F.R.I. PROJECT RECORD
NO. 1862

A GROWTH MODEL FOR RADIATA PINE GROWN IN CANTERBURY

M.E. LAWRENCE

REPORT NO. 8 MAY 1988

FRI/INDUSTRY RESEARCH COOPERATIVES

EXECUTIVE SUMMARY

This report documents the construction and evaluates the performance of a stand growth model for radiata pine in Canterbury. The model, one of a series of regional models, can be used to derive yield tables and reliably predict the effects of different management options, on stands grown on both the Canterbury foothills and plains.

A GROWTH MODEL FOR RADIATA PINE GROWN IN CANTERBURY

M.E. Lawrence

CONTENTS

A GROWTH MODEL FOR RADIATA PINE GROWN IN CANTERBURY

M.E. Lawrence

1.0 INTRODUCTION

The Canterbury region is bounded by the Kaikoura ranges to the north and the Waitaki river in the south, and stretches eastwards from the Southern Alps to the Pacific. The region contains approximately 56 000 ha of exotic forest, of which radiata pine (Pinus radiata D.Don) is the major species. This corresponds to about 20% of the resource in the South Island and 5% of the national exotic forest resource.

Until now, forest managers and researchers have been forced to rely on adaptations to growth models based on data from other regions to obtain estimates of growth and yield. The most common approach has been to use the Kaingaroa Growth Model (Elliott and Goulding, 1976) with a local height-age curve - initially the curves developed by Burkhart and Tennent (1977), and more recently, those developed by J. Leitch of the New Zealand Forest Service Resource Inventory Group (1983 unpubl.).

This new model has been completed under the auspices of the FRI/Industry Research Cooperatives, Stand Growth Modelling Cooperative. The methodology is identical to that described by Garcia (1979, 1983) and used for earlier models for Golden Downs, Hawkes Bay, North Island Sands, Auckland Clays and Kaingaroa. An attempt has been made to identify differences in growth between the Canterbury plains and the Canterbury foothills. The approach adopted was to initially pool the data from both sub-regions and fit the various components of the model. Once the "best" model had been constructed, the same model structure was fitted separately to the data from the plains and foothills, and the models tested for significant differences. The results of this analysis are described in Section 4.1, Foothills versus Plains.

2.0 DATA

Most of the data were obtained from the New Zealand Forest Service Permanent Sample Plot (PSP) system (McEwen, 1979). The remaining 6% came from sectional measurements collected to fill gaps in the main database. (Wilcox, 1987 unpubl.). All data up to and including the 1986 measurements were initially extracted. The forests represented in the data are Eyrewell, Balmoral, Burnham, Ashley, Hanmer, Geraldine, Waimate and Omihi State Forests, plus Bottle Lake and Selwyn Plantation Board forests. (The latter incorporated in the sectional measurement data).

Preliminary screening of the data consisted of identifying plots and measurements that fall into the categories listed below. These measurements were subsequently deleted from the database.

- measurements taken in December, January or February.
- poisoned thinned.
- regeneration in the stand.
- fertilized.
- more than 2 trees per plot windblown.
- mean DBH of windblown trees > mean DBH of remaining trees.
- basal area, stocking or height missing.
- less than 4 height trees.
- only crop trees measured.
- average height substituted for mean top height. (PMH is permitted as a valid estimator of MTH).
- less than two measurements per plot.
- measurement dates less than 7 months apart.

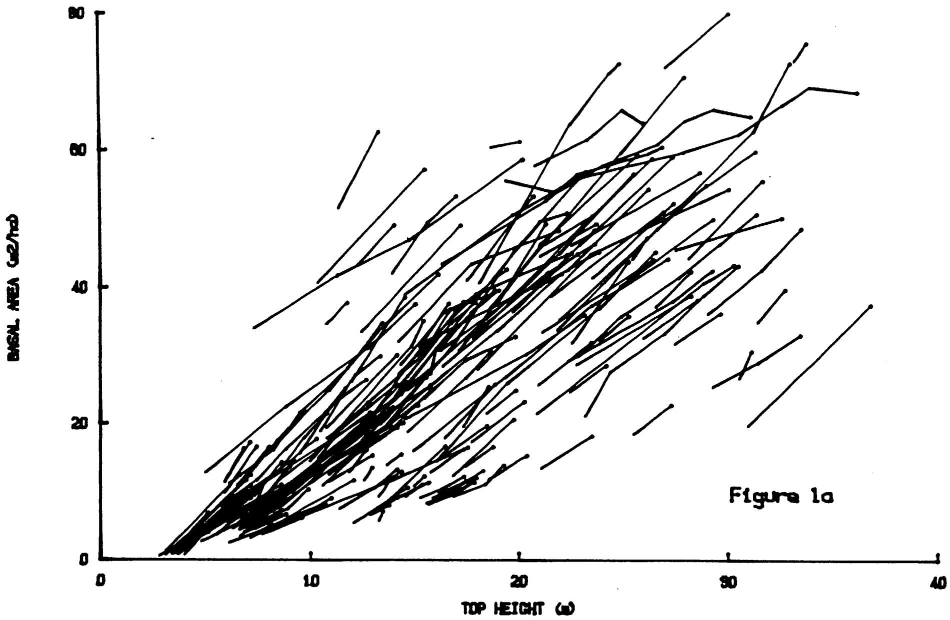
After preliminary screening, the individual measurements of basal area, height etc. for each plot were graphed and visually checked for errors or "abnormal" growth patterns. If at all possible, errors were corrected and the measurement included, otherwise the measurement was deleted from the database.

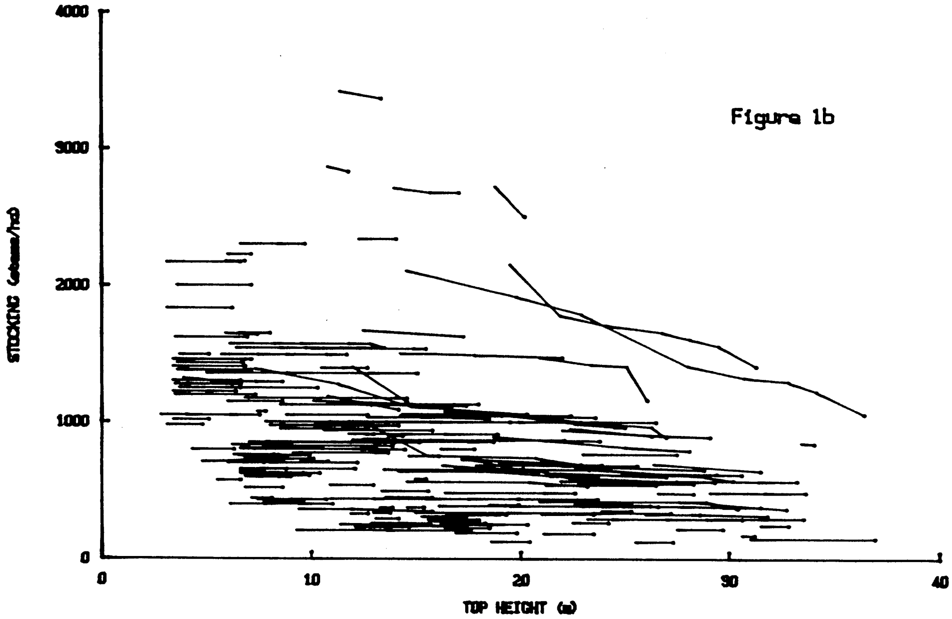
The final database, after checking, consisted of 767 measurements from 201 plots. Figures 1a, 1b and 1c illustrate the range of stand parameters used in the model. Table 1 provides a breakdown of plots and measurements by location.

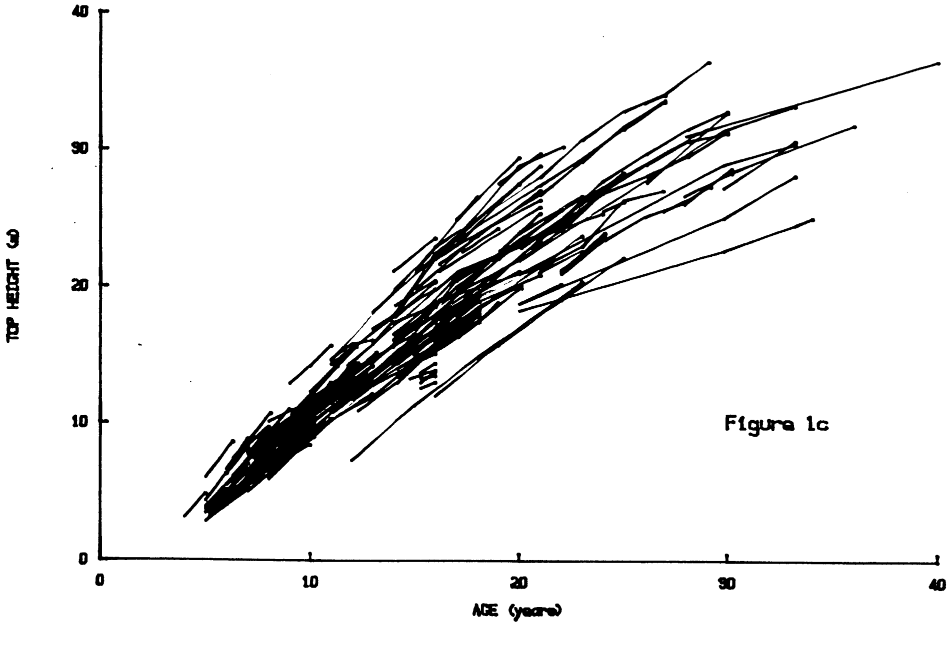
Table 1

Summary by Location

Forest	No. of plots	No. of measurements
Eyrewell	51	213
Balmoral	37	147
Burnham	12	45
Bottle Lake	3	33
Ashley	41	147
Hanmer	7	17
Omihi	17	50
Waimate	6	16
Geraldine	16	50
Sectional msts.	11	49


80
60
40
20
0
BASAL AREA (m2/ha)
0
10
20
30
40
TOP HEIGHT (m)
Figure 1a


4000
3000
2000
1000
0
STOCKING (stems/ha)
0
10
20
30
40
TOP HEIGHT (m)
Figure 1b


40
30
20
10
0
TOP HEIGHT (m)
0
10
20
30
40
AGE (years)
Figure 1c

All ages are calculated based on the month and year of measurement minus the establishment year. The corrections for month of measurement are given in Table 2 and are used to apportion growth in line with seasonal variation. (McEwen, 1979).

