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Theme: Radiata Management

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Heartwood and Green-Density Models for Pinus Radiata

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EXECUTIVE SUMMARY

The Problem

WQI members need access to models:
for estimating *P. radiata* heartwood diameter at any point on a stem and heartwood volume between any two points on a stem, as a function of age, position in stem and site; a heartwood model.

for estimating green density (inside bark) from the proportion of wood that is heartwood; a green-density model.

WQI Initiatives

WQI has already developed models for estimating log acoustic velocity [INT8 and INT13]. These are useful for predicting the potential availability of logs that are suitable for structural products from forest inventory.

This project arose from the realisation that heartwood is indirectly affecting the interpretation of stem velocity measurements via green density [STR27] and that a combination of green density and velocity would provide a better estimate of the structural potential of a forest resource.
Using a heartwood model is believed to be the most practical way of estimating green density in growing trees.

There is also interest amongst members in estimating heartwood diameter for the effect that it may have on appearance grade logs.

WQI evaluated an existing Future Forests Research (FFR) heartwood model [FFR08] and found that it was satisfactory in function and behaviour but had been developed using a limited dataset [RAWLEY2008].

This project provides new heartwood and green density models. These were developed using existing WQI, FFR and Scion data; all heartwood data that was readily available.

Results

New heartwood and green density models were developed and are presented in this report. The models explain 79% of the variation in heartwood diameter and 79% of the variation in green density and are ready for implementation.

Implications for WQI

WQI members now have access to heartwood and green density models that can be implemented in software, including YTGen and Atlas Forecaster, and used to estimate heartwood diameters, heartwood volume and green density. Their use is expected to lead to greater accuracy in the predictions of structural outturn from forest estate modelling.
Green density predictions are valuable as it was found in Str 27 that including the average green density, with the HM200 of the log to obtain an average stiffness, significantly improved (r-squared lifted from 0.47 to 0.65) the prediction of the average MoE of the boards cut from the log, compared to just using HM200 acoustic speed alone. The prediction of heartwood diameter will be useful in appearance logs for predicting whether the heartwood boundary extends outside the defect core (which downgrades the value as an appearance log).

INTRODUCTION

This work arose out of two streams of prior WQI work. In one, models were developed for estimating the velocities of logs in standing trees, [INT8] and [INT13], with the intention that these estimates would be useful for forecasting the availability of logs suitable for structural products from forest inventory data. In the other, the relationship between log velocity and modulus of elasticity (MOE) of sawn lumber was re-examined and it was concluded that this relationship varied with log green density, [STR27].

The conclusion was that forecasts of structural log availability could be improved by the incorporation of estimated green density. It was noted that in standing trees green density is largely a function of heartwood proportion. The intention was to build a model that estimated heartwood diameter at any point on a stem as a function of the variables that are typically available in forest inventory; age, predicted inside-bark diameter, position in stem and location. This model could be used to estimate the green density of logs and, in combination with the existing velocity models, to improve the prediction of structural out-turn.

WQI examined the suitability of an existing FFR heartwood model [FFR08] and concluded that, while the model was satisfactory in function and behaviour for the purposes envisaged by WQI, it had been developed using a limited dataset and could be improved upon through use of the combined WQI and FFR heartwood datasets.

METHODS

Heartwood Data

For this study, WQI and FFR made available measurements from 7653 disks from 1456 trees on 81 sites in New Zealand. Additional data were available from 4 sites in South Australia but were not used.

The disks were collected during the course of 25 studies going back over many years. These included:

- The national benchmarking study with data from 17 sites, all at age 25, spread throughout New Zealand [RES34]
- The effects of silviculture study on an age 23 Northland site; CHH Forsyth Downs spacing trial [INT3]
- An age 15 Canterbury plains site; Shellocks [INT2]
- West Coast sites at ages 8, 16 and 25 [STR4], [STR6]
- Two Central North Island sites at ages 26-28 [STR27]
- Data collected by WQI under STR1.9, from 120 trees at 6 South Island sites with ages ranging from 13-30.
- Data from 31 sites spread throughout New Zealand and collected during the *radiata* pine wood properties survey [FRI Bulletin 50]
- 18 independent Scion studies from which heartwood data was provided for this project by R. McKinley and D. Cown.

Age at measurement ranges from 8-47 with data spread by age and location as shown in Table 1.

Table 1 Number of trees by region and age

	Age					
Region	5-10	10-15	15-25	25-35	35-50	Total
Southland,Otago	0	20	40	35	5	100
Canterbury,West Coast	50	270	89	65	5	479
Nelson, Marlborough	0	20	50	30	10	110
Southern North Island, Hawkes Bay	0	0	36	25	0	61
Central North Island	0	30	242	310	39	621
Auckland, Northland	0	0	60	5	20	85
Total	50	340	517	470	79	1456

There is a reasonable spread by age and by location but not by age across locations. For example, all of the data from trees less than 10 years old comes from the West Coast.

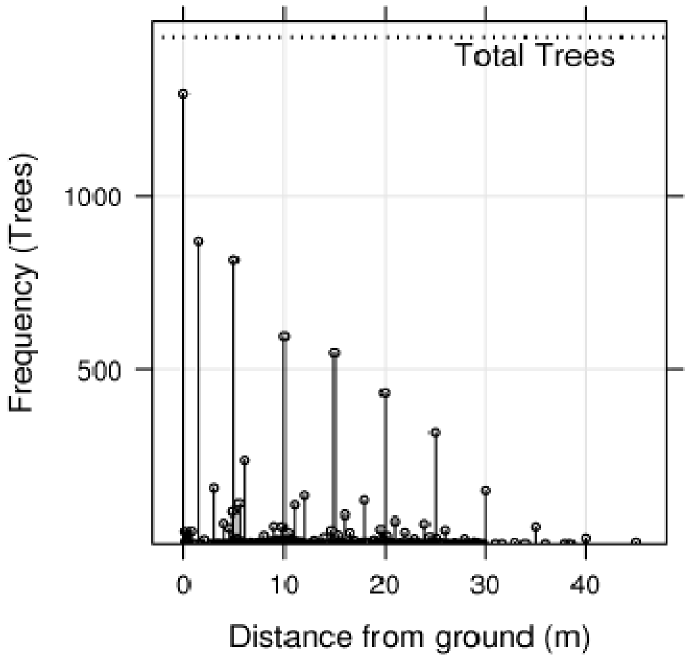
The nature of the data varies by study. Trees were not all sampled at the same heights. Neither do they all have the same numbers of disks. The typical measurement pattern was based on 5m intervals and 89% of trees had a measurement at ground-level. See Figure 1 for the distribution of disk positions.

Most trees did not have height measurements, which ruled out the possibility of using relative height in the heartwood model. Many did not have diameter at breast height.

Most studies did not provide heartwood ring counts or total ring counts. This precluded the development of a ring-based model.

