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Modelling Radiata Pine Wood Density in Relation to Site, Climate and Genetic Factors

Part 2: Modelling Within-stem Density

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Research Provider:
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EXECUTIVE SUMMARY

Wood density in *Pinus radiata* is often used as a surrogate for stem stiffness, and is central to the accounting of carbon sequestration in plantations. Therefore it is important to be able to map and model its distribution across New Zealand, and to understand how it is influenced by environmental variables and management. The objectives of this study were to collate basic density data from past studies:

1. To generate new regional maps of the development of outerwood basic density with stand age and silvicultural treatment based on geographical, site and genetic variables.
2. To have a comprehensive dataset of within-stem density variation to generate a new model to estimate tree component densities based on actual or predicted outerwood values, and to allow for the prediction of outerwood density forward and backwards in time.
3. To investigate the need for genetic adjustments in the prediction of wood density components to cater for the introduction of specific breeds.

This report covers Objective 2 and describes the methods that will be used to meet Objective 3. Objective 1 is covered in a separate companion report (Modelling Radiata Pine Wood Density in Relation to Site, Climate and Genetic Factors: Part 1 - Density Surfaces).

The Objective 2 model was developed using an extensive historical dataset. This model predicts within-stem variation in wood density for radiata pine, and the variation in density between trees in a typical stand. It has similarities to various earlier models, although it has a degree of flexibility not present in any previous model, and was developed using a more comprehensive wood density database. The data included studies of density measured radially at breast height in five-ring groups at 340 locations throughout New Zealand, and additional ring-level densitometry data. Also used were density data consisting of discs sampled vertically within trees from 219 stands from throughout the country.

The model includes the following main components:

1. Models to predict breast-height density by ring. Two versions were developed: (1) a simpler version predicting density as a function of ring number; and (2) a more complex model which utilises the strong relationship between ring width and density to account for the influence of stocking on density.
2. A model to predict the reduction in disc density that occurs with increasing relative height (i.e., height within a stem expressed as a ratio of total stem height).

A number of other model components were also developed for performing necessary functions, including converting from disc ring density to disc density, predicting log density from disc densities, and predicting variation in wood density among individual trees in a stand. The model is also designed to be able to incorporate genetic adjustments.

The developed model can be used to predict density of discs or logs cut from any position within a tree. It can utilise measured outerwood density in a stand to predict density by log height for that stand. Further, it can be used to predict density in that stand at any future harvest age, or the densities that would have been obtained, if the stand had been felled earlier.

The model can also be used in conjunction with the mapped surface of index age 20 outerwood density developed in Part 1 of this study. That is, for any specified geographical location, the model will predict wood density by log for stands of any specified management regime felled at any age. In collaboration with the Radiata Pine Breeding Company (RPBC), genetic adjustments are being developed for use in the model. Ultimately these will make it possible to predict the effects of site, silviculture and genetics on radiata pine log density for any location in New Zealand.

The model is available in two basic versions:

1. Stand-alone version. The inputs required are breast-height density at a known ring from pith (this can be obtained from the New Zealand mapped surface described in Part 1 of this study, or from a measurement), the stand age, and the mean height of the stand. The model predicts the density of logs cut from any position within the stem. In this mode, the system cannot predict the effect of silvicultural treatments such as stocking. It effectively predicts density for an "average" regime. Forest managers could use this version in situations where only the age and Site Index of a stand are known along with an estimate of outerwood density (either measured or obtained from the map).
2. Version for use in an integrated stand modelling system. This version has to be linked to a radiata pine growth model. The system requires, as inputs, the breast-height density at a known ring number (this can be obtained from the New Zealand mapped surface from Part 1 of this study), the usual growth model inputs (e.g., site productivity indices, stocking history, etc.), and the stand age. As in the stand-alone version, the model predicts density of logs cut from any position within the stem. The advantage of using this version is that it can predict the effects of different management regimes on wood density for any site in New Zealand. Potentially, this system can also predict density by ring number at any height in the tree. Forest managers would use this version when full details of a stand are known, including its management history, stocking, and productivity (specified either by a stand measurement or by the 300 Index and Site Index). This version of the model will be implemented in FFR Forecaster.

INTRODUCTION

Wood density is a critical variable relating to both stiffness and stability of wood products. Early work on radiata pine[1] established that there are significant regional trends in density, and also pronounced within-tree patterns. Subsequently, a variety of models for predicting wood density in radiata pine have been developed for various purposes and using different sets of data. These are described briefly in the following paragraphs. A comprehensive summary of radiata pine density models is contained in Cown *et al*.[2].

Early work resulted in various informal models describing trends in density with age and showing regional differences[3,4], and also led to some formal mathematical models, e.g., STANDQUA[5]. A model was derived from the WQI Benchmarking Study[6]; it concluded that density is dependent on both temperature and ring width, in line with previous findings[4].

A more comprehensive WQI model was also developed[7]. One component of this model predicts breast-height outerwood density from various site and stand variables. Another series of equations was developed[8] using data from WQI and other contributed sets. The output is a series of equations, suitable for implementation in YTGen or FFR Forecaster, which can predict disc density at any height in the stem using breast-height outerwood density as a model input. The model can also be used to grow a virtual stand forward or backward in time. Some validation work was carried out by WQI, and a revised version was produced using more WQI and CHH data[9].

Requirements for predicting carbon sequestration have led to at least one additional model allowing prediction of annual mass accumulation in stands in response to site and silviculture[10]. All the previous models to predict wood density were developed using various subsets of the extensive wood density data that have been collected in numerous studies, mainly by Scion (or previously, FRI) over many years. The model described in this report is in many ways similar to earlier models, especially the WQI model developed by Rawley[8,9], but it also has some unique features.

The model is available in two basic versions:

1. <u>Stand-alone version</u>. The inputs required are breast-height density at a known ring from pith (this can be obtained from the New Zealand surface described in Part 1 of this study, or from a measurement), the stand age, and the mean height of the stand. The model predicts the density of logs cut from any position within the stem. In this mode, the system cannot predict the effect of silvicultural treatments such as stocking. It effectively predicts density for an “average” regime. Forest managers could use this version in situations where only the age and Site Index of a stand are known along with an estimate of outerwood density (either measured or obtained from the map).