Table 2

Age Corrections

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
-0.3	-0.2	-0.1	0.0	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6

2.1 Height model data

The data used for modelling height growth were restricted to plots with three or more measurements that have a period of at least six months between successive measurements. The 590 measurements from the 120 plots used to develop the model are summarised in Table 3.

Table 3

Height Model Data Statistics*

Age: Initial Values			Age: Final Values		
Min	Mean	Max	Min	Mean	Max
5.0	11.0	29.8	8.0	17.6	35.0

Height: Initial Values			Height: Final Values		
Min	Mean	Max	Min	Mean	Max
2.8	11.4	33.4	7.5	18.9	38.1

Age difference (between msts)			Site index		
Min	Mean	Max	Min	Mean	Max
1.9	6.6	22.0	15.4	22.4	31.3

Average measurements/plot : 4.9

* per plot statistics

2.2 Basal area & stocking model data

In order to determine the model coefficients for basal area and stocking, the data is formed into measurement pairs consisting of the basal area, stocking and top height at the age of each measurement and the values of the same variables at the time of the next measurement, for that plot. There is no discontinuity between measurement pairs other than for thinnings or excessive levels of windthrow.

Table 4 summarises the 327 measurement pairs formed from 179 plots used for modelling basal area growth and stocking.

Table 4

Basal Area & Stocking Model Data Statistics

First Measurement of each Pair

Basal Area			Stocking			Height		
Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
0.6	23.2	73.1	124	903	5377	2.8	14.6	34.1

Age			Age Difference			Site		
Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
4.0	13.8	33.2	0.6	2.2	13.0	15.4	22.4	31.3

An estimate of site index is calculated for each plot from the output of the height model estimation procedure. If a plot was not used in the height model for some reason, the estimate is obtained by averaging the site indices calculated from the individual measurements for that plot.

2.3 Initial growth function data

The initial growth function is used when starting the growth model from age zero. The function is fitted to data derived from the first measurements of plots that have had no prior thinning. This data is grown either forward or back in time, using the new model, and the function fitted to the basal area and stocking at a pre-determined top height. Table 6 summarises the 49 estimates of basal area, stocking and top height on which the function was based.

Table 6

Initial Growth Function Statistics

Basal Area			Stocking			Height		
Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
0.6	7.5	45.9	973	1449	2485	2.3	5.7	12.3

2.4 Thinning function data

The thinning information is extracted from the basic data file and consists of the measurements taken immediately before and after a thinning. Table 5 summarises the 74 thinning measurements from which the function is derived.

Table 5

Thinning Function Data Statistics

Basal Area			Stocking			Height		
Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
6.0	26.4	59.2	125	1099	2832	6.2	14.9	31.5

Residual B.A.			Residual Stock			Ratio*		
Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
3.3	16.1	53.4	100	528	1038	0.93	1.05	1.31

* Ratio = Mean DBH after thinning / Mean DBH before thinning

2.5 Volume function data

The PSP system uses a general volume/basal area regression to calculate the volume for a measurement if there is a poor height sample in the plot. (The normal procedure is to use an individual tree volume function.) Accordingly, the data was restricted to plots where volume was calculated from a tree volume function, the default tree volume tables being used in all cases. The 675 measurements of volume used to develop the V/B regression are summarised in Table 7.

Table 7

Volume Function Data Statistics

Basal Area			Stocking			Height		
Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
0.2	25.0	83.0	100	902	6817	2.3	16.0	38.1

Volume			V/B		
Min	Mean	Max	Min	Mean	Max
0.6	174.9	833.6	2.3	5.7	11.9

3.0 ANALYSIS and RESULTS

The growth equations used in the model are described fully in Garcia (1979, 1983). Only those methods used to construct the the Canterbury model are described in this report.

3.1 Height growth

The site index equation has the general form :

$$\frac{dH^c}{dt} = b(a^c - H^c) \qquad (1)$$

By using a, b, or a linear combination of the two as a measure of site index, it is possible to estimate the remaining parameters by using a maximum likelihood technique.

Integrating (1) with H=0 at t=t0, where t=age in years, we can obtain the general equation for top height :

$$H = a[1-e^{-b(t-t0)}]^{1/c} \qquad (2)$$

and by adopting b as the measure of site index such that:

$$b = -\ln[1-(S/a)^c]/(20-t0) \qquad (3)$$

and where S is the site index in metres and t0 is the height at age 0 years. The final parameter estimates are:

$$a = 53.704$$
$$c = 0.74019$$
$$t0 = 1.2753$$

Top height (H) is in metres and age (t) is in years. The value of b is obtained from (3).

The height residuals from this equation are shown in Figure 2a. The residuals are fairly well distributed about the axis, the mean residual is 0.07m, and the residual mean square is 0.7. This is very consistent with height models developed for other regions in New Zealand.

Figure 2b compares the new model with the earlier unpublished curves of J. Leitch for the Canterbury foothills and plains. It can be seen that the curves are fairly similar up until age 20, whence they diverge quite markedly - even by age 30 years. Whereas the Leitch curves approach a unique asymptote for any given site index, the new curves tend to a common upper asymptote of approximately 54 metres.

Further visual comparison of the curves with the data would indicate that the Leitch curves are somewhat conservative, possibly due to a lack of data at the time they were constructed.

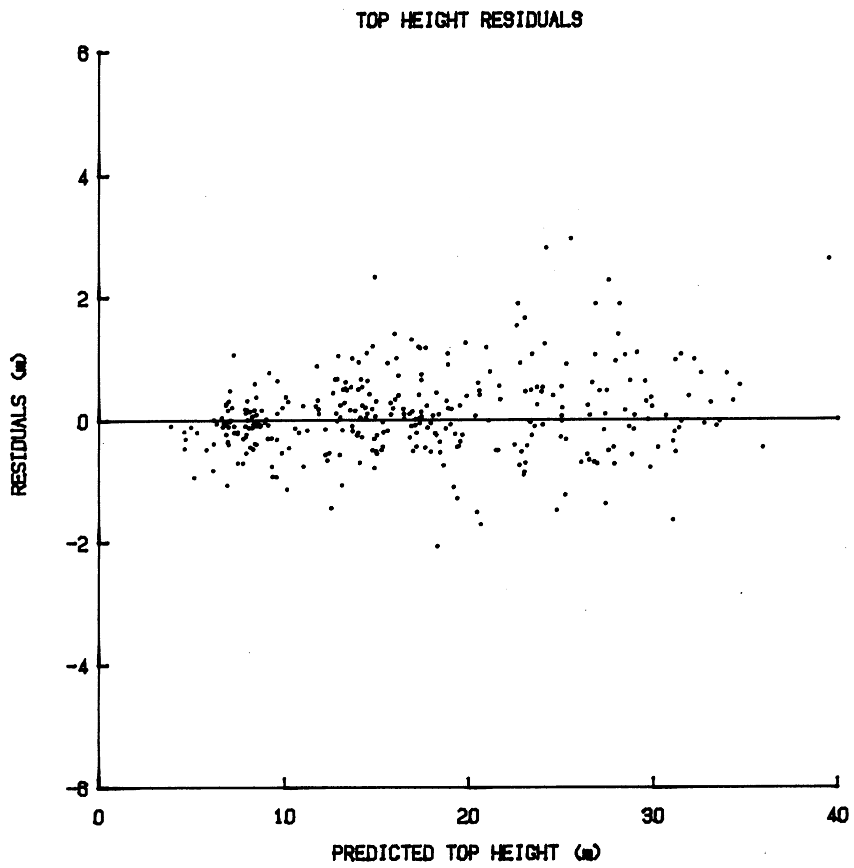


Figure 2a

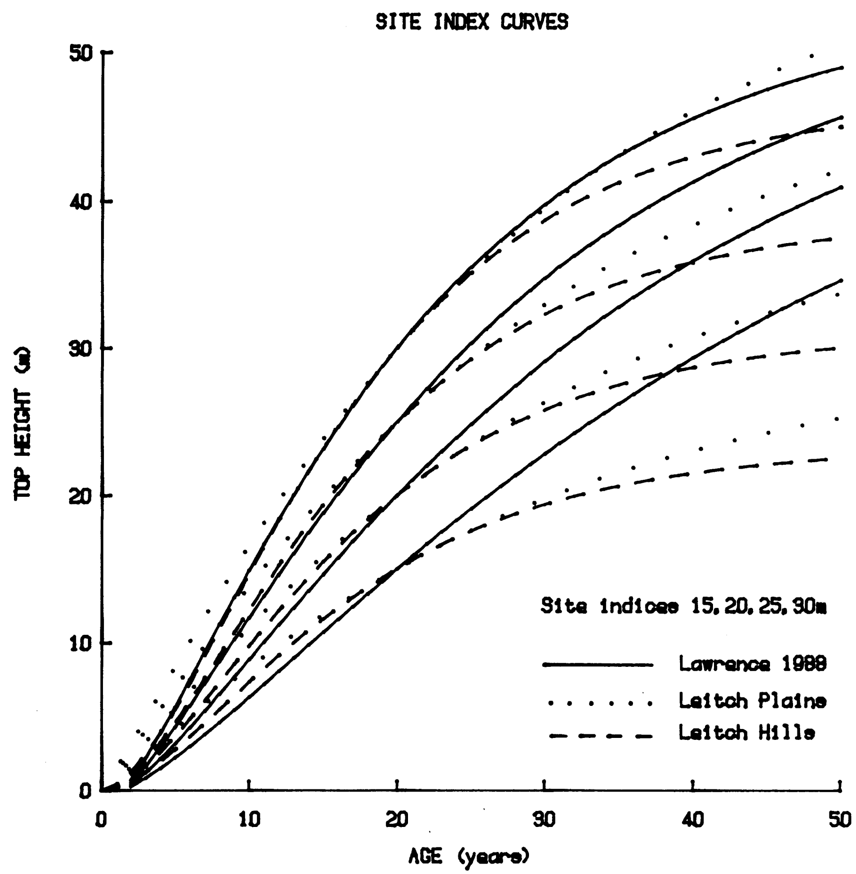


Figure 2b

3.2 Basal area and stocking equations

The equations to model basal area growth and changes in stocking are fitted once the height model has been solved and the resulting coefficients incorporated in the basal area/stocking analysis program. A number of variations of the model described below are tested, and the version with the highest function value (an estimate of the maximum log-likelihood) in relation to the number of parameters involved, is usually the "best" fit for that data set.

