



REPORT

Vol. 13 No. 18 1988

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NEW ZEALAND

AN INTRODUCTION TO NETWORK ANALYSIS

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ABSTRACT

Network analysis techniques offer a relatively simple way of solving many complex problems in the management of forests. Computer models are easily and cheaply available to assist in the solution of variable cost problems and some models will cope with fixed costs as well. The technique does not require a formal mathematical background. This publication gives a brief introduction to the use of network analysis and uses two examples to emphasise various points.

INTRODUCTION

Efficient management in today's world requires the cost of actions and operations to be predicted with reasonable certainty. The selection of the cheapest parts of any proposed action does not guarantee that the optimum overall solution has been achieved. When faced with complex problems having many possible solutions the usual solution method is to use trial and error methods tempered with judgement gained by long experience.

This publication gives an introduction to the solution of complex inter-related problems using Network Analysis techniques on a Personal Computer. Network analysis is not the only way to handle such problems, but it is an easily understood method using a mix of graphical representation of the problem, mathematical analysis (performed by the computer), enhanced by good common sense to ensure that the solution arrived at is sensible.

The first appearance of network planning techniques for personal computers in the forest industry was in the late 1970's when Carson and Dykstra presented three programs for a Hewlett-Packard 9830 calculator. Unfortunately they were quite machine specific and in any case the availability of personal computers in the New Zealand forest industry has only reached the stage required to make such programs useful in the last two to three years.

In 1986 Dr J Sessions presented a workshop at LIRA which explored the available network analysis programs and gave some forest industry employees the background information needed to appreciate the potential of the technique. His program is available, in the United States and some New Zealand companies have purchased a copy for their own use. Demonstration copies are also available although naturally they are limited in the size of problem they are able to solve. Although the Sessions' program is specifically aimed at the "Landing to Mill" transportation problem, it is adaptable to other problems providing the operator has an understanding of the solution techniques. Sessions' program can account for fixed and variable costs on links, as well as discounting cash flows for multiple period sales.

Subsequent courses by FRI and LIRA have reinforced the potential of network analysis as a forestry planning tool, and as LIRA has available a computer program for use on IBM style personal computers, there is a need for a few notes on the background and pitfalls of the technique.

The LIRA programs which have been available to members at a nominal charge are not particularly user friendly although they are powerful and robust in operation. A later version of these programs using only variable costs and having easy interactive full screen input and editing should be available from LIRA in October 1988.

USES

Transportation problems and fluid flow problems have been the traditional areas for network analysis. These areas however, are by no means, the only areas to which the technique is useful. A sound understanding of the general principles of network analysis and also the features and shortcomings of the particular program in use, are essential if the best use is to be made of the program. Some programs allow upper and lower limits on the flow in network links while others do not. Other programs allow for multiple time periods. Increasing sophistication inevitably makes the program easier to use but less adaptable to other types of problem.

In the forest industry a wide range of problems can be solved, or at least partially solved, by the sensible use of Network analysis techniques.

MODELLING USING NETWORK ANALYSIS TECHNIQUES

Terminology:

As in any new area it is necessary to first understand the terms used to come to any understanding of the technique.

In Figure 1 I refer to a road transportation network. It is important to emphasise that this is only an example and in no way limits the use of network analysis to the road transportation area. In this figure major New Zealand North Island cities and towns are "nodes" which are the start and end points of "links". A collection of interconnected links between nodes makes up a network. A node in this type of

network is any feature that may be treated as the point of departure or destination of some path through a portion of the network. For example:- nodes will often be towns, cities, road intersections, changes in classification of the road, changes in gradient, bridges, or changes in the ease or cost of travelling.

In studying this simple example, it may be that the objective is to find the shortest route between Wellington and Auckland. Although in this example the solution from practical experience is obvious, it would be possible to solve the network for a number of criteria such as:-

1. shortest route
2. shortest time
3. least fuel used
4. least tiring to the operator
5. most scenic, etc,

provided that the costs/benefits associated with the links reflect the problem. It is also possible to easily re-run this network problem with changes in the link criteria. The ease of a re-run is one of the operating advantages of network analysis.