2. <u>Version for use in an integrated stand modelling system</u>. This version must be linked to a radiata pine growth model. The system requires, as inputs, the breast-height density at a known ring number (this can be obtained from the New Zealand mapped surface from Part 1 of this study), the usual growth model inputs (e.g., site productivity indices, stocking history, etc.), and the stand age. As in the stand-alone version, the model predicts density of logs cut from any position within the stem. The advantage of using this version is that it can predict the effects of different management regimes on wood density for any site in New Zealand. Potentially, this system can also predict density by ring number at any height in the tree. Forest managers would use this version when full details of a stand are known, including its management history, stocking, and productivity (specified either by a stand measurement or by the 300 Index and Site Index). This version of this model will be implemented in FFR Forecaster.

Objective

The objectives of this study were to collate basic density data from past studies:

1. To generate new regional maps of the development of outerwood basic density with stand age and silvicultural treatment based on geographical, site and genetic variables.
2. To have a comprehensive dataset of within-stem density variation to generate a new model to estimate tree component densities based on actual or predicted outerwood values, and to allow for the prediction of outerwood density forward and backwards in time.
3. To investigate the need for genetic adjustments in the prediction of wood density components to cater for the introduction of specific breeds.

In Part 1 of this study[11], a nationwide surface map was developed for predicting outerwood density at age 20 years for radiata pine plantations established and managed using genetic material and regimes typical of the 1990s for any location in New Zealand.

This report (Part 2 of the study) describes the development of models to predict within-stem wood density in radiata pine, and also covers the methods that will be used to derive genetic adjustments to meet the third objective in the study. These genetic adjustments will be obtained from Radiata Pine Breeding Company (RPBC) trials for specific parents, crosses, seedlots, clones, or more simply, for density GF+ rating.

METHODS

Data

Data from a large number of historic studies were used to develop the within-tree density model. Data were of two main forms, namely density measured radially from pith to bark at breast-height, and vertically within the stem.

The data included studies of density measured radially at breast-height from radiata pine plantations at a large number of locations throughout the country. Typically, about 30 trees were sampled per site. Density was sampled at 340 locations throughout the country in five-ring groups from breast-height cores or blocks using conventional volumetric techniques. These data were used to develop a model predicting breast-height density as a function of ring number from pith. In eight stocking trials and 17 sites assessed by WQI for a “benchmark” seedlot, x-ray densitometry was used to measure both basic density and ring width by ring from breast-height cores. These data were used to develop a model for predicting breast-height density by ring number and ring width.

To develop a model for predicting density at any height within a tree, historic data from trees that had been destructively sampled from a large number of sites were used. These data consisted of densities of discs cut at breast-height, and at approximately 5-m height intervals along the stem, with typically about six trees sampled per site. Data were obtained from 219 stands from throughout the country.

Analysis

Various component models were developed from the extensive database described above. In most cases, these were developed using a nonlinear mixed modelling approach. These models were all fitted using the SAS Version 9.2 NLMIXED procedure. Details of the various model forms used for each component are described in the Results and Conclusion section.

RESULTS AND CONCLUSION

Breast-height Density by Ring

The following model predicts breast-height density by ring number. Because its only input requirement is the ring number and a local parameter, *L*, it can be used as a stand-alone model. The data used to develop this model consisted of radiata pine breast-height density data measured in five-ring groups from 340 sites throughout New Zealand. Densities were averaged across all trees at each site for each five-ring group prior to fitting the model.

(1) $D_{BH} = a + f \times L + (b - a + L) \times (1 - \exp(-c \times Ring))^d$.

D_{BH} is basic density at breast-height and *Ring* is ring number from pith. The model is calibrated for a particular site using a local parameter, *L*. As will be explained later, *L* is derived either from a density surface or map, or by using a density measurement from a sample of trees. The model coefficients are given in Table 1, and Figure 1 shows the model plotted against the data. Model (1) explains 93% of the variation in breast-height density in the dataset. This can be partitioned into variation between sites, which explains 45% of the variation, and the pith to bark trend described by the model which explains a further 48%.

Table 1. Parameter estimates for Model (1).

Parameter	Estimate	s.e.
a	339.9	2.5
b	459.2	2.5
c	0.1447	0.0092
d	2.44	0.29
f	0.657	0.088

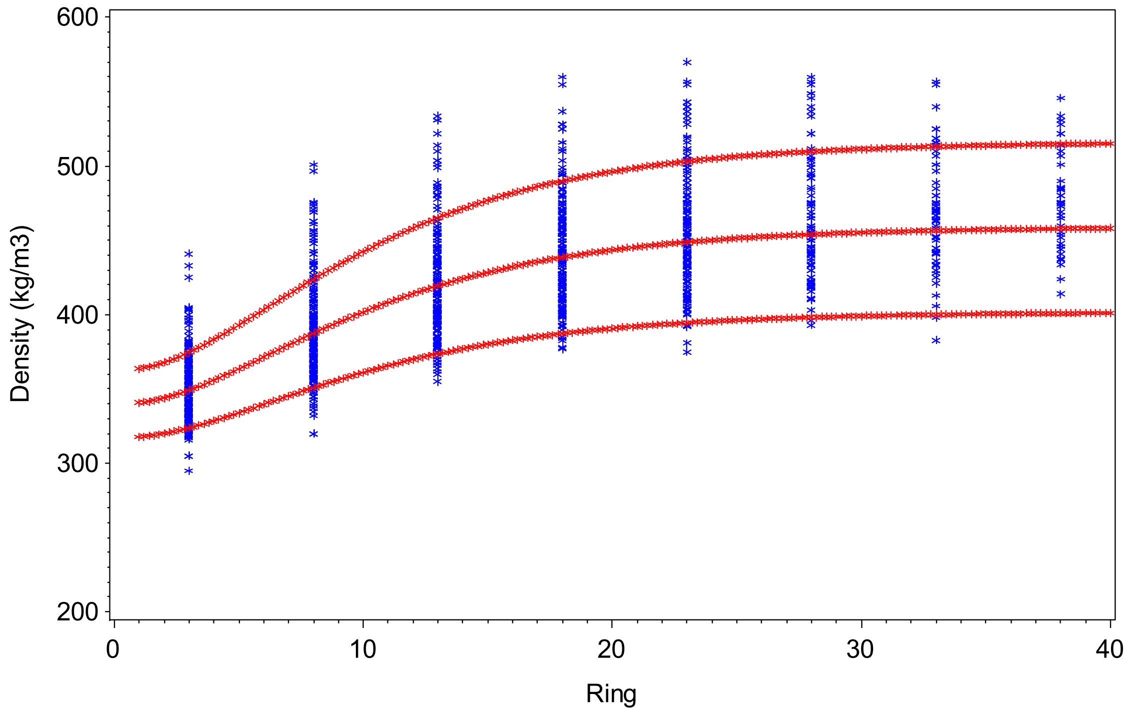

Figure 1. Pith to bark radial trend in breast-height density. Blue asterisks show mean densities by five-ring group at each site. The red lines show the density predicted by Model (1) for an average site (middle line, *L* = 0) and for sites at the 5th and 95th percentiles (lower and upper lines, *L* = -34.6 and *L* = +34.6, respectively).