Although it is usually desirable to constrain stocking to be non-increasing, it was found that the Canterbury model is a much better fit if this constraint is removed. This does, however, cause some predictions of stocking to increase over time. Fortunately this only occurs well outside the range of data used for constructing the model, and also outside most applications. Appendix 2 contains tables which highlight the areas of concern. In practice, the computer program contrains the stocking artificially and prints a warning message when the constraint has been applied.

The effect of site index is modelled through a change in the time scale and assumes site index has no effect on the relationships between the state variables. (Using (3), time 't' is scaled to become bt, and the variables change according to the scaled term). This assumption has held true in most other regions studied so far, (except Hawkes Bay), and was found satisfactory in this case also (see Figure 4a).

The specific form of the model then becomes:

$$\frac{dB^{c11}N^{c12}H^{c13}}{dT} = a11B^{c11}N^{c12}H^{c13} + a12B^{c21}N^{c22}H^{c23} + a13H^{c33} + b1$$

$$\frac{dN^{c22}}{dT} = a21B^{c11}N^{c12}H^{c13} + a22B^{c21}N^{c22}H^{c23} + a23H^{c33} + b2$$

$$\frac{dH^{c33}}{dT} = -H^{c33} + b3$$

where T (scaled time) = bt as previously described.

The parameter estimates were obtained by maximising the log-likelihood function, using a general purpose optimisation algorithm. (Biggs, 1971,1973; N.O.C., 1976). A number of model variations were tried, commencing with simple formulations containing few unconstrained parameters. Although the model selected (AFA) causes some implementation problems because the stocking is not constrained, its fit is superior to any other forms tried. Table 8 contains the results obtained from the optimisation algorithm for the various models studied.

Table 8

Basal area & stocking models : Function values

Model	Function	No.vars.	Description
M2	-402.0	9	Stocking non-increasing a22,a23,a24,b2,c21,c23=0
AF	-415.9	14	All parameters unconstrained
AFC	-414.6	12	AF with c21 and c23=0
AFA	-417.1	13	AF with a23=0

The parameter estimates for the model (AFA) are:

$$C = \begin{pmatrix} 0.29232 & -0.089178 & 0.56826 \\ 0.011092 & -0.72293 & -0.05909 \\ 0.00000 & 0.00000 & 0.74019 \end{pmatrix}$$

$$A = \begin{pmatrix} -0.75726 & 0.079329 & -0.12289 \\ 0.25581 & -0.11830 & 0.00000 \\ 0.00000 & 0.00000 & -1.0000 \end{pmatrix}$$

$$P = \begin{pmatrix} 1.00000 & -146.178 & -0.808157 \\ 0.000309909 & 1.00000 & -0.0000583326 \\ 0.000000 & 0.000000 & 1.00000 \end{pmatrix}$$

$$b = \begin{pmatrix} 19.722 \\ -0.00055855 \\ 19.078 \end{pmatrix} \quad a = \begin{pmatrix} 27.2903 \\ 0.0431278 \\ 19.0784 \end{pmatrix} \quad \begin{matrix} \text{lambda1} = -0.787584 \\ \text{lambda2} = -0.0879782 \\ \text{lambda3} = -1.00000 \end{matrix}$$

where $a = -A^{-1} . b$

Note: Lambdas 1,2 and 3 are the eigenvalues of A and the rows of P are the left eigenvectors such that:

$$A = P^{-1} \Lambda P$$

and lambdas 1,2 and 3 are the elements on the diagonal of ^ .
A more complete explanation is given in Garcia (1983), p.68.

Units : basal area (m2/ha) ; stocking (stems/ha) ; height (m)

The residuals for basal area and stocking from this model are shown in Figures 3a and 3b respectively. The basal area residuals are well distributed, with a mean of only 0.01 m2/ha. The residual mean square is 2.09, implying a mean deviation of only 1.4 m2/ha over the whole data set. Although the stocking residuals are particularly low below 400 stems/ha, there are several measurements above 1000 stems/ha where the model has overpredicted by approximately 15%. Overall, the mean residual of 1.25 stems/ha and mean square of 33.8 are lower than those found elsewhere.

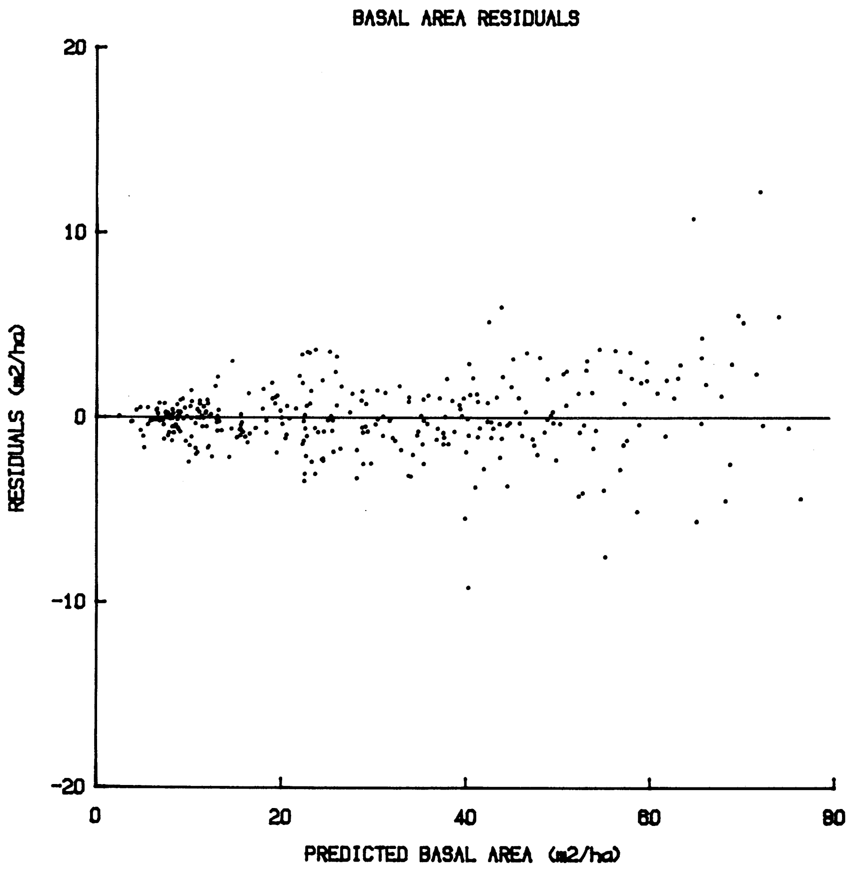


Figure 3a

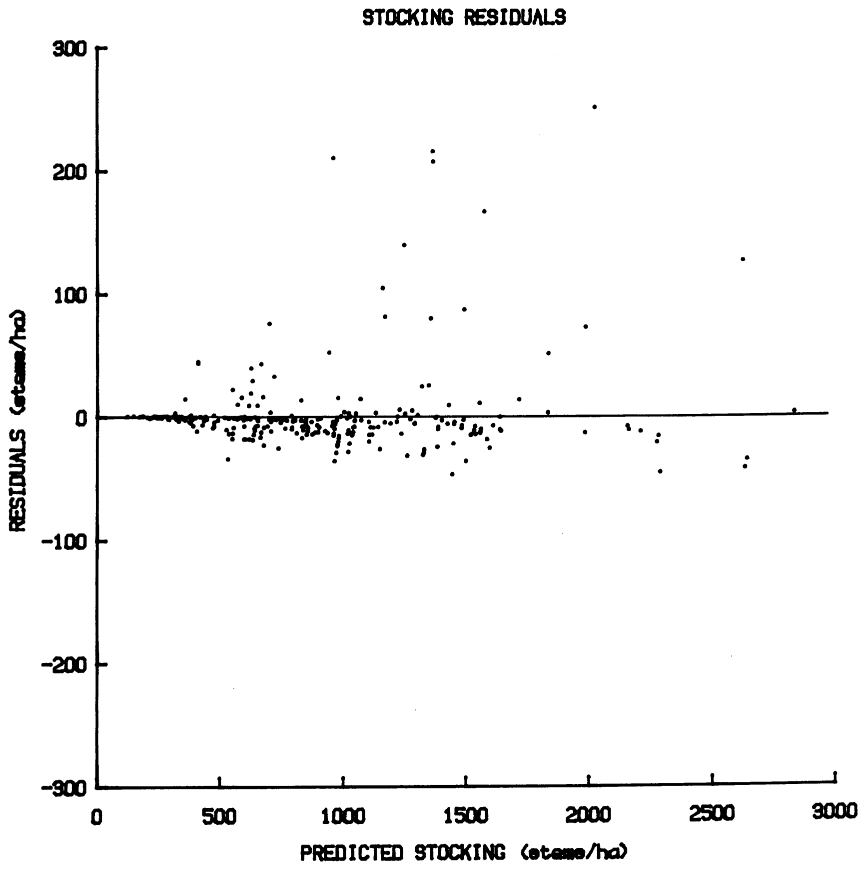


Figure 3b

3.3 Initial growth function

The data for the initial growth function is obtained by using the growth equations to grow measurements of basal area and stocking forward or back in time to a range of top heights; typically 4m to 8m. The starting points are taken from plots which have had their first measurement at age 10 or less, and where the first measurement has not been preceded by a thinning. A function to estimate basal area, given the stocking (and fixed top height), is then fitted to the data.