The usual aim in performing a network analysis is to find the minimum total cost path through the network. Various network models are available for other specific functions such as the longest path, and so on, but by far the most useful in solving the usual range of problems is the minimum total cost or maximum total value type.

The numbering system used in node identification is quite arbitrary, but the links are referenced by the nodes at which it begins and ends. Links are often bi-directional.

Often links will have different values associated with travel in opposite directions, even though they represent the same length of road. For example:- If the links have distance values, then link 1-2 is the same as link 2-1. If

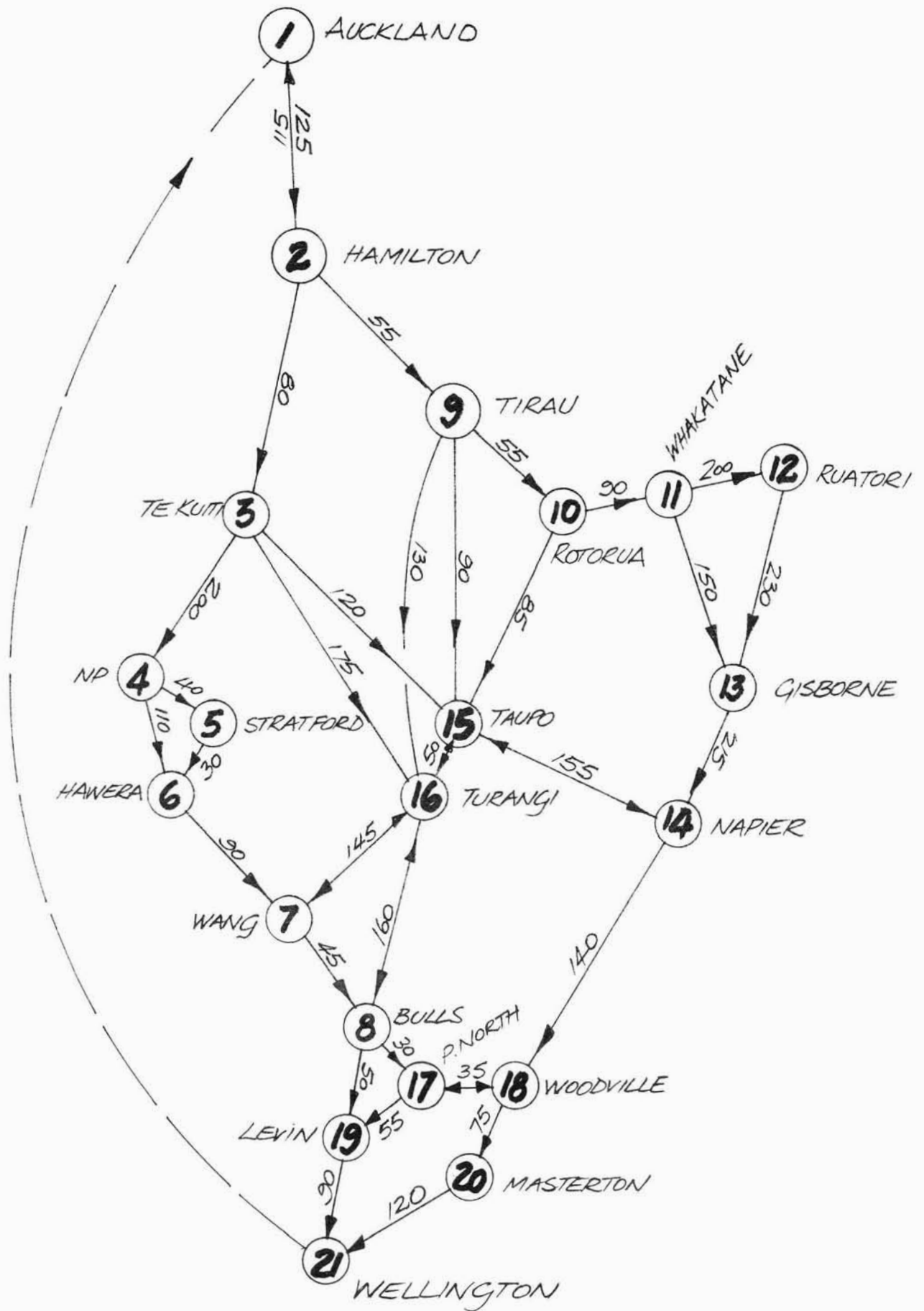


Figure 1 - Road Transportation Network

the value associated with the links is travel time then the link 1-2 may have a value of 125 whereas link 2-1 has a value of 115.