Although Model (1) performs well for typical radiata pine stands, it does not account for the effects of stocking on density. However there is clear evidence from stocking trials that wood density is influenced by tree spacing. For example, in a stocking trial at Tarawera density at ring 10 was about 400 kg/m^3 at 200 stems/ha, but 500 kg/m^3 at 2000 stems/ha (Figure 2). One method of taking account of stocking is to model wood density as a function of ring width. To a good approximation, for any given site density in radiata pine is a linear function of log(ring width), and largely explains the effects of stocking on density (Figure 3). However, ring width does not account for the variation in wood density between sites, and does not account for the effects of air temperature on density, for example (Figure 4). Therefore, as with Model (1), the density model using ring width as its main independent variable also requires a local parameter to account for site differences.

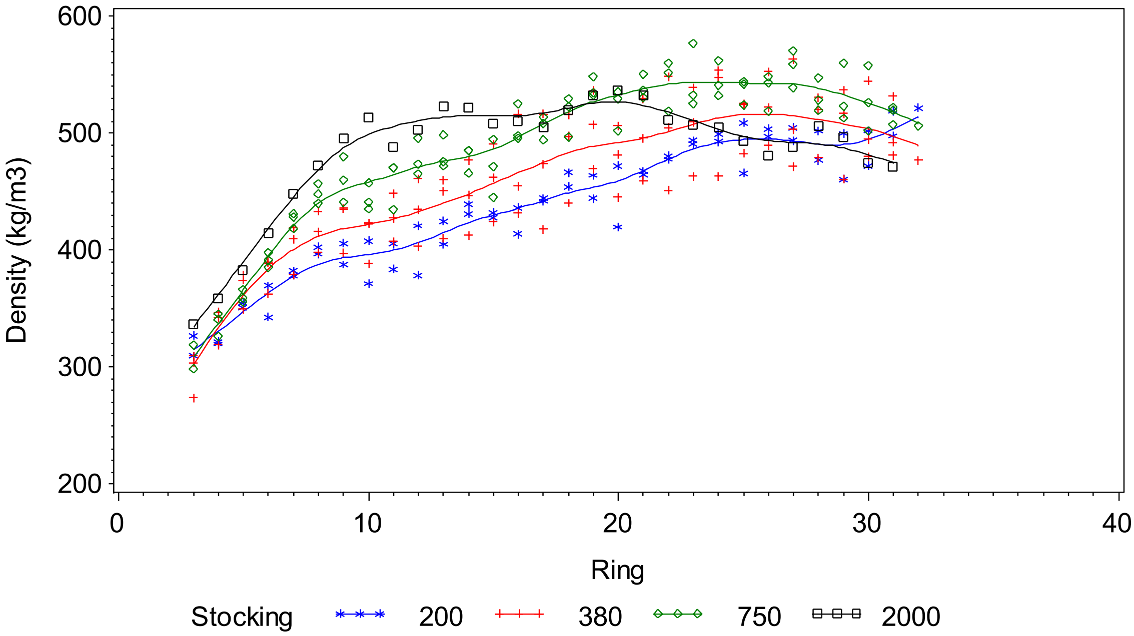

Figure 2. Breast-height wood density versus ring number for four final crop stocking levels from a trial in Tarawera Forest.

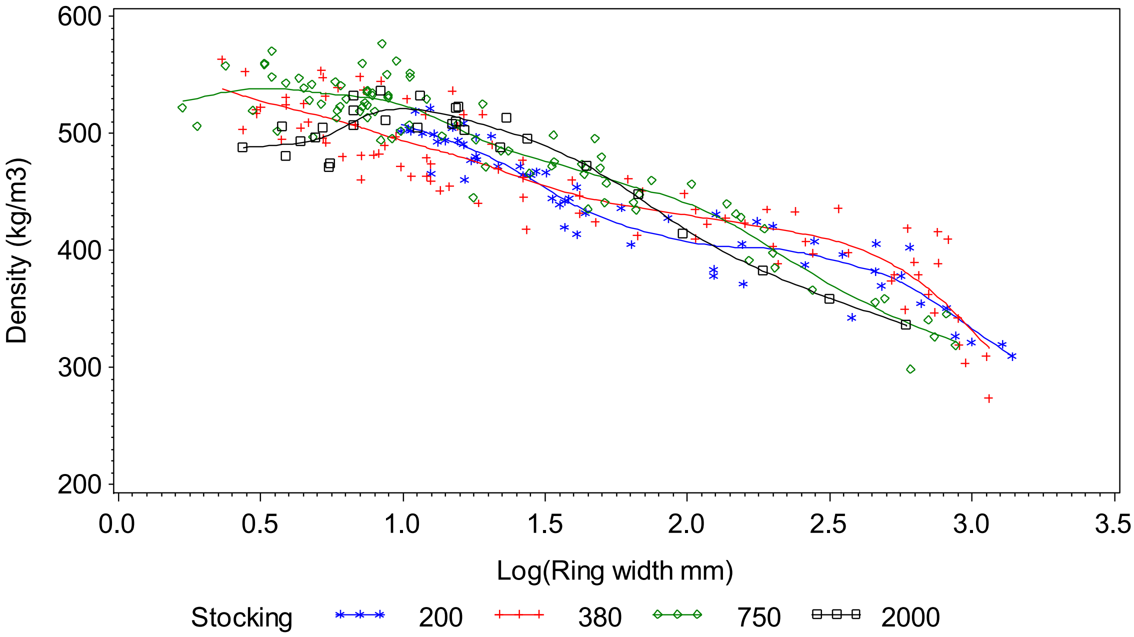

Figure 3. Breast-height wood density versus log(ring width) for four final crop stocking levels from a trial in Tarawera Forest.