The function has the general form:

$$B = \begin{cases} (a/c - b)N & \text{for } N<=c \\ a \ln N - bN + a(1-\ln c) & \text{for } N>c \end{cases} \qquad (4)$$

where B = basal area (m2/ha)
N = stocking (stems/ha)

Adopting the mean square of the residuals as a measure of fit, the values of a, b and c were obtained from a non-linear least squares procedure. The results for top height 4m are:

a = 28.637
b = 0.00858
c = 2836.1

and the function to predict basal area at top height 4m is:

$$B = 0.001517*N \qquad \text{if } N <= 2836.1 \qquad (5)$$

The residuals are shown in Figure 4b. Although there is no obvious bias in the results, the wide spread, possibly a result of differences in establishment, is some cause for concern.

3.4 Thinning function

The thinning function is used to predict the basal area remaining after thinning, given the basal area and stocking before thinning and the number of stems removed. The general form of the equation is :

$$\frac{d\ln B}{d\ln N} = aB^{b} N^{c} H^{d} \qquad (6)$$

where a,b,c and d are the parameters to be estimated.

Using a non-linear, least squares procedure, a number of variations were fitted to the integrated form of (6) by setting b, ,c or d, or a combination of them equal to zero. It was found that that by setting b=0, a simpler function could be used without significantly effecting the error of prediction. The final function then becomes :

$$B = BO * \exp(a/c * H^{d} * (N^{c} - NO^{c})) \qquad (7)$$

And in (7): B = basal area after thinning (m2/ha)
BO = basal area before thinning (m2/ha)
H = top height (m)
N = stocking after thinning (stems/ha)
NO = stocking before thinning (stems/ha)

and a = 7.28047
c = -0.2305
d = -0.24443

The residuals from (7) are shown in Figure 5a. The residual mean square was 1.128 m2/ha, marginally better than a full model containing all four paramters.

3.5 Stand volume function

The usual form of the stand volume function in New Zealand is:

$$V/B = a + bH \qquad (8)$$

where V = net volume (m3/ha)
B = basal area (m2/ha)
H = top height (m)

and the stand volume/basal area ratio is the dependent variable in a simple linear regression. Beekhuis (1966) claimed that (8) could not be valid immediately before and after thinning, because the thinned trees are generally shorter than the residuals so that the V/B ratio after thinning is greater than before. Taking this into account, the function was fitted using stepwise regression with independent variables H and N, and the transformations N*H/B, H/SQR(N) and H*N (where N = stems/ha).

The resulting function is:

$$V/B = (1.1882+0.2684*H+0.3682*H/SQR(N)) \qquad (9)$$

and the residuals are shown in Figure 5b. The additional H/SQR(N) term is highly significant, t=13.44, and the S.E. for (9) is 0.019m3/ha.

