second comparison perhaps provides more insight, though the number of approximations involved means any inferences should be regarded as indicative only. Two main groups can be identified in Figure 1; those models representing the Central North Island pumice plateau, Hawkes Bay and Southland, and those representing Nelson, Canterbury and the Auckland clay soils. This is in accord with Eggleston's results based on comparing predictions from the models.

The degree of overlap between the Hawkes Bay and Central North Island models, where the mean of the Central North Island model falls within the confidence ellipse generated from the Hawkes Bay model, is not unexpected and confirms preconceived notions of growth in the two adjacent regions.

The reasons for the overlap between the Canterbury and Auckland clays models are not so obvious, other than both regions suffer from more severe site problems than is perhaps the case elsewhere. It can be seen, however, that there is far more variability in the parameter estimates for the Auckland clays model than any other, making it difficult to draw any firm conclusions.

Finally, the benefits of pooling all the data for a national model appear somewhat dubious. Although the parameter confidence limits are agreeably small, such a model would only provide an estimate of the 'average' growth, without being particularly accurate for specific locations when compared to the existing regional models.

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