This distinction is important and useful when differentiating between the cost of hauling logs on a favourable grade in one direction and an unfavourable grade in the opposite direction.

Drawing the Network

Before any inputs are made to the computer (in any program available at the present time) it is necessary to

draw the network on paper. While this is being done it is sensible to put arrows on the links to show possible directions of flow and the costs associated with that flow. If constraints such as upper and lower bounds are allowed, this is the time to put them on also. A scheme should be adopted in this drawing and it is important that it be unambiguous and easy to follow, say in six months time when you may want to review the solution. My own scheme, which is recommended, is demonstrated in Figure 1. If there are upper and lower constraints they must be quite unambiguous and clearly not the flow or cost components.



Figure 2 - Labelling Scheme on Links

Note how the figures when being read right way up refer to flow from left to right. When flow is in only one direction there can be no ambiguity and liberties can be taken in the labelling scheme.

Computer graphics are generally not available on personal computer programs as both the complexity and memory

requirements are too high. Either before or after the drawing stage it is necessary to make a table of values and it makes sense to have the table drawn in the same format as the computer input requires it. For example:- If the problem is to find the shortest route from Wellington to Auckland then for the LIRA program the input suitable for the computer would look like Table 1.

*Table 1 - Input Table for the Network
in Figure 1*

ORIG	DEST	LOWER BOUND	UPPER BOUND	VAR COST

1	2	0	1000	125
2	3	0	1000	80
2	9	0	1000	55
3	4	0	1000	200
3	15	0	1000	120
3	16	0	1000	175
4	5	0	1000	40
4	6	0	1000	110
5	6	0	1000	30
6	7	0	1000	90
7	8	0	1000	45
8	16	0	1000	145
8	17	0	1000	30
8	19	0	1000	50
9	10	0	1000	55
9	15	0	1000	90
9	16	0	1000	130
10	11	0	1000	90
10	15	0	1000	85
11	12	0	1000	200
11	13	0	1000	150
12	13	0	1000	230
13	14	0	1000	215
14	15	0	1000	155
14	18	0	1000	140
15	16	0	1000	50
15	14	0	1000	155
16	15	0	1000	50
16	7	0	1000	145
16	8	0	1000	160
17	18	0	1000	35
17	19	0	1000	55
18	17	0	1000	35
18	20	0	1000	75
19	21	0	1000	90
20	21	0	1000	120
21	1	1	1	0

Many of the links in this table should have flow possible in both directions. In the interests of a compact table for this example I have only given a few links this feature and these are listed twice in the table eg. link 15-16 and link 16-15.

When drawing the network it is important to know if the computer input requires a closing or "feedback" loop from the destination to the start node. Some

programs automatically provide this closing link. The LIRA programs require them to be entered just as any other link. They are a necessary part of the setup for a solution. Without this link the program cannot find a closed network system on which to optimize the solution. If the program allows, it is a useful check to provide a realistic upper and lower limit on the flow in the closing link. The closing link may have flow constraints which force a flow in the feedback link. In Table 1 above for instance I have forced 1 unit of flow through the feedback link to make the solution to the problem give the total cost for 1 unit of flow.

It is also important to place all the known information on the drawing even though some may appear trivial or unnecessary. "The obvious solution won't include this link so I will leave it out" is not a good approach. The computer analysis will take care of the redundant links and it is important to keep all the possible elements of any problem available for the computer analysis.