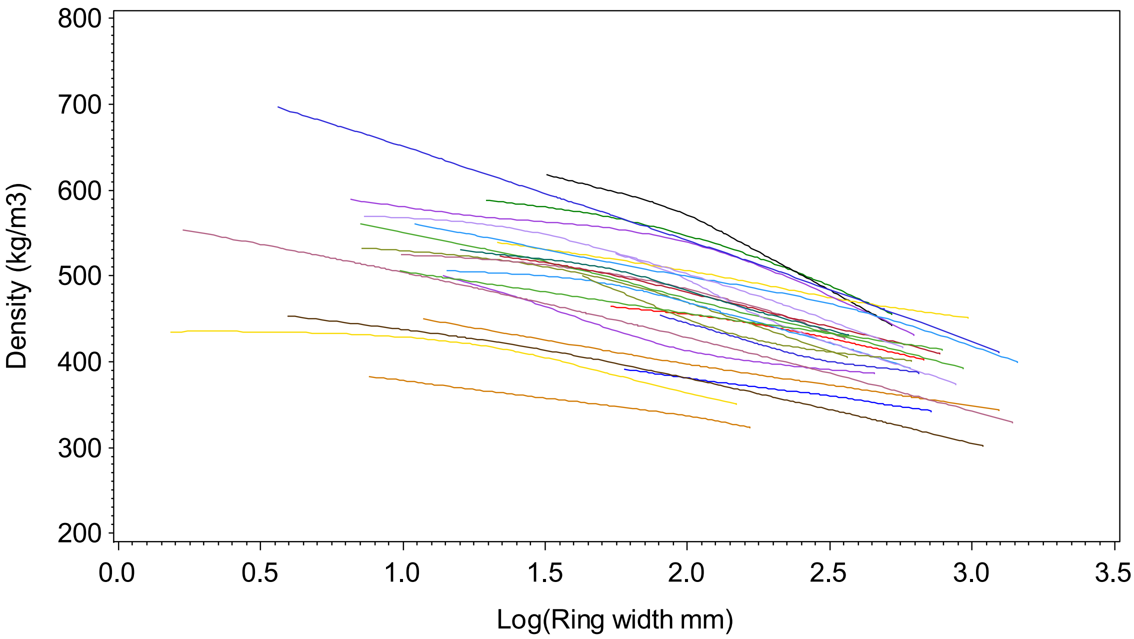

Figure 4. Breast-height wood density versus log(ring width) at 27 trial sites in New Zealand. The lines show smoothing curves fitted for each site.

A model using ring width as its principal independent variable was fitted to densitometry data consisting of breast-height density averaged by plot for each ring at 27 sites. Data from 10 of these sites were obtained from stocking trials, with each trial consisting of several replicate plots covering a range of final crop stockings. For these data, the site effects explained 62.6% of the variation in the density, adding ring number to the model (using the Model (1) equation) explained 88.7%, but a model using log(ring width) as the independent variable explained 91.5% of the variation. However, the best fit was achieved using a model including both ring width (in mm) and ring number as independent variables, which explained 93.1% of the variation. This model is as follows:

(2) $D_{BH} = a + L + b \times (1 + c \times L) \times \ln(Rwidth) - d \times \exp(-f \times Ring)$.

Parameter estimates for this model are given in Table 2. Predictions from this model are compared with actual stockings for the Tarawera trial in Figure 5 (for clarity, only the two extreme stocking treatments are shown).

Table 2. Parameter estimates for Model (2).

Parameter	Estimate	s.e.
a	579.8	17.7
b	-58.61	3.59
c	0.002763	0.000224
d	80.17	4.73
f	0.1845	0.0260

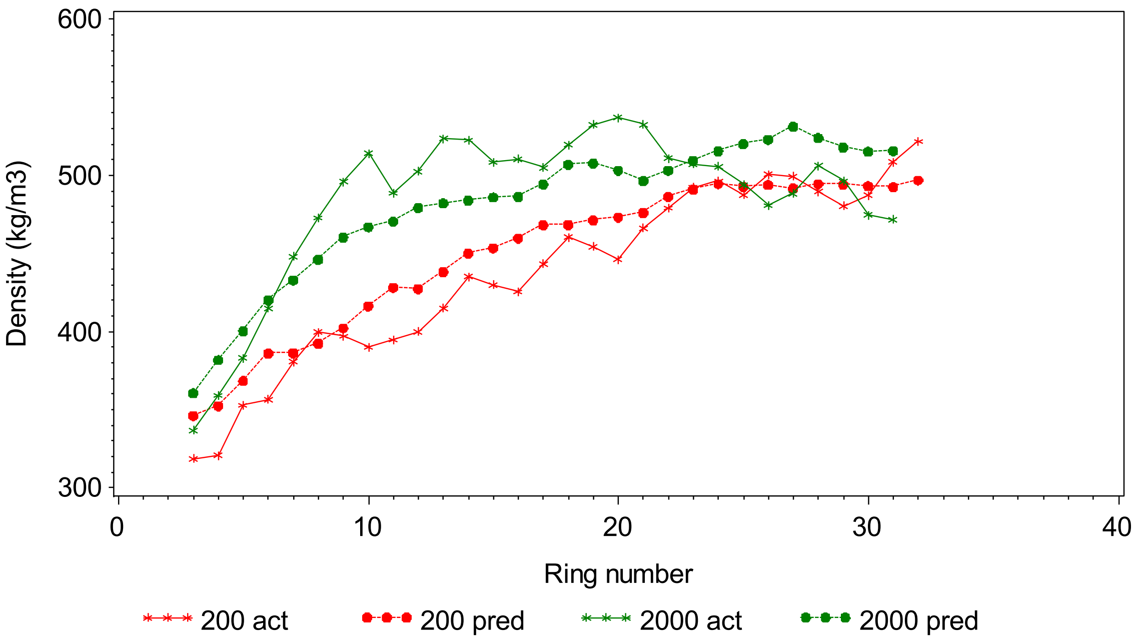

Figure 5. Actual breast-height wood density versus ring number and density predicted using Model (2) for the two extreme stocking treatments (200 and 2000 stems/ha) from a final crop stocking trial in Tarawera Forest.

It is conceivable that soil nitrogen which is also known to influence wood density can be explained by ring width. However, as we assume that fertility effects are already accounted for in the density map developed in Part 1 of this study, we do not need to account for these effects in the within-tree density model.

Disc Density by Height in Stem

Mean wood density reduces within a stem with height in radiata pine. This occurs largely because the number of rings decreases with height, meaning that proportionately more wood is contained within the lower density inner rings. The data used to derive a model of longitudinal variation in this study consisted of measurements of disc density made at approximately 5-m intervals in trees from stands at 219 sites throughout New Zealand. In addition to wood density, measurements of diameter and height were also made for each disc.