Some of the data came with latitude or northing information. For the rest an approximate northing was determined by reference to forest names in the data and a map.

Figure 1 Frequency of disk positions



Heartwood Patterns

Figure 2 shows the pattern of heartwood diameter within a single age 25 tree. In the lower part of the tree the heartwood diameter is fairly constant then declines toward the top of the tree. As a percentage of inside-bark diameter, there is an initial increase, then a decline, but the initial increase is largely a function of butt flare in the inside-bark diameter rather than a pattern in the heartwood. Trees vary but the general pattern in Figure 2 is fairly typical.

Figure 3 shows the same information for 20 trees selected at random from the data with one tree in each of 20 age-by-size classes. The age classes run from 5-10, 10-15, 15-25, 25-35 and 35-50. The size classes are bounded by the 15th, 50th and 85th percentile of diameter within each age class. That means that a tree in the smallest size class at age 35-50 will be larger than one in the same size class at ages 0-10. The smallest size class is shown at the bottom of Figure 3. Diameter, in this context, is maximum disk diameter; typically the diameter of the disk at ground-level.

Figure 2 Heartwood diameter and inside-bark diameter for a single tree

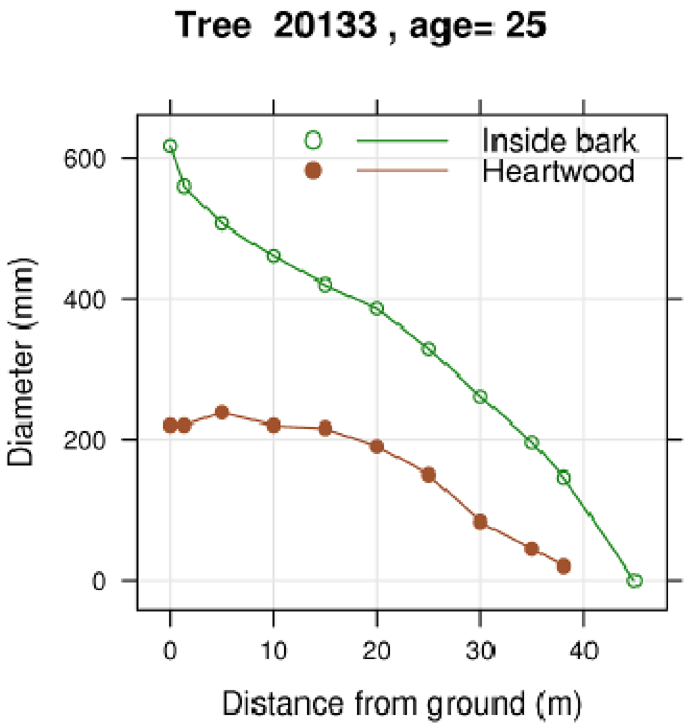


Figure 3 Heartwood profiles by age and tree size

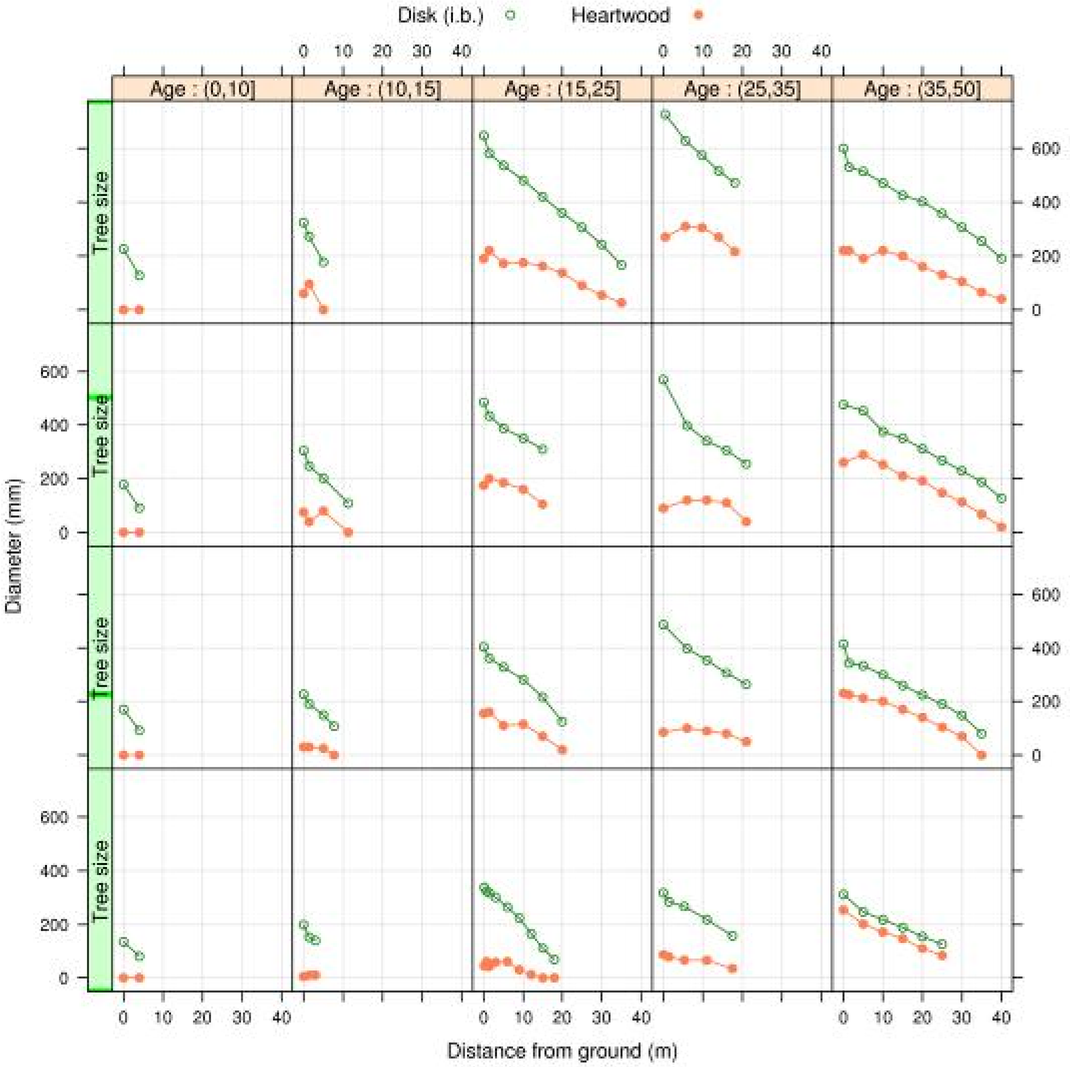


Figure 4 shows that inside-bark diameter is another good predictor of heartwood diameter. In fact, a simple linear model of heartwood diameter on inside-bark diameter accounts for 69% of the variation in heartwood diameter.