AVAILABLE ALGORITHMS*

The shortest path algorithm is probably the most common available and can be expressed as minimum cost , time or distance. The other common algorithms such as optimal flow, maximum flow and so on, all have some distinct advantages for particular problems. The LIRA programs are based on the Out of Kilter algorithm which has as its features:-

1. It finds a feasible minimum cost flow
2. It is suitable for networks in which there are loops with a net negative cost.
3. It will attempt to find the shortest path which includes any link although this is not guaranteed. Exploratory examination though can usually tell if this is so.

* An algorithm is a rule or procedure used to solve a problem

4. It will find the maximum possible flow by assigning a very large upper bound on the closing link, a zero lower bound, and a large negative cost to the flow.

5. Super sources and super sinks can be applied to allow for multiple origin or destination networks.

6. Non Linear costs can be assigned to links as long as the total cost as a function of flow is a non linear convex function. A single link is replaced by a series of links with the appropriate cost associated. The out of kilter algorithm will assign flow to the least cost link first and then to the next cheapest and so on.

7. It is possible to have unbalanced flows at nodes. This feature is not normally used but may be useful for some special problems.

8. It lends itself readily to a sensitivity analysis as any changes do not mean a complete recalculation of the network. In large networks this can be a real advantage where computing time runs to many hours. The LIRA program does not at this stage take advantage of this feature.

9. It is useable with variable costs only on links to give an optimal solution.

10. It is possible by the use of various tricks to allow for fixed costs on any link although this is not implemented in the LIRA program. A commercially

available version will be marketed in New Zealand in late 1988 which has this feature.

In finding the shortest path (or least cost) most programs expect input in terms of cost. As a negative cost is equivalent to revenue, there is a simple mechanism to cope with both costs and returns in any problem.

Optimal Solutions

Most programs which are easily available can only cope with variable costs associated with any link. This would typically mean costs expressed as \$/m³ for flows expressed as m³, or \$/km when the links are in km.

Fixed costs associated with the particular link can be handled by converting them to a variable cost by taking the fixed cost and dividing it by the expected flow. This of course is fine if the flow in fact goes through the link but if it doesn't then that link has been unduly penalized in the calculation of the optimal path. Sessions' program takes this in to account by including the variabilisation of fixed costs in the program. The inclusion of fixed costs in any network analysis algorithm changes the solution from optimal to one that is merely feasible. Often the solution is near optimum but some care is required to ensure that the solution from this type of program is sensible. An earlier LIRA network program allowed for fixed costs on any link but considerable trial and error in assigning weighting factors was required and care was needed in interpreting the output.

Table 2 - Sample Output for the Problem

ORIG	DEST	LOWER BOUND	UPPER BOUND	VAR COST	FLOW	ACCUM COST
1	2	0	1000	125	1	125
2	9	0	1000	55	1	180
9	16	0	1000	130	1	410
16	8	0	1000	160	1	570
8	19	0	1000	50	1	620
19	21	0	1000	90	1	710
21	1	1	1	0	1	710

The usual form of output from network problems is similar to that shown in Table 2. The output from some programs show all links regardless of flow or not. The output from programs allowing fixed costs to be associated to links may be slightly different again, but the basic elements of the above example will always be present.

After the solution has been found it is often desirable to highlight the optimal route on the original diagram. It is always essential to make sure that the

solution given by the computer makes sense. While there should be no problems with an optimal solution algorithm the use of fixed costs and the associated trial and error solution makes careful checking of these type of solutions essential.

A FURTHER EXAMPLE

Below is a simpler example with a forestry bent. It is the example used by Carson and Dykstra in their original paper and is presented here in the same format.