Tree heights were not generally measured in these studies. However it was possible to estimate approximate heights of trees for many studies using the relationship between disc diameter and height for each stem. Quadratic regression models were fitted for predicting diameter from height, and tree height was estimated by extrapolating these models to a diameter of zero. These regression models were fitted using discs cut at greater than 5-m height, as above this height, stem taper could be well approximated by a quadratic model. Tree heights were estimated in this way only when the diameter of the highest disc was less than 200 mm, and where the greatest disc height was greater than 70% of the estimated tree height. This procedure made it possible to estimate tree height for 150 of the 219 sites. Using the estimated tree heights, the relative height of each disc could be calculated as (disc height)/(tree height).

By plotting disc density against relative height, it was apparent that density has a sigmoidal pattern with relative height which is well approximated by a cubic function. A model of this form was fitted to the data. As with the radial density models, this model is calibrated for a particular site using a local parameter, *L*. The variation between sites explained 45.4% of the variation in density between all discs in the dataset, and the model explained 73.2%, meaning that the vertical pattern predicted [redacted] absolute height as the independent variable, it explained 71.0% of the variation, indicating that relative height is a better predictor of disc density than absolute height. Furthermore, it was obvious when examining plots of density versus absolute and relative height, that the vertical trend of density was much more consistent between sites when using relative height as the independent variable. The final model is of the following form and predicts disc density (kg/m^3) as a function of relative height (*H*). Parameter estimates are shown in Table 3, and the model is shown plotted against the data in Figure 6.

(3) $$D_{disc} = (a + L) + (b + f \times L) \times (H - 0.4) + c \times (H - 0.4)^2 + d \times (H - 0.4)^3.$$

Table 3. Parameter estimates for Equation (3).

Parameter	Estimate	s.e.
a	390.3	2.2
b	-42.30	2.67
c	31.93	4.07
d	-293.0	16.6
f	-0.5181	0.0307

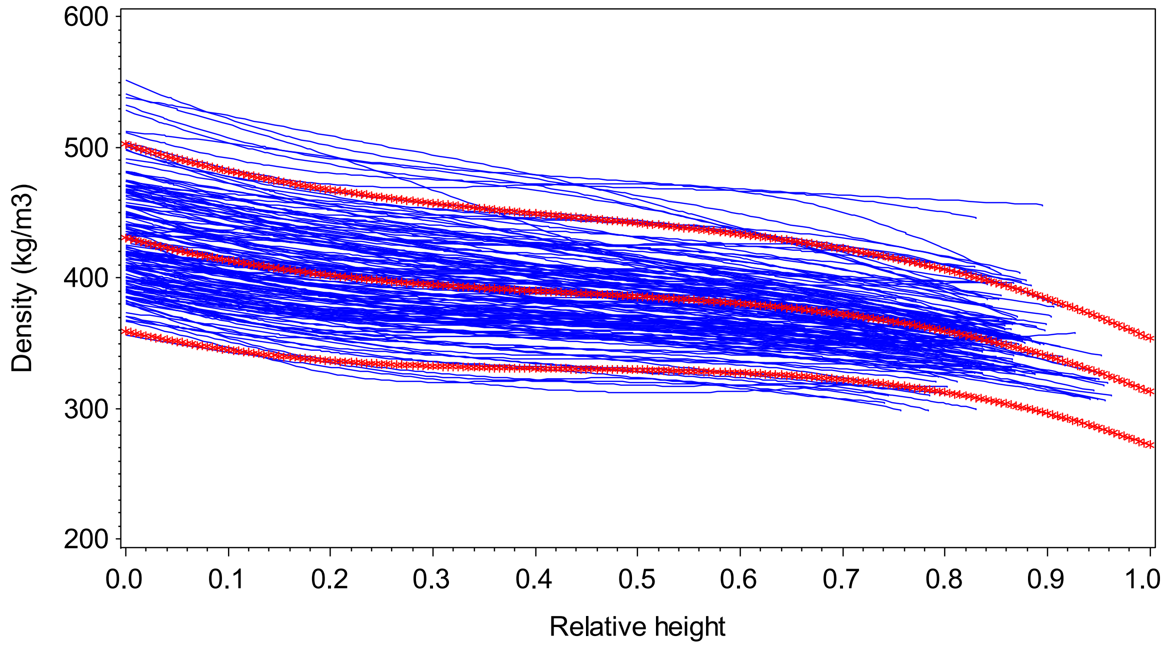

Figure 6. Disc density versus relative height. The blue lines are smoothing curves fitted to the disc densities from each of 150 sites. The red lines show disc density predicted by Model (3) for an average site (middle line, *L* = 0) and for sites at the 5th and 95th percentiles (lower and upper lines, *L* = -59.3 and *L* = +59.3, respectively).

Conversion Between Ring Density and Disc Density

The density models described in this report are intended to be linked together, as will be described in the next section. When linking the breast-height radial density model (either Equation (1) or (2)) with the longitudinal model (Equation (3)), it is necessary to convert from a measure of the density of the outer five rings (outerwood density) to disc density at breast-height. When the model is used in conjunction with a growth model, this procedure will be achieved by predicting the density at each ring using Equation (2), and averaging these densities weighting by the ring area as predicted from the growth model.

However, when using the model in stand-alone mode, ring widths will not be known, and an alternative direct approach is needed to convert from ring to disc density. Based on the studies where outerwood density was assessed using the outer five rings, rather than 50-mm cores, the ratio of disc density to outerwood density is shown to be fairly constant with age, averaging 0.942 with standard error 0.0032 (Figure 7). A simple conversion to disc density could therefore be achieved by multiplying the outerwood density by this mean value. However, the ratio would be expected to be closer to one, both at very young ages (e.g., at about age 6 years when the disc will consist of only about five rings), and very old ages (because the pith to bark density profile flattens beyond age 20 years). This behaviour is modelled by the following function, which was fitted to the data as a nonlinear regression, and can be used to convert from outerwood to disc density at breast-height:

(4) $D_{disc}/D_{OW} = 1 - 1 / (a + b \times (Age - 6) + c \times (Age - 6)^2)$.

Table 4. Parameter estimates for Equation (4).