****

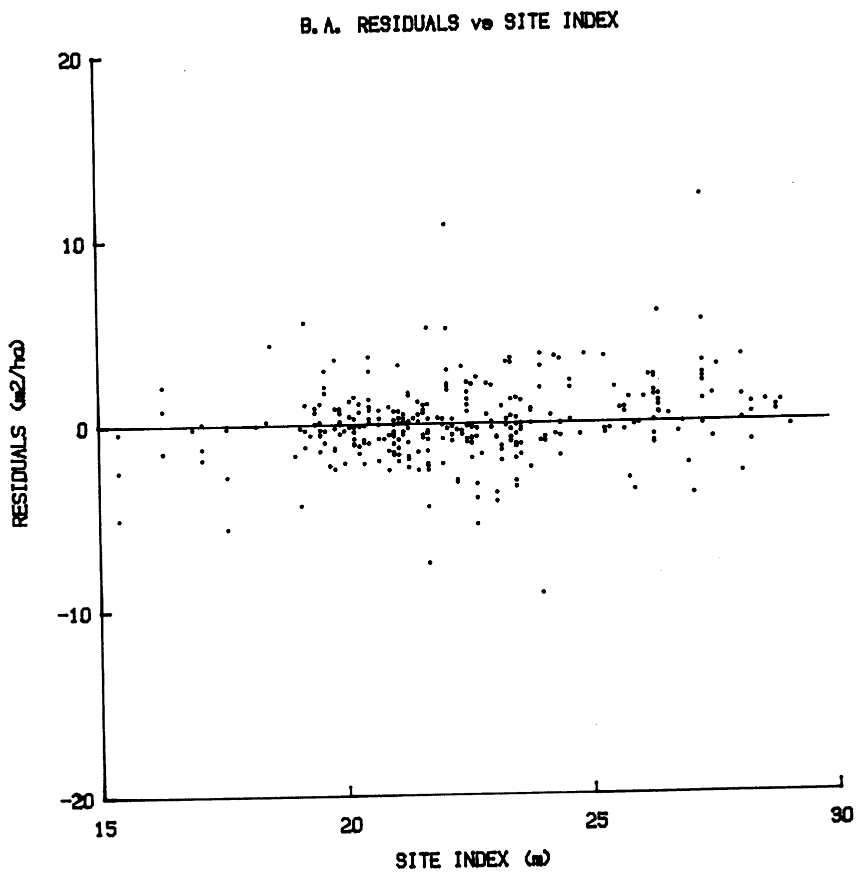


Figure 4a

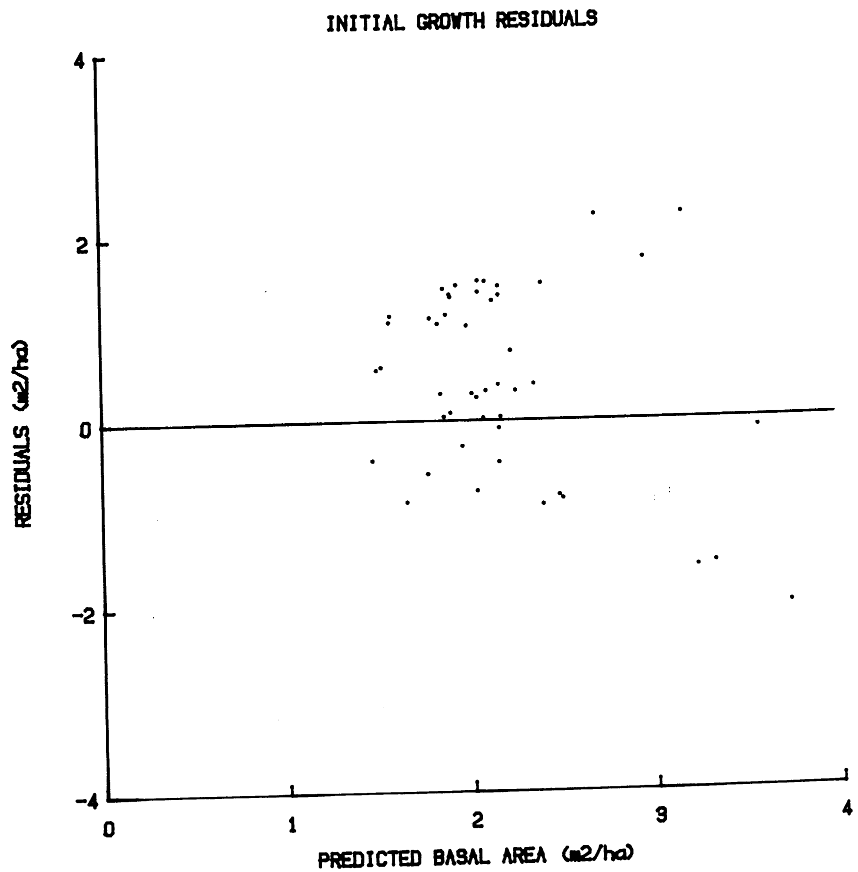


Figure 4b

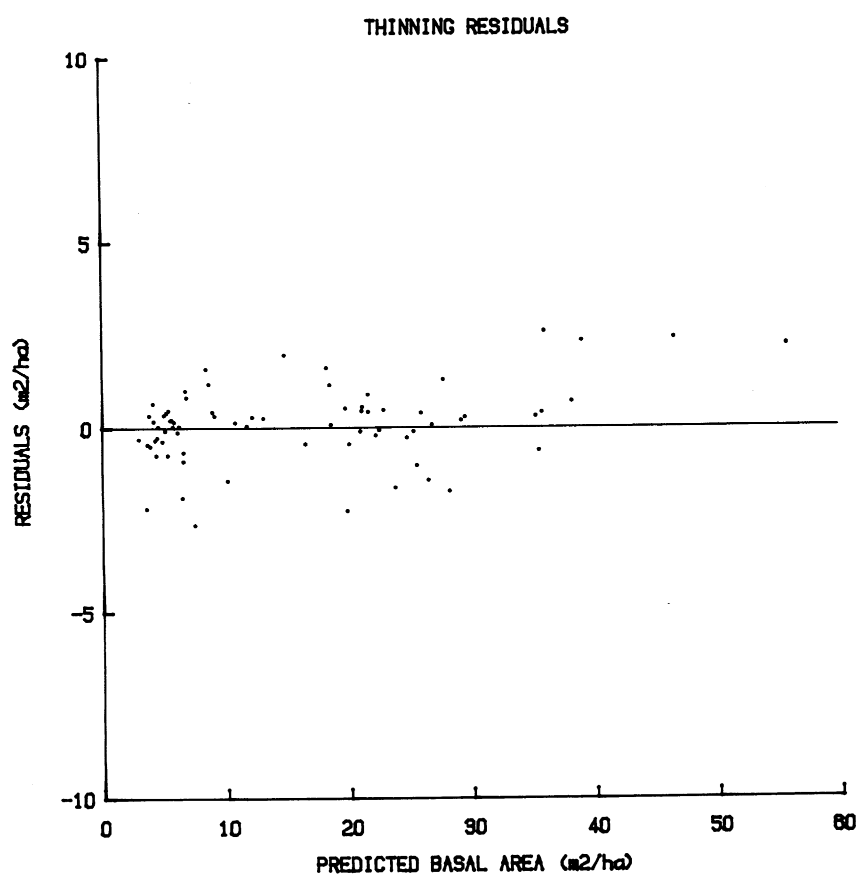


Figure 5a

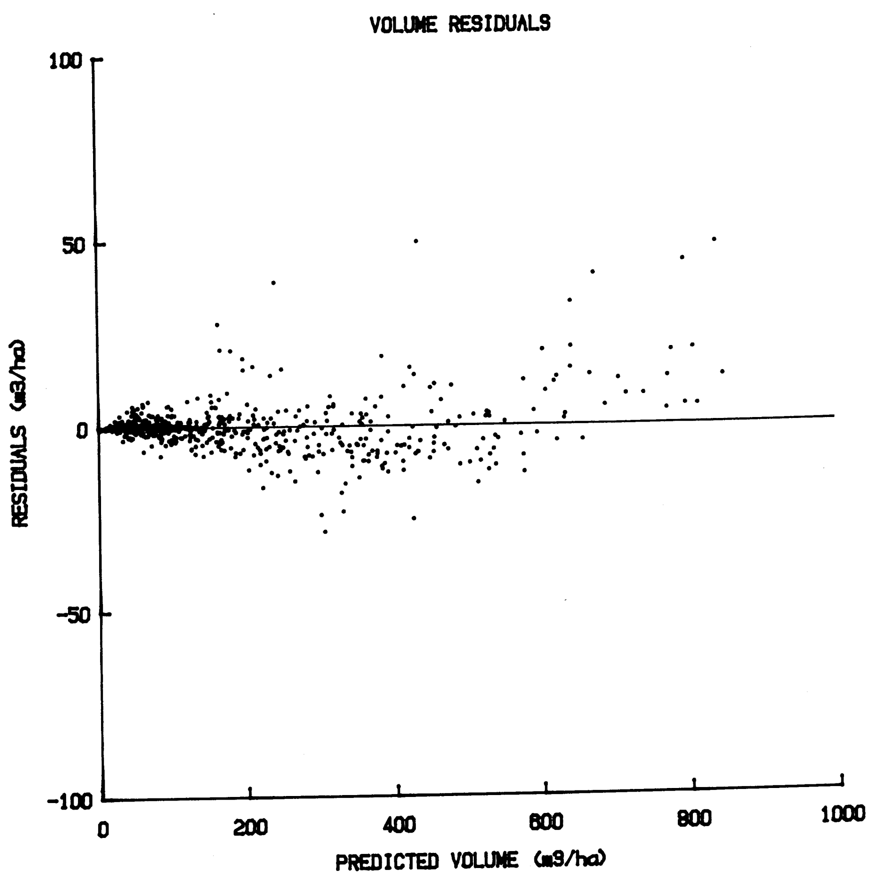


Figure 5b

4.0 EVALUATION and DISCUSSION

The mean residuals for the major variables are given in Table 9. Except for the basal area*height transformation, they are all considerably lower than the other models constructed using this methodology.

Table 9

Measurement Pair Residuals
(Mean time interval 2.23yrs)

	Mean	RMS*
Basal area (m2/ha)	0.0131	2.09
Stocking (stems/ha)	1.25	33.8
Top height (m)	0.0745	0.687
Mean DBH (cm)	-0.0305	0.725
Average spacing (m)	-0.000977	0.0512
Basal area * height (m3/ha)	5.66	62.8

* Residual Mean Square

The basal area residuals are illustrated in Figures 3a and 4a. It is interesting to note that despite a belief that the maximum carrying capacity is around 55-60 m2/ha on the plains and 60-75 m2/ha on the foothills, there are seven plots on the plains and twelve in the foothills, each with several measurements, exceeding these limits. The plots are spread over most of the forests, range in age from 18 - 35 years, and contain stockings between 543 and 5554 stems/ha. The plot numbers have been marked with an asterisk in Appendix 1, List of Plots.

The stocking residuals are particularly low, especially at stockings less than 400 stems/ha. Although the "interim" Canterbury model released in December 1987 was considered to overpredict at high stockings (1000 stems/ha +), there is no real evidence that this model, which behaves similarly, is incorrect.

The model was found to fit the data best if stocking was not constrained to be non-increasing. This is especially true at lower stockings and explains the good fit mentioned previously. For most purposes, this has no effect on the predictions of stocking, however the program has been constrained artificially so that there is no increase during use. A warning message is printed on the screen if the increase is greater than one stem/ha in any one year. Appendix 2 contains tables which illustrate where the increase will occur.

The functions for estimating basal area at top height 4m (Figure 4b) and basal area after thinning (Figure 5a) show no obvious bias, although there is a fairly wide scatter of points for the initial growth function. For this reason, it is recommended that regimes be run from known starting points in preference to commencing from age zero.

The total stand volume equation residuals are shown in Figure 5b). Below 600m3/ha , the function is a good fit. Above this point there is a tendency for the function to over-predict, however it should be noted that the over-predictions are of the order of 3-5%.

Table 10 compares yield predictions from the new model with those obtained from KGM2 using Leitch's height-age curves for the Canterbury foothills and plains. The models have been run for three different management regimes, containing zero, one or two thinnings, at an average site index of 22m. The regimes are:

Regime 1: Plant 1250 and leave

Regime 2: Plant 830, Thin to 250 @ 6m

Regime 3: Plant 830, Thin to 550 @ 6m, Thin to 275 @ 11m

Table 10

Comparison between KGM2 plains and foothills, and CANT
(All values at age 30, for site index 22m)

	KGM2 plains	KGM2 hills	CANT		
	Volume	Volume	Volume	%Diffp	%Diffh
R1	770	704	722	-6.2	2.6
R2	536	485	504	-6.0	3.9
R3	531	476	528	-0.6	10.9

where Diffp, Diffh = (CANT-KGM)/KGM*100

Table 10 compares regimes based on the current model (KGM2) and the new model. It can be seen that the new model falls between the two predictions for the foothills and plains. This should not be construed as a "convenient average", but considered a more accurate reflection of the actual growth on both sites. These differences are discussed further in Section 4.1, Foothills versus Plains.

The new model for Canterbury fits the data well and performs reliably. Although there is a difference between the new height model and the earlier curves formulated by Leitch, the new model model should be more accurate and reliable over the range of sites and ages commonly found in the region. As for similar models elsewhere, the prediction of growth from age zero may introduce some error, and it is therefore suggested that plot or crop type means be used as starting points whenever possible.

4.1 FOOTHILLS versus PLAINS

There has been a long held belief that there are differences in growth between the Canterbury foothills and plains. The procedure adopted to test for differences was first to construct the "best" growth model with all the data from the foothills and the plains combined. The database was then split into the two groups and the same model structure used in the Combined model re-fitted to the two separate sets of data. This entailed re-fitting the height model as well as the basal area and stocking components.

Several methods were used to compare the resulting three models (Hills, Plains and Combined). For the two major components, the height model and the basal area and stocking model, the log-likelihood estimates from the "fitting" procedure can be compared directly by adding the log-likelihoods from the Hills and Plains and subtracting the value for the Combined. The result can then be compared with the Chi-squared value with the appropriate degrees of freedom. This indicated that for both the height component and the basal area and stocking component, there was a statistically significant difference between the two sets of data.

To obtain some idea of the magnitude of the differences, the height models for the Hills and Plains were re-run to obtain the "best" model for each. In theory, this should be at least as good, and probably better, than the Combined model formulation. The resulting height models were then compared with the Combined by graphically overlaying a series of site index curves from each.

For the Plains, there was almost no divergence between the sets of curves - less than 0.5m maximum at age 40 years. For the Hills, there were some differences below age 6. Unfortunately, the model behaved poorly between age 0 and age 6, probably as a result of the lack of data for this model (only 37 plots). There would appear to be little practical reason to adopt separate height-age models in preference to the Combined approach.

A third comparison was made by contructing the Hills and Plains models, producing yield tables based on three regimes over four site indices, and comparing the estimates of basal area, stocking and volume at age 30 years. The results of this comparison are presented in Table 11.

The results indicate that although the models differ statistically, there is little practical difference between them. It is important to note that the differences are greatest on sites outside the range for that particular set of data. Hence, the biggest difference between the Combined and the Hills is at low site indices (15-20m) which are uncommon in the foothills. Similarly, the largest difference between the Combined and the Plains is at high site indices (25-30m) which are rare on the plains.

Table 11

Comparison of Hills, Plains & Combined Models
(Percent difference from Combined at age 30)

Site index 15m:

	Basal area			Stocking			Volume		
	%H	%P	Comb	%H	%P	Comb	%H	%P	Comb
Regime 1	-5	-2	52.2	-3	2	1126	-8	-2	394
Regime 2	-11	0	30.7	0	0	273	-14	1	240
Regime 3	-12	-2	33.5	0	0	249	-15	-1	262

Site index 20m:

	Basal area			Stocking			Volume		
	%H	%P	Comb	%H	%P	Comb	%H	%P	Comb
Regime 1	0	-4	67.1	-5	4	1033	-1	-4	625
Regime 2	-4	-4	43.9	-1	1	270	-5	-3	423
Regime 3	-5	-6	46.7	1	1	245	-6	-6	452

Site index 25m:

	Basal area			Stocking			Volume		
	%H	%P	Comb	%H	%P	Comb	%H	%P	Comb
Regime 1	4	-5	79.3	-6	7	921	4	-5	866
Regime 2	1	-7	56.2	-1	3	264	1	-7	634
Regime 3	0	-9	58.4	1	3	240	0	-9	662

Site index 30m:

	Basal area			Stocking			Volume		
	%H	%P	Comb	%H	%P	Comb	%H	%P	Comb
Regime 1	7	-6	89.1	-7	12	798	8	-6	1102
Regime 2	4	-10	68.0	-1	6	254	5	-10	868
Regime 3	4	-12	69.6	2	5	232	5	-12	891

The analyses partly confirm the suspicions of those who have worked in the region, and are best summarised as follows:

- there is some evidence that there are differences in growth pattern between the Foothills and the Plains other than those indicated by site quality, but they are not likely to be great
- for all practical purposes, the Combined model is a much simpler approach that will provide good estimates on both the Foothills and the Plains.
- any error in the estimate for the Combined model will increase as the model is applied outside the range of sites normally found in that sub-region.
- there is insufficient data (112 measurement pairs for the Foothills and 215 for the Plains) to construct separate models with as much confidence as that gained from using the Combined approach.