Figure 4 Heartwood diameter against inside-bark diameter for all disks

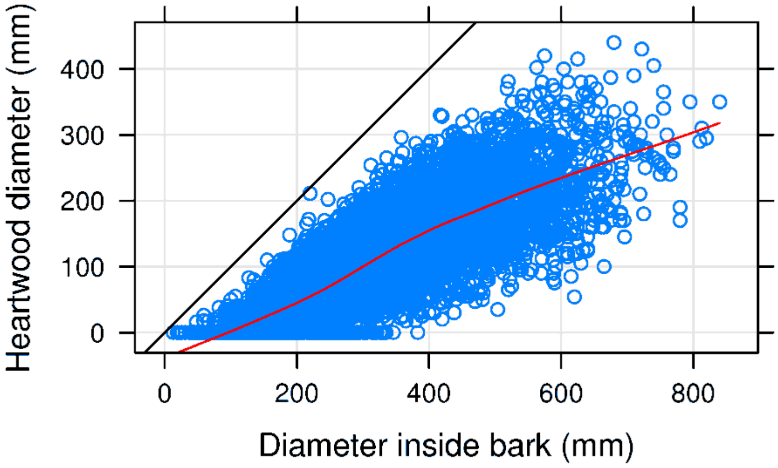


Figure 5 shows that the relationship between heartwood diameter and inside-bark diameter holds well for individual trees. These are the same trees that are shown in Figure 3. The relationship appears to become steeper with age and falls over at the base of the tree because of butt flare. Ground-level is on the right hand side of each of the graphs. The effect of age is easier to see in Figure 6 where it is clear that the intercept decreases and the slope increases with age. Heartwood diameter typically hits zero at a disk diameter between 100 and 200mm.

The observation by Cown et al. [FRI Bulletin 50] that heartwood development starts at about ring 10 is supported by the data. The trees aged less than 10 years had no heartwood.

Figure 5 Heartwood diameter against inside-bark diameter for individual trees

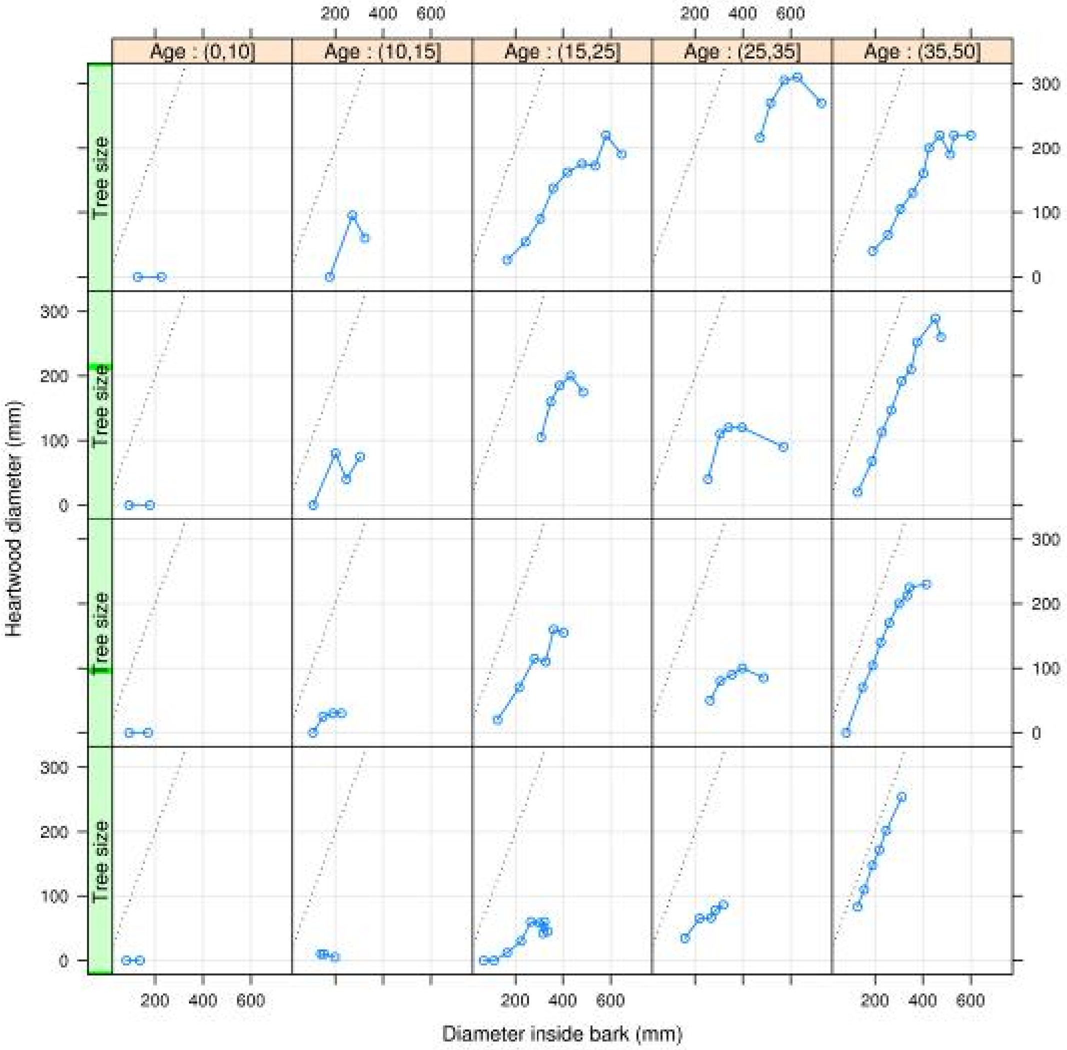
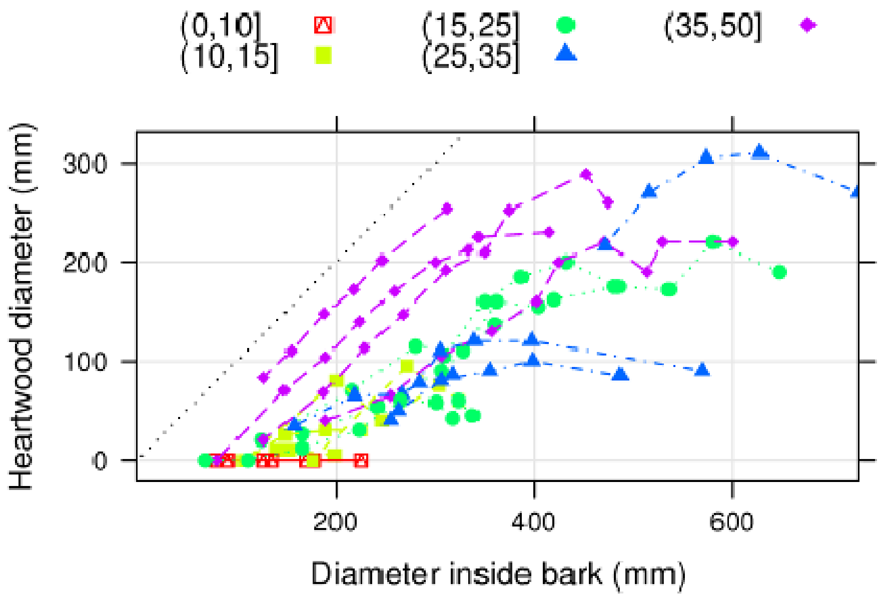
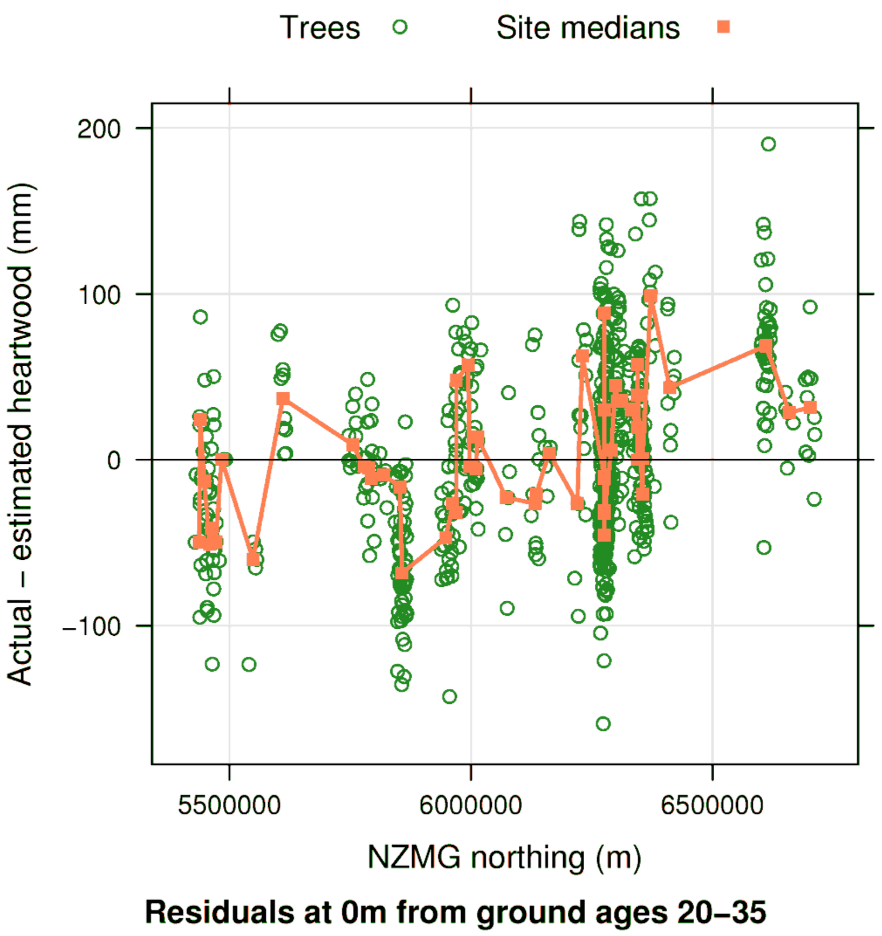