Parameter	Estimate	s.e.
a	44.0	13.3
b	-2.34	1.10
c	0.0467	0.0218

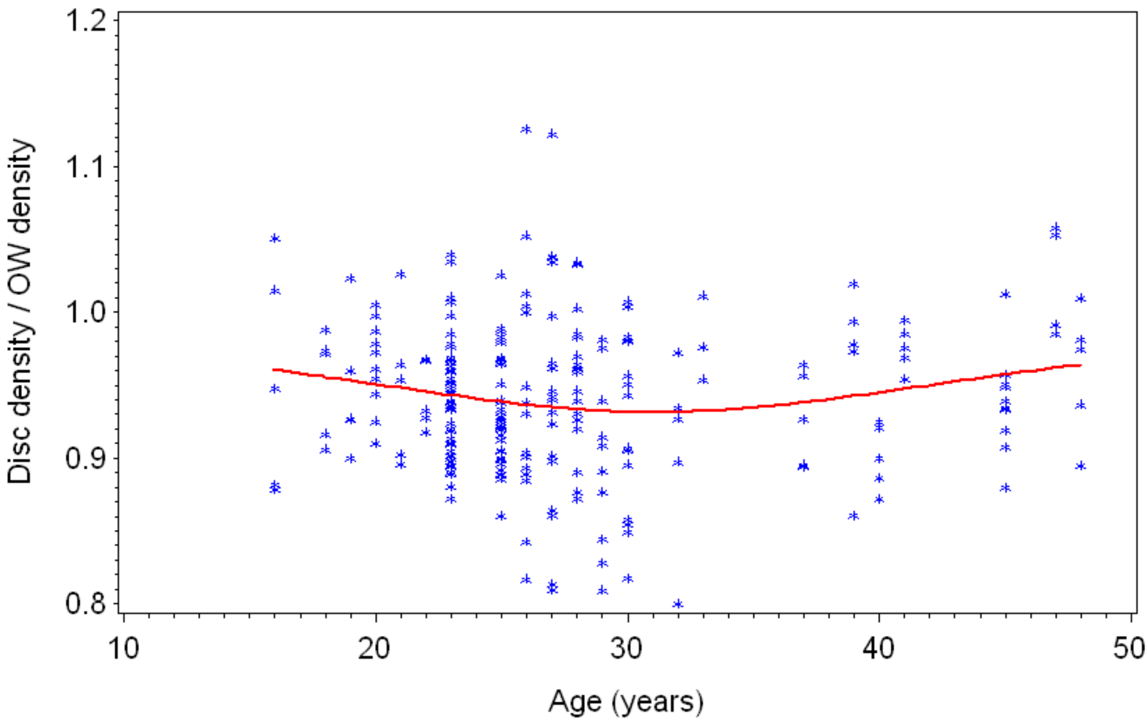

Figure 7. The ratio between disc density and density of the outer five rings (at breast-height) versus age. The line shows Equation (4).

Predicting Log Densities from Disc Densities

The model described above predicts the basic density of discs cut at any height within a stem. As described in the previous section, this model can be linked to the breast-height radial density model to predict disc density at any height and age. Furthermore, when linked to a growth [redacted] models to estimate density by ring and height. This would be achieved by estimating the disc densities annually, and using the predicted ring width at any height to derive density of each ring.

However, in practice a typical modelling system would usually be required to predict the average density of logs, and should be able to do so for logs cut from any position within a stem. The model described above can be used to predict the density of discs representing each end of the log for any log position. It is therefore necessary to have a way of estimating the average density of a log from the disc density at each end of the log. The simplest approach would be to simply average the density of each end. However, as logs are usually tapered, slightly more weight should be assigned to the density at the large end than the small end when calculating average log density.

If both disc diameter and density vary linearly with distance along a log (a reasonable assumption over the length of a typical log), the average density of the log is:

(5) $$D_{\text{log}} = W \times D_{Ldisc} + (1\text{-}W) \times D_{Sdisc}$$

where D_{Ldisc} and D_{Sdisc} are the disc densities at the large and small end of the log respectively, and where the weighting factor W is a function of the ratio of small to large end diameter of the log (*SED/LED*):

(6) $$W = ((SED/LED)^2/4 + (SED/LED)/2 + 3/4)/((SED/LED)^2 + SED/LED + 1)$$

This result can be established using basic calculus (see Appendix). Using the above result, log density will be obtained in the growth model implementation of the density model using the predicted diameter of the stem at each end of the log. In a stand-alone version of the density model, the weight W could be calculated by making reasonable assumptions on the values of log diameter, length and taper. As an example, in a 5-m log with 400 mm *SED* and 6-mm/m taper, the value of W calculated using Equation (6) is 0.512. It can be seen from this example that a simple average of the two log end densities (i.e., assuming $W = 0.5$) would generally be adequate.

Variation in Wood Density Among Individual Trees in a Stand

An analysis of the variation in wood density between individual trees within a stand was performed using outerwood density data. The advantage of these data is that trees were generally sampled randomly within each stand. In contrast, trees felled to obtain disc densities were not always sampled randomly. Often they were sampled using a two-stage sampling system with the final selection intended to cover the range of density (based on outerwood density samples of a larger initial sample) and/or DBH within the stand. Therefore, although disc data can be used for developing models, they should not be used for analysing variation between trees.

However, examining variation in outerwood density is also problematic because the small size of core samples means that they are likely to be more variable than larger scaled samples such as discs or logs. To overcome this problem, we used data from 82 stands which were sampled using two or more cores per tree. With these data it was possible to separate out the small-scale variability of cores within a tree from the general variation between trees using a statistical analysis technique called variance component analysis performed using the SAS Version 9.2 MIXED procedure.

The first step in this analysis was to examine the form of the statistical distribution of outerwood density between trees. As found in earlier studies, this analysis showed that wood density is normally distributed (Figure 8).