****

5.0 ACKNOWLEDGEMENTS

The author would like to acknowledge the contributions of a number of people to this project:

Mrs Judy Klitscher and Mrs Judy Dunlop for ensuring that all relevant plot information was available on the Permanent Sample Plot system and for providing summaries as required.

Mr Shaf van Ballekom and staff at Timberlands Southern Region and North Canterbury District offices.

Mr Phil Wilcox for providing additional data to "fill in gaps" in the database.

Drs Oscar Garcia and Chris Goulding, for their critical evaluations and advice.

****

6.0 REFERENCES & OTHER RELEVANT PUBLICATIONS

Bailey, R.L. 1972: Development of unthinned stands of Pinus radiata in New Zealand. Unpublished Ph.D. Dissertation, University of Georgia, Athens, USA.

Beekhuis, J. 1966: Prediction of yield and increment in Pinus radiata stands in New Zealand. New Zealand Forest Service Technical Paper No.49.

Bell, A. C. 1981: Using the Kaingaroa growth model for heavy early thinning regimes. New Zealand Forest Service, Resource Inventory Group, Technical Note No.13. (Unpublished).

Biggs, M.C. 1971: Minimization algorithms making use of non-quadratic properties of the objective function. Journal of the Institute of Mathematics and its Applications 8: 315 - 327.

------- 1973: A note on minimization algorithms which make use of non-quadratic properties of the objective function. Journal of the Institute of Mathematics and its Applications 12: 337 - 338.

Burkhart, H.E.; Tennent, R.B. 1977: Site index equations for radiata pine in New Zealand. New Zealand Journal of Forestry Science 7(3): 408-416.

Clutter, J.L.; Allison, B.J. 1974: A growth and yield model for Pinus radiata in New Zealand. In Fries, J. (Ed.) Growth models for tree and stand simulation. Royal College of Forestry, Stockholm, Sweden.

Dunningham, A.G.; Lawrence, M.E. 1987: An "interim" stand growth model for radiata pine grown on the Central North Island pumice plateau. Ministry of Forestry, Forest Research Institute, Project Record No.1497. (Unpublished).

Elliott, D.A.; Goulding, C.J. 1976: The Kaingaroa growth model for radiata pine and its implications for maximum volume production (abstract). New Zealand Journal of Forestry Science 6(2): 187.

Garcia, O. 1979: Modelling stand development with stochastic differential equations. Pp. 315-317 in Elliott, D.A. (Ed.) "Mensuration for management planning of exotic forest plantations". New Zealand Forest Service, Forest Research Institute Symposium No.20.

------- 1983: A stochastic differential equation model for the height growth of forest stands. Biometrics 39: 1059-1072.

------- 1984: New class of growth models for even-aged stands - Pinus radiata in Golden Downs Forest. New Zealand Journal of Forestry Science 14(1): 65-88.

------- 1987: Mortality in the Golden Downs model. Ministry of Forestry, Forest Research Institute, Project Record No.1457. (Unpublished).

Goulding, C.J. 1979: Validation of growth models used in forest management. New Zealand Journal of Forestry 24(1): 108-124.

Hayward, W.; Jenkin, I.; New, D. 1987: Evaluation of the performance of growth models for Kaingaroa. Ministry of Forestry, Forest Research Institute, Project Record No.1732. (Unpublished).

James, R.N. 1983: The prediction of growth and wood quality parameters for plantation grown Pinus radiata D.Don in New Zealand. Unpublished Ph.D. Thesis, University of Oxford, Oxford, England.

Klitscher, J.A.K. 1987: Mortality trends in Pinus radiata in the Rotorua region. New Zealand Forestry 31(4): 23-25.

Lewis, E.R. 1954: Yield of unthinned Pinus radiata in New Zealand. New Zealand Forest Service, Forest Research Institute, Forest Research Notes 1(10).

Manley, B.R. 1981: A distance-independent tree growth model for radiata pine in New Zealand. Unpublished Ph.D. Thesis, University of Washington, Seattle, USA.

Manley, B.R. 1986: Performance of the Kaingaroa growth model for low-stocking regimes. New Zealand Forest Service, Forest Research Institute Bulletin No.113.

Manley, B.R.; Knowles,R.L. 1980: Growth of radiata pine under the direct sawlog regime. New Zealand Journal of Forestry Science 25(1): 15-34.

McEwen, A.D. 1979: N.Z. Forest Service computer system for permanent sample plots. Pp. 235-252 in Elliott, D.A. (Ed.) "Mensuration for management planning of exotic forest plantations". New Zealand Forest Service, Forest Research Institute Symposium No.20.

N.O.C. 1976: "OPTIMA" - Routines for Optimisation Problems". Numerical Optimisation Centre, The Hatfield Polytechnic.

Shula, R.B. 1987: Auckland Clays radiata pine growth and yield model: development, history and performance. Ministry of Forestry Forest Research Institute, Project Record No.1384. (Unpublished).

------- 1987: CLAYSFERT - Auckland Clays growth model with Phosphorus fertiliser effects. Development, history and performance. Ministry of Forestry, Forest Research Institute Project Record No.1688. (Unpublished).

Tennent, R.B. 1981: A simulation model for Pinus radiata in New Zealand. Unpublished Ph.D. Thesis, University of Georgia, Athens, USA.

West, G.G.; Knowles, R.L.; Koehler, A.R. 1982: Model to predict the effects of pruning and early thinning on the growth of radiata pine. New Zealand Forest Service, Forest Research Institute Bulletin No.5.

Wilcox, P.L. 1987: The retrospective sampling technique. Ministry of Forestry, Forest Research Institute, Project Record No. 1583. (Unpublished).

7.0 APPENDICES

7.1 APPENDIX 1 : LIST OF PLOTS

V = Volume function
H = Height model
I = Initial growth function
G = Basal area & stocking equations
T = Thinning function
* = High basal area plots

Eyrewell

	CY(103. 00. 01. 0)	V	G	H	T	
	CY(103. 00. 02. 0)	V	G	H	T	
	CY(103. 00. 03. 0)	V	G	H	T	
*	CY(103. 00. 04. 0)	V		H		
	CY(103. 00. 05. 0)	V	G	H	T	
	CY(103. 00. 06. 0)	V	G	H	T	
	CY(103. 00. 07. 0)	V	G	H	T	
*	CY(103. 00. 08. 0)	V	G			
	CY(103. 00. 10. 0)	V	G	H		
	CY(103. 00. 13. 0)	V	G	H	T	
	CY(103. 00. 14. 0)	V	G	H	T	
	CY(103. 00. 15. 0)	V	G	H		
	CY(103. 00. 16. 0)	V	G			
	CY(103. 00. 20. 0)	V	G			
	CY(103. 00. 21. 0)	V	G	H		
	CY(103. 00. 22. 0)	V	G	H		
	CY(103. 00. 23. 0)	V	G			
	CY(103. 00. 24. 0)	V	G	H		
	CY(385. 01. 01. 1)	V	G			
	CY(385. 01. 01. 2)	V	G	H		
	CY(385. 02. 01. 1)	V	G			
	CY(385. 02. 01. 2)	V	G	H		
	CY(399. 01. 01. 1)	V				
	CY(399. 01. 01. 2)	V				
	CY(489. 22. 01. 0)	V	G	H	T	
	CY(489. 22. 02. 0)	V		H		
	CY(489. 35. 05. 0)	V	G	H	T	
	CY(489. 35. 07. 0)	V	G	H	T	
	CY(489. 37. 06. 0)	V	G	H	T	
	CY(489. 39. 01. 0)	V	G	H	T	
	CY(489. 40. 03. 0)	V	G	H	T	
	CY(489. 42. 05. 1)	V	G	H		I
	CY(489. 43. 06. 0)	V	G	H	T	
	CY(489. 45. 11. 0)	V	G		T	I
	CY(489. 45. 12. 0)	V	G	H	T	I
	CY(489. 46. 02. 0)	V	G	H	T	I
	CY(489. 48. 03. 0)	V	G	H		I
	CY(489. 49. 05. 0)	V	G	H		I
	CY(489. 49. 06. 0)	V				I
	CY(489. 49. 07. 0)	V	G	H		I
	CY(489. 49. 09. 0)	V	G	H		
	CY(489. 49. 10. 0)	V	G	H		I
	CY(508. 00. 02. 0)	V	G	H		I
	CY(508. 00. 06. 0)	V	G	H		I
	CY(508. 00. 09. 0)	V	G	H		I
	CY(560. 05. 01. 0)	V	G	H		I
	CY(560. 05. 02. 0)	V	G	H	T	I
	CY(560. 05. 03. 0)	V	G		T	
	CY(560. 05. 04. 0)	V	G			
	CY(560. 05. 05. 0)	V	G	H		I
	CY(560. 05. 06. 0)	V	G	H	T	I

Balmoral

CY(283. 00. 01. 0)	V	G			
CY(283. 00. 02. 0)	V	G			
CY(283. 00. 03. 0)	V	G			
CY(283. 00. 04. 0)	V	G			
CY(283. 00. 07. 0)	V	G			
CY(283. 00. 08. 0)	V	G			
CY(283. 00. 18. 0)	V	G			
CY(337. 00. 01. 0)	V	G	H		
CY(337. 00. 02. 0)	V	G	H		
CY(337. 00. 03. 0)	V	G	H		
CY(337. 00. 04. 0)	V	G	H		
CY(337. 00. 05. 0)	V	G			
CY(337. 00. 06. 0)	V	G			
CY(337. 00. 07. 0)	V	G	H		
CY(337. 00. 08. 0)	V	G	H		
CY(337. 00. 09. 0)	V	G	H		
CY(395. 00. 02. 0)	V	G		T	
CY(395. 00. 10. 0)	V				
CY(459. 00. 07. 0)	V	G	H	T	I
CY(459. 00. 08. 0)	V	G	H	T	I
CY(459. 00. 09. 0)	V	G	H	T	I
CY(459. 00. 10. 0)	V	G	H	T	I
CY(459. 00. 11. 0)	V	G	H	T	I
CY(459. 00. 12. 0)	V	G	H	T	I
CY(459. 00. 13. 0)	V	G	H	T	I
CY(459. 00. 14. 0)	V	G	H	T	I
CY(459. 00. 15. 0)	V	G	H		I
CY(459. 00. 16. 0)	V	G	H		I
CY(459. 00. 17. 0)	V	G	H	T	I
CY(459. 00. 18. 0)	V	G	H	T	I
CY(459. 00. 19. 0)	V	G	H	T	I
CY(459. 00. 20. 0)	V	G	H	T	I
CY(459. 00. 21. 0)	V	G	H		I
CY(459. 00. 22. 0)	V	G	H		I
CY(459. 00. 23. 0)	V	G	H	T	I
CY(459. 00. 24. 0)	V	G	H	T	I
CY(459. 00. 25. 0)	V	G	H	T	I