Figure 6 Heartwood diameter against inside-bark diameter by age



A further observation by Cown et al [FRI Bulletin 50] that there is north-south trend in heartwood development is also supported by the data. Figure 7 plots the residuals from a model of heartwood diameter predicted from inside-bark diameter, position and age. The graph is limited to disks at ground-level and trees aged between 20 and 35. For the same inside-bark diameter at the same age, heartwood diameter at ground-level increases from south to north. This trend was included as a difference between North Island and South Island in the model of Ping et al [FFR08] and this two way stratification accounts for most of the variation. Cook Strait is at about 6000000m in the graph.

Figure 7 Residuals against northing for model without northing



In summary, the pattern of heartwood diameter in a tree starts with zero heartwood at about 150mm and increases roughly linearly with increasing stem diameter. It departs from this trend near ground-level because of butt flare. The slope of the line and its intercept are influenced by age and latitude.

Heartwood Model

A model that incorporates the trends observed in the previous section is provided in Equation 1.

Equation 1 Heartwood diameter

$$hwd = \beta_0 + \beta_1 dib + \frac{\beta_2}{h+0.5} + \beta_3 northing + \beta_4 age + \frac{\beta_5 dib}{h+0.5}$$
$$+ \beta_6 dib \cdot northing + \beta_7 dib \cdot age + \beta_8\, age \cdot h + \varepsilon$$

where:

hwd	=	Heartwood diameter (mm)
dib	=	Inside-bark diameter (mm)
h	=	Distance from ground (m)
northing	=	New Zealand map grid northing (m)
age	=	Tree age (years)
$\beta_0 \cdots \beta_8$	=	Parameters to be estimated
ε	=	Prediction error

Parameters were estimated as fixed effects in a linear mixed model and are provided in Table 2. Equation 1 together with the parameters in Table 2 and ε=0 can be used to estimate the expected heartwood diameter for a given set of inputs.

Table 2 Parameter estimates for Equation 1

Parameter	Estimate	Term
β_0	##	Intercept
β_1	##	dib
β_2	##	1/(h+0.5)
β_3	##	northing
β_4	##	age
β_5	##	dib/(h+0.5)
β_6	##	dib*northing
β_7	##	dib*age
β_8	##	age*h

The error term in Equation 1 has a component that is systematic within each tree. In other words, if the heartwood is over-predicted at the base of the tree then it will tend to be over-predicted at the top as well. This systematic within-tree error was estimated by including slope and intercept terms as random effects at the tree-level within the model. This was done to provide the opportunity for a stochastic model with an error term at the tree-level. With Equation 1 all of the trees in an inventory with the same DBH and height will have the same heartwood diameters. In some situations it is better to allow the profile of heartwood diameters to vary between trees to reflect natural variation and this is what the random effects in the model allow.

Equation 2 provides the stochastic version of Equation 1. All terms existing in Equation 1 have the same meaning and use the parameter estimates provided in Table 2.

Equation 2 Stochastic version of heartwood model.

$$hwd = \beta_0 + \beta_1 dib + \frac{\beta_2}{h+0.5} + \beta_3 northing + \beta_4 age + \frac{\beta_5 dib}{h+0.5}$$
$$+ \beta_6 dib \cdot northing + \beta_7 dib \cdot age + \beta_8\, age \cdot h$$
$$+ \gamma_0 + \gamma_1 dib + \varepsilon$$

Table 3 Random effects

Parameter	Standard deviation	Correlation	Description
γ_0	##		Tree-level intercept adjustment $\sim N(0, \#\#)$
γ_1	##	##	Tree-level slope adjustment $\sim N(0, \#\#)$

The terms γ_0 and γ_1 are both normally distributed with mean of zero and standard deviations of ## and ## respectively. They are correlated with correlation coefficient of ##. In a stochastic implementation they would be selected once per-tree. The systematic within-tree variation accounts for half of the residual variance of the model. The remaining non-systematic error term has a standard deviation of 18.6 mm. See “implementation” section for how a stochastic model would work in practice.