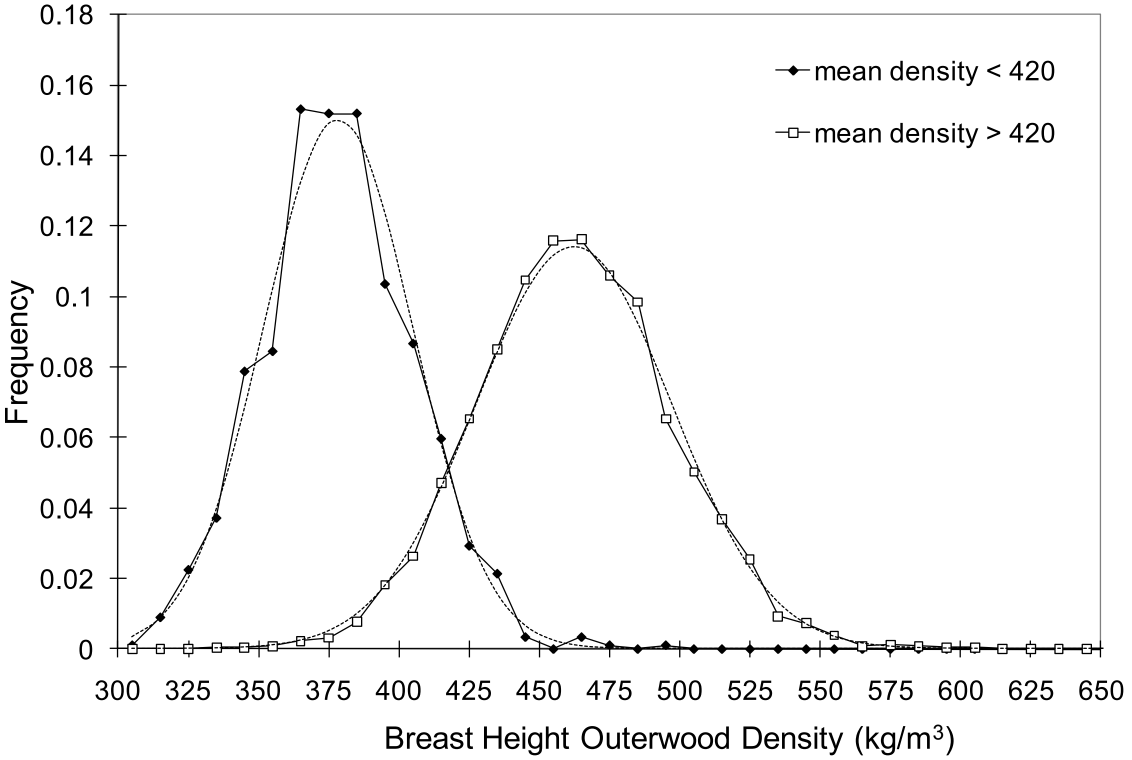

Figure 8. Distribution of outerwood density in low (<420kg/m^3) and high density (>420kg/m^3) stands. For illustrative purposes, densities were subtracted from stand means, and plotted around the mean for each density class. Dashed lines show fitted normal distributions.

Secondly, it is important when modelling wood density in a stand to take account of any relationship between easily predicted tree characteristics and density. For this reason, the relationship between DBH and outerwood density was investigated by calculating correlation coefficients between these two variables for each stand. This analysis revealed almost no relationship between tree diameter and density within a stand. The correlation coefficients averaged -0.07, and were symmetrically distributed about this mean. A close examination of the few stands that showed a significant association between DBH and density suggested that this occurred only in older, highly stocked stands, containing small suppressed trees. Such trees tended to be of higher than average density, producing a weak negative correlation between density and DBH in these stands. Note that this result applies to individual trees within a stand. As shown earlier in the report, it does not apply to stands growing at different stockings at the same site, where average stand density is related to average ring width.

To quantify the variation, between and within-tree variance components were estimated for each stand. Stands were classified on the basis of mean density into classes 300-350, 350-400, 450-450, 450-500 and 500-550 kg/m^3, and the variance components were averaged for each class. Variance components were expressed as coefficients of variation (standard deviation as a percentage of the mean), and are shown plotted against mean density of each class in Figure 9. The variation between trees expressed as a coefficient of variation averaged 6.46% and did not vary with mean density. In contrast, within-tree variation (i.e., variation between multiple breast-height outerwood cores taken from the same tree) increased with mean density, or equivalently, with age. In other words, as trees become larger and older, the variation in outerwood density at different locations around the stem increases. However, more importantly, the variation in mean density does not increase when expressed as a percentage of the mean density.

In summary, for modelling purposes it can be assumed that the wood density of trees within a stand is normally distributed with a coefficient of variation of 6.46%, and is not related to tree diameter.

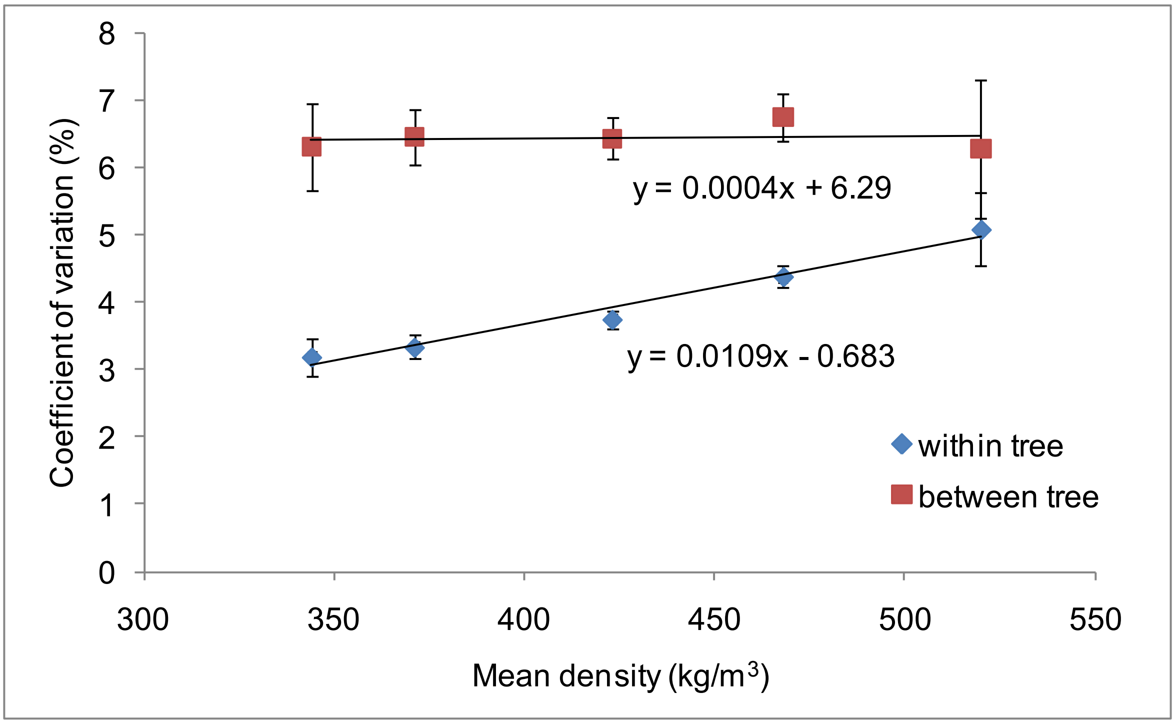

Figure 9. Variation between and within tree (expressed as coefficients of variation) for stands in five mean density classes. Error bars show standard errors of each coefficient of variation.