Burnham

CY(115. 00. 01. 0)	V	G	H	T	
CY(115. 00. 01. 1)	V	G			
CY(115. 00. 02. 0)	V	G	H	T	
* CY(115. 00. 03. 0)	V	G	H		
CY(116. 00. 01. 0)	V	G		T	
CY(116. 00. 01. 1)	V	G			
CY(116. 00. 02. 0)	V	G		T	
CY(116. 00. 02. 1)	V	G			
CY(116. 00. 03. 0)	V	G		T	
* CY(116. 00. 04. 0)	V	G	H		
CY(451. 00. 14. 0)	V	G	H	T	I
CY(451. 00. 15. 0)	V	G	H	T	I

Bottle Lake

*	CY(101. 00. 01. 0)	V	G	H		
	CY(101. 00. 02. 0)	V	G	H	T	
	CY(101. 00. 03. 0)	V	G	H	T	

Ashley

*	CY(100. 00. 01. 0)	V	G	H		
	CY(100. 00. 02. 0)	V	G	H	T	
	CY(100. 00. 03. 0)	V	G	H	T	
	CY(104. 04. 01. 0)	V	G			
	CY(104. 04. 02. 0)	V				
	CY(104. 04. 03. 0)	V	G			
	CY(104. 04. 04. 0)	V				
	CY(159. 00. 02. 0)	V	G		T	I
	CY(159. 00. 03. 0)	V				
	CY(383. 00. 09. 0)	V	G			
	CY(383. 00. 12. 0)	V	G			
	CY(383. 00. 13. 0)	V				
	CY(383. 00. 15. 0)	V				
	CY(383. 00. 17. 0)	V				
	CY(383. 00. 18. 0)	V				
	CY(423. 00. 02. 0)	V				
	CY(423. 00. 14. 0)	V				
	CY(449. 00. 01. 0)	V	G			
	CY(471. 00. 02. 0)	V	G		T	
	CY(471. 00. 05. 0)	V	G			
	CY(471. 00. 07. 0)	V	G		T	
	CY(471. 00. 08. 0)	V	G		T	
	CY(471. 00. 09. 0)	V	G		T	
	CY(471. 00. 10. 0)	V	G		T	
	CY(477. 00. 07. 0)	V	G			
	CY(477. 00. 10. 0)	V	G			
	CY(477. 00. 12. 0)	V	G			
	CY(478. 00. 01. 0)	V	G	H	T	
	CY(478. 00. 05. 0)	V	G	H		
	CY(494. 00. 02. 0)	V	G	H	T	
	CY(494. 00. 03. 0)	V	G	H	T	
	CY(494. 00. 05. 0)	V	G	H	T	
	CY(494. 00. 08. 0)	V	G	H		
	CY(494. 01. 02. 0)	V	G	H	T	
	CY(560. 01. 01. 0)	V	G	H		
	CY(560. 01. 02. 0)	V	G	H		
	CY(560. 01. 03. 0)	V	G			I
	CY(560. 01. 04. 0)	V	G	H		
	CY(560. 01. 05. 0)	V	G	H		
	CY(581. 00. 02. 0)	V				I
	CY(581. 00. 08. 0)	V				I

Hanmer

*	CY(87. 00. 03. 0)	V				
	CY(87. 00. 03. 1)	V	G			
	CY(560. 02. 01. 0)	V	G	H		
*	CY(560. 02. 02. 0)	V	G	H		
	CY(560. 02. 03. 0)	V		H		
	CY(560. 02. 04. 0)	V	G			
	CY(560. 02. 05. 0)	V	G			

Geraldine

*	CY(425. 00. 11. 0)	V	G	H		
	CY(425. 00. 13. 0)	V	G	H		I
*	CY(425. 00. 14. 0)	V	G			
*	CY(425. 00. 16. 0)	V	G	H		I
*	CY(432. 00. 01. 0)	V	G	H		
	CY(432. 00. 02. 0)	V	G	H		
	CY(432. 00. 03. 0)	V	G	H		
	CY(432. 00. 04. 0)	V	G	H		
	CY(432. 00. 05. 0)	V	G	H		
	CY(432. 00. 06. 0)	V		H		
	CY(432. 00. 07. 0)	V	G			
	CY(432. 00. 07. 1)	V	G			
	CY(432. 00. 08. 0)	V	G		T	I
	CY(560. 03. 03. 0)	V	G	H	T	I
	CY(560. 03. 04. 0)	V	G			
	CY(560. 03. 05. 0)	V	G			I

Omihi

*	CY(189. 00. 01. 0)	V	G	H	
*	CY(189. 00. 03. 0)	V	G	H	
	CY(189. 00. 04. 0)	V	G	H	
	CY(189. 00. 05. 0)	V	G	H	
	CY(189. 00. 06. 0)	V		H	
*	CY(189. 00. 07. 0)	V	G	H	
	CY(447. 00. 02. 0)	V	G		T
	CY(447. 00. 06. 0)	V	G		
	CY(447. 00. 08. 0)	V	G		
	CY(447. 00. 15. 0)	V	G		T
	CY(447. 00. 17. 0)	V	G		
	CY(447. 00. 21. 0)	V	G		
	CY(447. 00. 24. 0)	V	G		T
	CY(447. 00. 25. 0)	V	G		T
	CY(447. 00. 31. 0)	V	G		
	CY(447. 00. 34. 0)	V			
	CY(479. 00. 01. 0)	V	G		

Waimate

CY(472. 00. 01. 0)	V	G		T	I
CY(472. 00. 02. 0)	V	G		T	I
CY(472. 00. 02. 1)	V	G			
CY(560. 04. 01. 0)	V	G	H		
CY(560. 04. 02. 0)	V	G	H		
CY(560. 04. 03. 0)	V	G			I

Sectional measurement data

	CY(598. 01. 02. 0)	G	H
	CY(598. 01. 03. 0)	G	H
*	CY(598. 01. 04. 0)	G	H
	CY(598. 02. 02. 0)	G	H
	CY(598. 03. 01. 0)	G	H
	CY(598. 03. 02. 0)	G	H
	CY(598. 03. 03. 0)	G	H
*	CY(598. 04. 02. 0)	G	H
*	CY(598. 04. 03. 0)	G	H
	CY(598. 05. 01. 0)	G	H
*	CY(598. 05. 03. 0)	G	H

7.2 APPENDIX 2 : OPERATING RANGE OF THE MODEL

The tables in this Appendix indicate those regions of the model where predictions of stocking increase over time. For a range of top heights, basal areas and stockings, a -1, 1 or blank is used to indicate the behaviour of the model as follows:

-1	predictions of stocking are increasing.
1	predictions of stocking are increasing, but are less than 1 stem/ha/year and therefore not apparent to the user.
blank	predictions of stocking behave "normally".

HEIGHT = 4

	BASAL AREA																
SPH	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1														
20	1	1															
30	1																
40	1																
50	1																
60	1																
70	1																
80	1																
90	1																
100	1																
110	1																
120	1																
130	1																
140	1																
150	1																
160	1																
170	1																
180	-1																
190	-1																
200	-1																
210	-1																
220	-1																
230	-1																
240	-1																
250	-1																
260	-1																
270	-1																
280	-1																
290	-1																
300	-1																
310	-1																
320	-1																
330	-1																
340	-1																
350	-1																
360	-1																
370	-1																
380	-1																
390	-1																
400	-1																
410	-1																
420	-1																
430	-1																
440	-1																
450	-1																
460	-1																
470	-1																
480	-1																
490	-1																
500	-1																

HEIGHT = 6

SPH	BASAL AREA 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1											
20	1	1	1														
30	1	1	1														
40	1	1															
50	1	1															
60	1	1															
70	1	1															
80	1	1															
90	-1	1															
100	-1	1															
110	-1	1															
120	-1	1															
130	-1	1															
140	-1	1															
150	-1	1															
160	-1	1															
170	-1	1															
180	-1	1															
190	-1	1															
200	-1	1															
210	-1	1															
220	-1	1															
230	-1	1															
240	-1	1															
250	-1	1															
260	-1	1															
270	-1	-1															
280	-1	-1															
290	-1	-1															
300	-1	-1															
310	-1	-1															
320	-1	-1															
330	-1	-1															
340	-1	-1															
350	-1	-1															
360	-1	-1															
370	-1	-1															
380	-1	-1															
390	-1	-1															
400	-1	-1															
410	-1	-1															
420	-1	-1															
430	-1	-1															
440	-1	-1	1														
450	-1	-1	1														
460	-1	-1	1														
470	-1	-1	1														
480	-1	-1	1														
490	-1	-1	1														
500	-1	-1	1														