It is apparent from Figure 7 that there is a component of the error that is site-specific. This was not included in the model, or rather it was included at the tree-level, because the expectation in the typical use of a stochastic model would be that the between-tree variation should average out to zero for a particular site.

Fit of Heartwood Model

The model in Equation 1 has a coefficient of determination (R^2) of 0.79, calculated using fixed effects only. Ninety-five percent of estimated heartwood diameters are within 75mm of the measured value. There are no disturbing patterns in the residuals; see Figure 8 and Figure 9, in which the residuals are calculated using the fixed effects only. When both fixed and random effects are included in the predicted value the residuals are even better behaved.

Figure 10 shows the predicted heartwood diameter by position for the trees from Figure 3. Predicted values that are less than zero have not been set to zero. The separation between the actual and predicted curves in Figure 10 illustrates the systematic tree-level error.

Figure 11 shows what happens if the tree-level terms in the model are included in the prediction. This is not a real reflection of the goodness of fit of the model because we can never know how far a given tree will depart from the average. Figure 11 is included because the difference between it and Figure 10 illustrates the sort of "noise" that the random effects in Equation 2 would introduce in a stochastic model.

Figure 8 Heartwood residuals by position and age

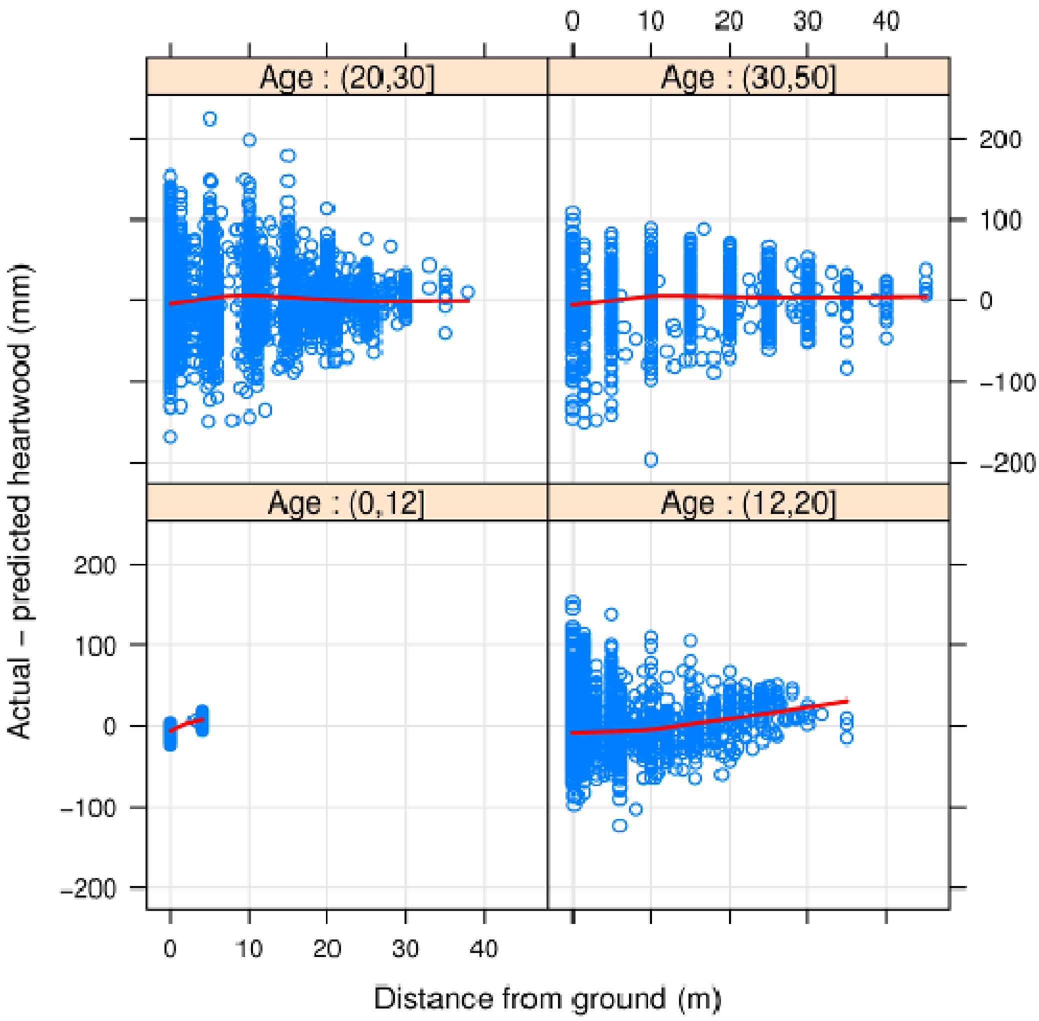


Figure 9 Heartwood residuals by island

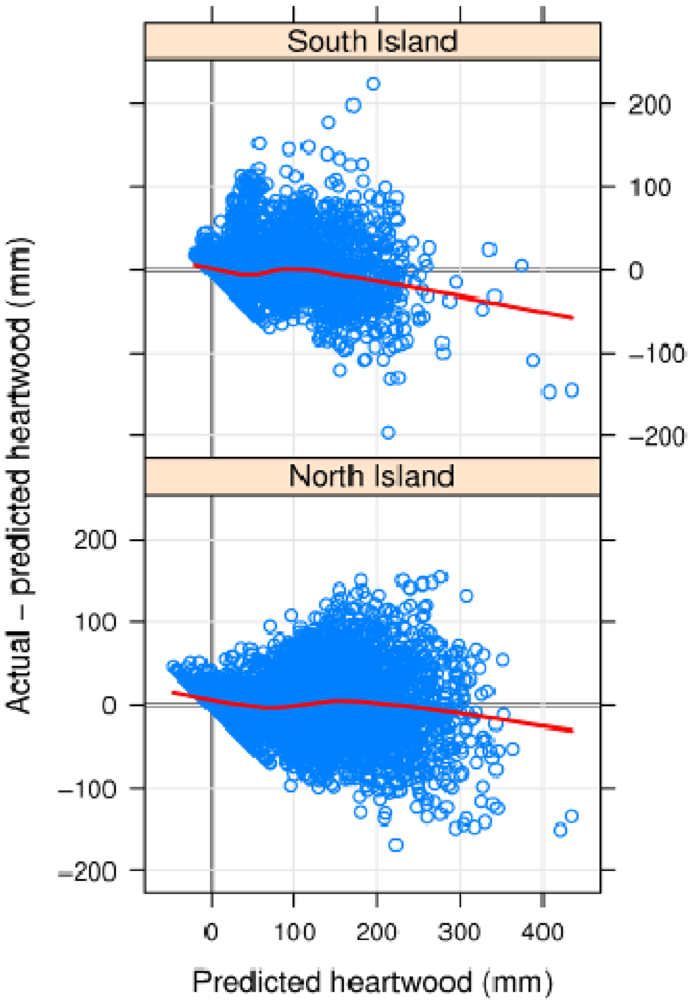


Figure 10 Predicted heartwood profiles for selected trees

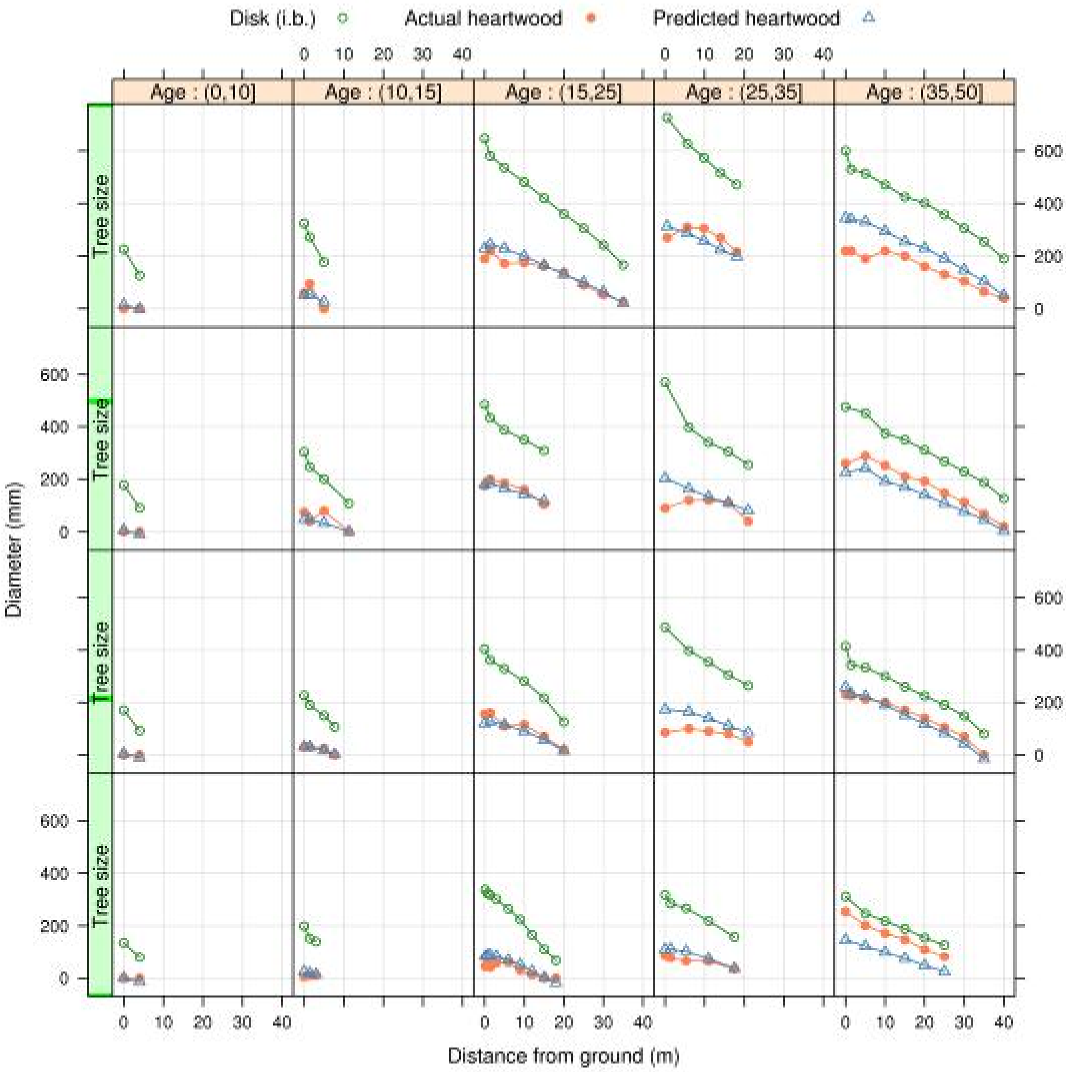
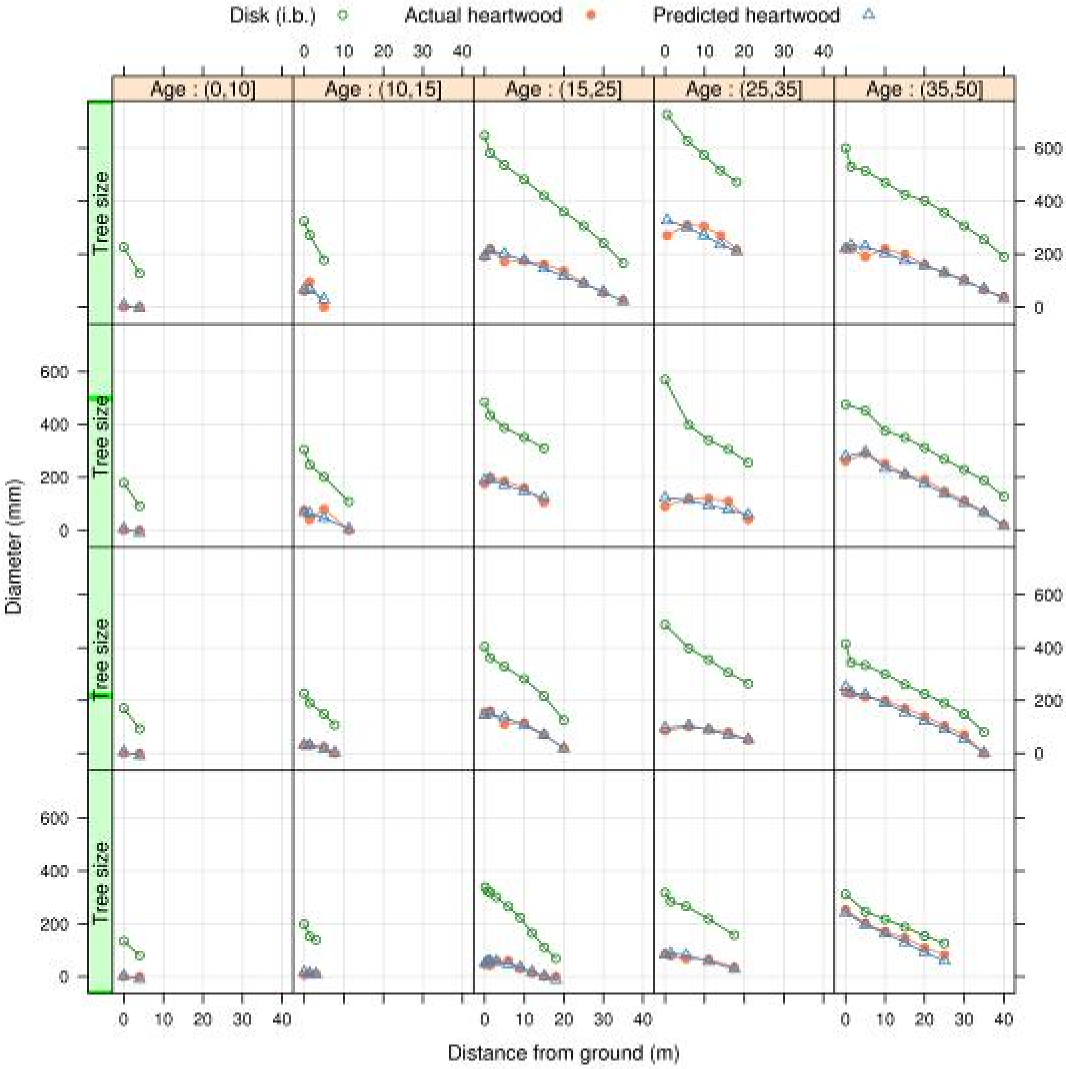