Genetic Adjustments

Genetic effects can be incorporated into the modelling framework described above by including them as adjustments to the local parameter in the radial density Models (1) or (2). Measurements of wood density in RPBC trials are currently being used to derive such genetic adjustments.

The procedure for doing this is fairly straightforward. The local parameter L for use in Model (1) can be calculated for each genotype from measurements of wood density in a genetic trial or a series of trials at a specific ring number using:

(7) $L = [D_{BH} - a - (b - a) \times (1 - \exp(-c \times Ring))^{d}] / [f + (1 - \exp(-c \times Ring))^{d}]$

Similarly, for Model (2), L is calculated using:

(8) $L = [D_{BH} - a - b \times \ln(Rwidth) + d \times \exp(-f \times Ring)] / [1 + c \times \ln(Rwidth)]$

Equation (8) requires a standard ring width for the measured density ring number averaged across all genotypes for each trial. This ring width is obtained from the mean DBH using a growth model.

Local parameters L estimated using Equations (7) and (8) are averaged for each genotype (these could be for estimated by parent, seedlot, clone, or could be averaged for breeding values or GF+ density values). Adjustments for use in Models (1) or (2) are then calculated as the ratio of the mean L value of the genotype to the mean L value for a standard genotype. As the within-tree density prediction model is intended to be used in conjunction with the mapped density surface of radiata pine planted in the 1990s produced in Part 1 of this study, this standard genotype should ideally be typical of those planted in the 1990s.

Implementation of Models

The density models described above use local parameters to calibrate them to particular sites or stands. For the breast-height radial density models, this parameter is calculated from the density at a specified ring number. This can be obtained either from a measurement of density (e.g., from outerwood density cores taken from sample trees), or more commonly, using the nationwide surface described Part 1 of this study.

For the breast-height radial density Model (1), the local parameter is calculated using the following equation:

(9) $L = [D_{BH} - a - (b - a) \times (1 - \exp(-c \times Ring))^{d}] / [f + (1 - \exp(-c \times Ring))^{d}]$.

In this equation, *Ring* is the ring number from pith, and the coefficients *a*-*f* are from Table 1. For the national surface, the index density is defined as outerwood density (density of the outer five rings) at age 20 years. The mean ring number of age 20 outerwood cores is calculated using the following equation:

(10) $Ring = 18 – (7.8 - 0.329 \times SI + 0.00388 \times SI^{2})$.

The bracketed term in this equation is the number of years for a stand of given Site Index to achieve a mean height of 1.4 m. It was derived by running the 300 Index growth model at a range of Site Indices, and fitting a quadratic regression to the age at which mean height was predicted to reach 1.4 m.

For the breast-height radial density Model (2), the calculation of the calibration parameter is made using the following equation:

(11) $L = [D_{BH} - a - b \times \ln(Rwidth) + d \times \exp(-f \times Ring)] / [1 + b \times c \times \ln(Rwidth)]$.

The coefficients *a*-*d* in this equation are from Table 2. This equation requires a measurement of density at a known ring number, and also a measurement of ring width at the ring number. Both of these could be obtained from outerwood cores taken from a sample of trees, in which case the mean ring number and ring width of the density sample would be used.

However, if the nationwide density surface map is used to calibrate the model, it is necessary to use a ring width suitable for use with this surface. Note that Model (2) is intended to be used in conjunction with the 300 Index Growth Model. Therefore the 300 Index and Site Index must be either specified by the user, or estimated from a growth measurement. In either case, *Ring* is estimated using Equation (10). Ring width is calculated using the following equation:

(12) $RWidth = 11.29 + 0.263 \times I_{300} – 0.340 \times SI + 0.00543 \times SI^{2} – 0.00807 \times I_{300} \times SI$.

This equation was developed by predicting mean ring width of the outer five rings at age 20 years using a series of 300 Index Growth Model runs performed for 300 Index ranging from 15 to 40 m^3/ha, and Site Index ranging from 20 to 40 m using a regime assumed to be typical of those contributing to the database used to develop the surface map (initial stocking 833 stems/ha, pruned and thinned to 400 stems/ha). Equation (12) was then fitted as a regression model to the predicted ring widths from these runs.

Once the local parameter, *L*, has been estimated, the breast-height density can be predicted by ring using either Model (1) or (2), as appropriate. When Model (2) is used in conjunction with a growth model, the ring widths of each ring are predicted by the growth model, and these can be used to estimate the breast-height disc density as an area-weighted average. When Model (1) is used, breast-height disc density is estimated using Equation (4).

To then predict disc density at any height within a stem, Model (3) is used. This requires a local parameter, *L*, which is calculated using the following equation:

(13) $L = (D - a - b \times h - c \times h^2 - \mathrm{d} \times h^3) / (1 + f \times h)$.

where *D* is the breast-height disc density, and h = 1.4 / *TreeHeight* - 0.4, and the coefficients *a*-*f* are from Table 3.

To predict mean density of logs cut from a specified location in the stem, the disc densities are calculated for each log end, and log density is calculated using Equation (5).

Finally, to model the distribution of density for logs cut from a particular position in the stem, the mean density is calculated using the above procedures. The distribution between individual logs cut from different stems in the stand is then modelled on the assumption that they are normally distributed about this mean with a coefficient of variation of 6.46%.

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APPENDICES

Appendix 1 – Derivation of method to calculate density of log from densities of end discs

Without loss of generality, assume that log length is 1 and *LED* is 1.

Let R = *SED* / *LED* and let densities of each end of log be d_{LED} and d_{SED}.

The density at any position x along the log is $d_{LED} - x(d_{LED} - d_{SED})$ and the diameter of the log at position x is $1-(1-R)x$. The density of the log is the integral over the length of the log of density multiplied by the square of the diameter, divided by the integral of the diameter, i.e.,

$$\int_0^1 (d_{LED} - x(d_{LED} - d_{SED}))(1-(1-R)x)^2 dx \Big/ \int_0^1 (1-(1-R)x)^2 dx$$

The solution to this integral (steps not shown) is:

$$((R^2 + 2R + 3)d_{LED} - (3R^2 + 2R + 1)d_{SED})/(4R^2 + 4R + 4) = Wd_{LED} + (1-W)d_{SED},$$

where, $W = (R^2 + 2R + 3)/(4R^2 + 4R + 4)$