HEIGHT = 8

SPH	BASAL AREA 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1						
20	1	1	1	1	1	1											
30	1	1	1	1	1												
40	1	1	1	1	1												
50	1	1	1	1													
60	1	1	1	1													
70	-1	1	1	1													
80	-1	1	1	1													
90	-1	1	1	1													
100	-1	1	1	1													
110	-1	1	1	1													
120	-1	1	1	1													
130	-1	1	1	1													
140	-1	-1	1	1													
150	-1	-1	1	1													
160	-1	-1	1	1													
170	-1	-1	1	1													
180	-1	-1	1	1													
190	-1	-1	1	1													
200	-1	-1	1	1													
210	-1	-1	1	1													
220	-1	-1	1	1													
230	-1	-1	1	1													
240	-1	-1	1	1													
250	-1	-1	1	1													
260	-1	-1	-1	1													
270	-1	-1	-1	1													
280	-1	-1	-1	1													
290	-1	-1	-1	1													
300	-1	-1	-1	1													
310	-1	-1	-1	1													
320	-1	-1	-1	1													
330	-1	-1	-1	1													
340	-1	-1	-1	1													
350	-1	-1	-1	1													
360	-1	-1	-1	1													
370	-1	-1	-1	1													
380	-1	-1	-1	1													
390	-1	-1	-1	1													
400	-1	-1	-1	1													
410	-1	-1	-1	1													
420	-1	-1	-1	1													
430	-1	-1	-1	1													
440	-1	-1	-1	1													
450	-1	-1	-1	1													
460	-1	-1	-1	1													
470	-1	-1	-1	1													
480	-1	-1	-1	1													
490	-1	-1	-1	-1													
500	-1	-1	-1	-1													

HEIGHT = 10

BASAL AREA

SPH	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1					
30	1	1	1	1	1	1	1	1	1								
40	1	1	1	1	1	1	1	1									
50	1	1	1	1	1	1	1	1									
60	-1	1	1	1	1	1	1										
70	-1	1	1	1	1	1	1										
80	-1	1	1	1	1	1	1										
90	-1	1	1	1	1	1	1										
100	-1	-1	1	1	1	1	1										
110	-1	-1	1	1	1	1	1										
120	-1	-1	1	1	1	1	1										
130	-1	-1	1	1	1	1											
140	-1	-1	1	1	1	1											
150	-1	-1	1	1	1	1											
160	-1	-1	-1	1	1	1											
170	-1	-1	-1	1	1	1											
180	-1	-1	-1	1	1	1											
190	-1	-1	-1	1	1	1											
200	-1	-1	-1	1	1	1											
210	-1	-1	-1	1	1	1											
220	-1	-1	-1	1	1	1											
230	-1	-1	-1	1	1	1											
240	-1	-1	-1	-1	1	1											
250	-1	-1	-1	-1	1	1											
260	-1	-1	-1	-1	1	1											
270	-1	-1	-1	-1	1	1											
280	-1	-1	-1	-1	1	1											
290	-1	-1	-1	-1	1	1											
300	-1	-1	-1	-1	1	1											
310	-1	-1	-1	-1	1	1											
320	-1	-1	-1	-1	1	1											
330	-1	-1	-1	-1	1	1											
340	-1	-1	-1	-1	1	1											
350	-1	-1	-1	-1	1	1											
360	-1	-1	-1	-1	1	1											
370	-1	-1	-1	-1	1	1											
380	-1	-1	-1	-1	-1	1											
390	-1	-1	-1	-1	-1	1											
400	-1	-1	-1	-1	-1	1											
410	-1	-1	-1	-1	-1	1											
420	-1	-1	-1	-1	-1	1											
430	-1	-1	-1	-1	-1	1											
440	-1	-1	-1	-1	-1	1											
450	-1	-1	-1	-1	-1	1											
460	-1	-1	-1	-1	-1	1											
470	-1	-1	-1	-1	-1	1											
480	-1	-1	-1	-1	-1	1											
490	-1	-1	-1	-1	-1	1											
500	-1	-1	-1	-1	-1	1											

HEIGHT = 12

SPH	BASAL AREA 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
50	1	1	1	1	1	1	1	1	1	1	1	1	1				
60	-1	1	1	1	1	1	1	1	1	1	1	1					
70	-1	1	1	1	1	1	1	1	1	1	1	1					
80	-1	1	1	1	1	1	1	1	1	1	1						
90	-1	-1	1	1	1	1	1	1	1	1	1						
100	-1	-1	1	1	1	1	1	1	1	1	1						
110	-1	-1	1	1	1	1	1	1	1	1							
120	-1	-1	-1	1	1	1	1	1	1	1							
130	-1	-1	-1	1	1	1	1	1	1	1							
140	-1	-1	-1	1	1	1	1	1	1	1							
150	-1	-1	-1	1	1	1	1	1	1	1							
160	-1	-1	-1	-1	1	1	1	1	1	1							
170	-1	-1	-1	-1	1	1	1	1	1								
180	-1	-1	-1	-1	1	1	1	1	1								
190	-1	-1	-1	-1	1	1	1	1	1								
200	-1	-1	-1	-1	1	1	1	1	1								
210	-1	-1	-1	-1	1	1	1	1	1								
220	-1	-1	-1	-1	-1	1	1	1	1								
230	-1	-1	-1	-1	-1	1	1	1	1								
240	-1	-1	-1	-1	-1	1	1	1	1								
250	-1	-1	-1	-1	-1	1	1	1	1								
260	-1	-1	-1	-1	-1	1	1	1	1								
270	-1	-1	-1	-1	-1	1	1	1	1								
280	-1	-1	-1	-1	-1	1	1	1	1								
290	-1	-1	-1	-1	-1	1	1	1	1								
300	-1	-1	-1	-1	-1	1	1	1	1								
310	-1	-1	-1	-1	-1	1	1	1	1								
320	-1	-1	-1	-1	-1	-1	1	1	1								
330	-1	-1	-1	-1	-1	-1	1	1	1								
340	-1	-1	-1	-1	-1	-1	1	1	1								
350	-1	-1	-1	-1	-1	-1	1	1									
360	-1	-1	-1	-1	-1	-1	1	1									
370	-1	-1	-1	-1	-1	-1	1	1									
380	-1	-1	-1	-1	-1	-1	1	1									
390	-1	-1	-1	-1	-1	-1	1	1									
400	-1	-1	-1	-1	-1	-1	1	1									
410	-1	-1	-1	-1	-1	-1	1	1									
420	-1	-1	-1	-1	-1	-1	1	1									
430	-1	-1	-1	-1	-1	-1	1	1									
440	-1	-1	-1	-1	-1	-1	1	1									
450	-1	-1	-1	-1	-1	-1	1	1									
460	-1	-1	-1	-1	-1	-1	1	1									
470	-1	-1	-1	-1	-1	-1	1	1									
480	-1	-1	-1	-1	-1	-1	1	1									
490	-1	-1	-1	-1	-1	-1	1	1									
500	-1	-1	-1	-1	-1	-1	1	1									

HEIGHT = 14

								BASAL AREA									
SPH	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
70	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
80	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
90	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
100	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	
110	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1		
120	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1		
130	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1			
140	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1			
150	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1			
160	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1				
170	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1				
180	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1				
190	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1				
200	-1	-1	-1	-1	-1	1	1	1	1	1	1	1					
210	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1					
220	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1					
230	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1					
240	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1					
250	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1					
260	-1	-1	-1	-1	-1	-1	1	1	1	1	1						
270	-1	-1	-1	-1	-1	-1	1	1	1	1	1						
280	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
290	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
300	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
310	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
320	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
330	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
340	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
350	-1	-1	-1	-1	-1	-1	-1	1	1	1	1						
360	-1	-1	-1	-1	-1	-1	-1	1	1	1							
370	-1	-1	-1	-1	-1	-1	-1	1	1	1							
380	-1	-1	-1	-1	-1	-1	-1	1	1	1							
390	-1	-1	-1	-1	-1	-1	-1	1	1	1							
400	-1	-1	-1	-1	-1	-1	-1	1	1	1							
410	-1	-1	-1	-1	-1	-1	-1	1	1	1							
420	-1	-1	-1	-1	-1	-1	-1	1	1	1							
430	-1	-1	-1	-1	-1	-1	-1	1	1	1							
440	-1	-1	-1	-1	-1	-1	-1	1	1	1							
450	-1	-1	-1	-1	-1	-1	-1	1	1	1							
460	-1	-1	-1	-1	-1	-1	-1	1	1	1							
470	-1	-1	-1	-1	-1	-1	-1	1	1	1							
480	-1	-1	-1	-1	-1	-1	-1	1	1	1							
490	-1	-1	-1	-1	-1	-1	-1	1	1	1							
500	-1	-1	-1	-1	-1	-1	-1	1	1	1							

HEIGHT = 16

BASAL AREA

SPH	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50	–1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	–1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
70	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
80	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
90	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
100	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
110	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1	1
120	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1	1
130	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1
140	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1
150	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1	1
160	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1
170	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	1
180	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	
190	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	1	
200	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1	1	
210	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1		
220	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1		
230	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1	1		
240	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1			
250	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1	1			
260	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1			
270	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1	1			
280	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1				
290	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1				
300	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1				
310	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1				
320	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1				
330	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	1				
340	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
350	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
360	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
370	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
380	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
390	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
400	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
410	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1					
420	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
430	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
440	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
450	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
460	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
470	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
480	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
490	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						
500	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1						

HEIGHT = 18

SPH	BASAL AREA 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
70	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
80	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
90	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
100	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
110	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
120	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1
130	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1
140	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1
150	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1
160	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
170	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
180	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
190	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
200	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
210	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
220	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
230	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
240	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
250	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	
260	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	
270	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	
280	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1		
290	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1		
300	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1		
310	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
320	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
330	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
340	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
350	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
360	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
370	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
380	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
390	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
400	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
410	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
420	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
430	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
440	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
450	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
460	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
470	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
480	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
490	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1						
500	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1						

HEIGHT = 20

	BASAL AREA																
SPH	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
70	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
80	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
90	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
100	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
110	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1
120	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1
130	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1
140	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
150	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
160	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
170	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
180	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
190	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
200	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
210	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
220	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
230	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
240	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
250	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
260	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
270	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
280	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
290	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	
300	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	
310	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	
320	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1		
330	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1		
340	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1		
350	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1		
360	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
370	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
380	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
390	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1			
400	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
410	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
420	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
430	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
440	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1				
450	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
460	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
470	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
480	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
490	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					
500	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1					