Figure 11 Predicted heartwood profiles with random effects included in prediction[1]



Heartwood Percentages

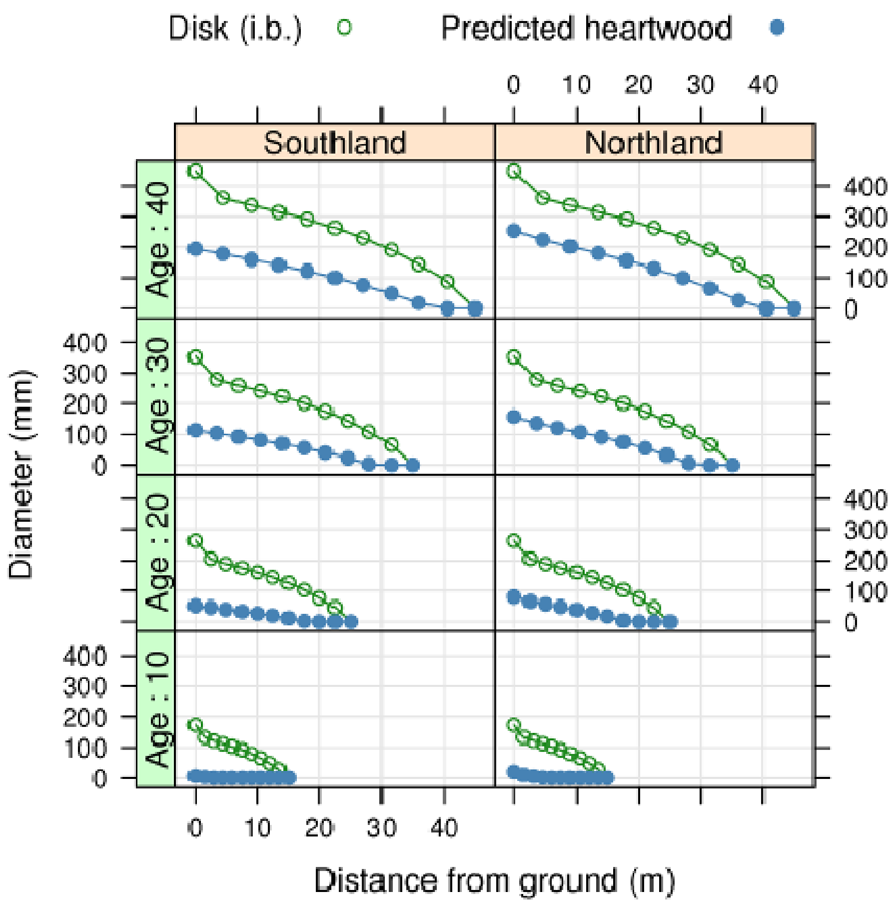
The model estimates heartwood diameter rather than heartwood cross-sectional area or heartwood proportion. As a consequence, the estimate of heartwood volume is slightly biased. Heartwood as a proportion of total volume can be approximated by calculating the total cross-sectional area of all heartwood and dividing by the total cross-sectional area of all disks. Using this method, the estimated heartwood is 14% of total volume and actual heartwood is 15% of total. The overall bias in estimation of heartwood volume is 1% of total volume or 8% of estimated heartwood volume. This bias will tend to be reduced by use of a stochastic model because gains on the upside more than offset losses on the downside.

Behaviour of Heartwood Model

Model behaviour was tested using all possible combinations of DBH from 50 mm to 1000 mm in 50 mm steps, height from 5m to 60m in 5m steps, distance from ground in 10% steps from 0% to 100% of height, age in 5 year steps from 5 to 60 and northing set at the equivalent of either Southland or Northland. Many of these combinations are unlikely to be found in the wild but some that might be are shown in Figure 12.

[1] This is cheating. See text.

Figure 12 Predicted heartwood for some sample trees



Wrongful Predictions

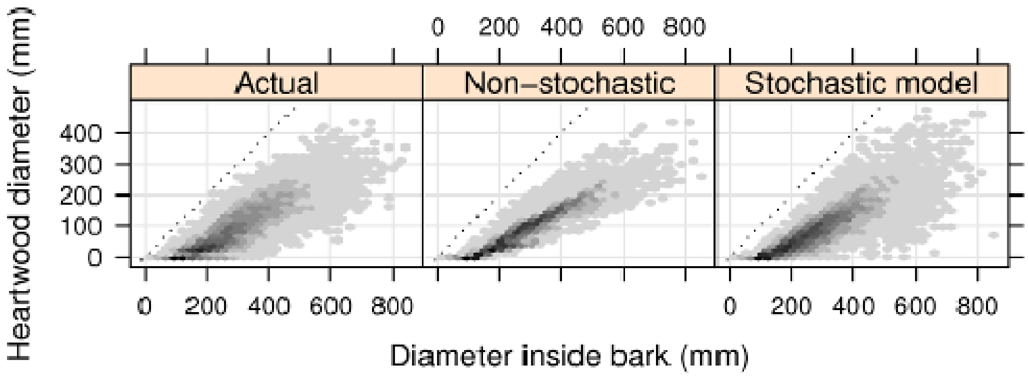
1. The model will predict heartwood diameters less than zero. When this happens predictions should be set to zero.
2. The model may predict non-zero heartwood diameters for trees that are aged less than 10, particularly if used stochastically. None of the disks at age 8 in the dataset had heartwood. The minimum age at which heartwood appeared was age 12 but there was no data between ages 8 and 12. In keeping with the theory that heartwood does not start forming at ground-level before age 10 [FRI Bulletin 50] I recommend a somewhat arbitrary limit of age 10 below which positive heartwood predictions are set to zero.
3. The model is not internally constrained to prevent predicted heartwood diameter exceeding inside-bark diameter and in extraordinary situations it may produce such deviant values. For example, near the top of a 5m high tree with a DBH of 100 mm at age 60 in Invercargill the heartwood is predicted to be greater than DIB. As DBH decreases below 100 mm, the age at which the heartwood pops through the cambium decreases. A simple and imperfect solution based on visual examination of Figure 4 would be to set the heartwood diameter to 90% of the inside-bark diameter if the prediction exceeds this value.
4. The slope of the relationship between heartwood diameter and inside-bark diameter continues to rise with age, with no limit. This will eventually lead to over-prediction of heartwood diameter. It is recommended that for ages greater than 50 the age be set to 50 in Equation 1.
5. Likewise, the values for northing used as input should be constrained to the range of mainland New Zealand.

Stochastic Model

Figure 13 compares the distribution of actual heartwood diameters against those predicted using the non-stochastic model (Equation 1) and the stochastic model (Equation 2). The stochastic model does not introduce heartwood predictions that are obviously wrong when compared to the distribution of actual heartwood diameters.

Predictions less than zero have been set to zero but the upper limit was not constrained.

Figure 13 Comparison of actual heartwood diameter with predictions from non-stochastic and stochastic models



Green Density Model

Two of the studies that provided heartwood data also provided disk green density (kg/m^3). These were:

- West Coast sites at ages 8, 16 and 25 [STR4], [STR6]
- Two Central North Island sites at ages 26-28 [STR27]

These studies provided paired green density and heartwood proportions for 404 disks on 186 trees, counted after removal of a small number of disks that exhibited extreme departure from the relationship between green density and heartwood proportion.

The relationship between green density and heartwood proportion can be modelled using a simple linear relationship;

Equation 3 Green density model

$$gd = \beta_0 + \beta_1 \left(\frac{hwd}{dib}\right)^2$$

Where:

hwd	=	Heartwood diameter (mm)
dib	=	Inside-bark diameter (mm)
β_0, β_1	=	Parameters to be estimated

Parameters were estimated using linear least squares and are presented in Table 4. There was no significant difference between studies or between ages, which is reassuring.

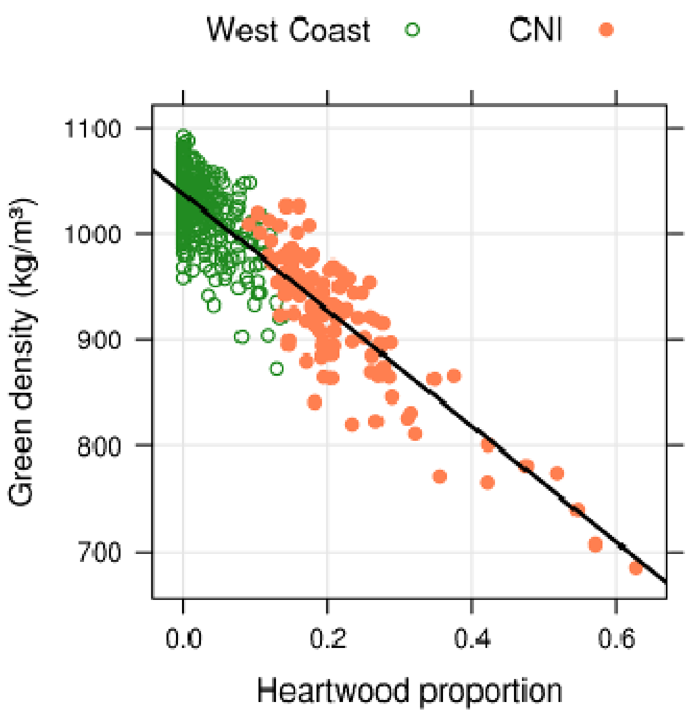
The coefficient of determination for the model (R^2) is 79% and the residual standard error is 30 kg/m^3.

Figure 14 provides a graphical depiction of the green density model.

Table 4 Parameter estimates for Equation 3

Parameter	Estimate	Term
β_0	##	Intercept
β_1	##	$\left(\frac{hwd}{dib}\right)^2$

Figure 14 Green density model



IMPLEMENTATION

The intention of this section is to provide clarification on implementation.

The two models were developed with the intention that they be used in conjunction with existing growth models and taper and volume functions to provide point estimates of heartwood diameter and green density on a stem. The point estimates can be converted to estimates of yield at the log level, both heartwood volume and green tonnes, for summarisation to per-hectare estimates.

Heartwood

Heartwood diameter is estimated using either Equation 1 for a deterministic estimate or Equation 2 for a stochastic estimate.

In the latter case the random components can be calculated as:

$$\gamma_1 0 = \#\# \, z_1 1$$
$$\gamma_1 1 = \#\# \, (\rho z_1 1 + \sqrt{(1 - \rho^2)} \cdot z_1 2)$$

Where:

ρ = ## (Table 3)

z_1, z_2 = Independent random numbers from standard normal distribution (mean=0,sd=1)

γ_0 and γ_1 are calculated for each tree but need only be calculated once for each tree.

Model inputs should be limited so that:

- If age > 50 then age = 50
- If northing > 6760000 then northing = 6760000
- If northing < 5380000 then northing = 5380000

Outputs should be limited so that:

- If hwd < 0 then hwd = 0
- If hwd > 0.9 * dib then hwd = 0.9 * dib
- If age <= 10 then hwd = 0

The heartwood volume of a log or other arbitrary stem section can be calculated by numerical integration of heartwood cross-sectional area calculated from heartwood diameter (hwd). There is no convenient analytical integration.

Green Density

Green density at a point on a stem can be estimated using Equation 3 with the heartwood diameter estimated as described in the previous section.
The total green kg between any two arbitrary points on a stem must be calculated using numerical integration. There is no convenient analytical integration.

CONCLUSION

The heartwood model was developed using a large data-set including data from ages 8-50 and most parts of New Zealand. The model explains 79% of the variation in the data and is reasonably robust in the sense that it is doesn't produce wild answers within the range of expected tree sizes and site conditions. The biggest weaknesses of the model arise from the data:

- The predictors are all correlated so that separating the exact effect of each is inexact. For example, the sample trees are older in the North Island than in the South Island.
- Figure 7 suggests that some locations will be under-predicted and some will be over-predicted but no attempt has been made in this project to quantify the average error that might be experienced if the model is applied in a specific location.

The green density model was based on a smaller subset of the data but explained 79% of the variation in green density in that subset and showed no signs that more data was needed; site and age were not significant predictors of green density.

Both models are good enough to be useful and are ready for implementation.

ACKNOWLEDGEMENTS

Thanks go to Russell McKinley and Dave Cown for assembling and providing the two FFR datasets and to Marco Lausberg for assembling the WQI datasets.

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