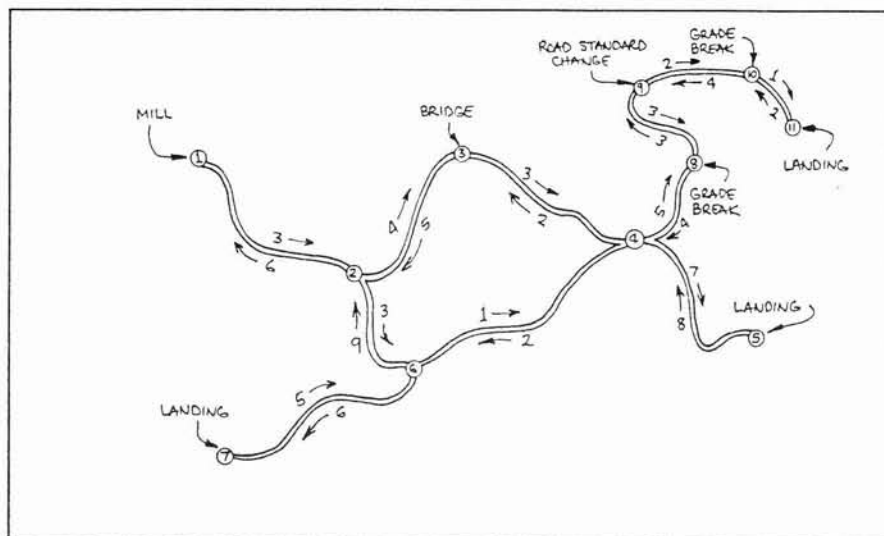


Figure 3 - Source map for Example 2

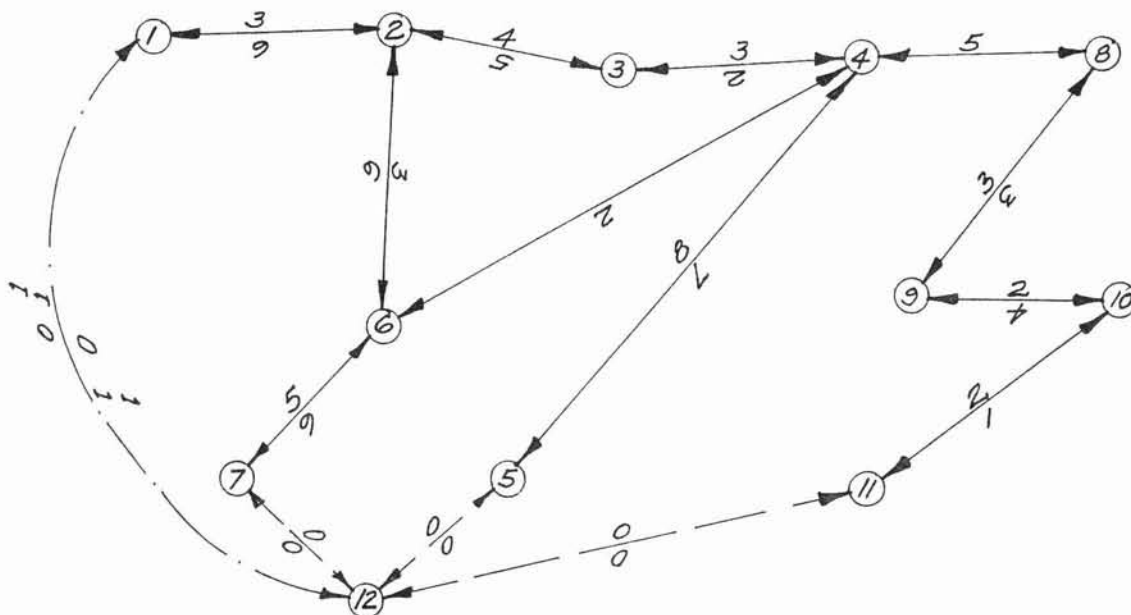


Figure 4 - Network Diagram

Table 4 - Input and Output for
Example 2

INPUT INFORMATION				
ORIG	DEST	LOWER BOUND	UPPER BOUND	VAR COST
1	2	0	100	3
2	1	0	100	6
2	6	0	100	3
6	2	0	100	9
2	3	0	100	4
3	2	0	100	5
3	4	0	100	3
4	3	0	100	2
4	5	0	100	7
5	4	0	100	8
4	6	0	100	2
6	4	0	100	1
6	7	0	100	6
7	6	0	100	5
4	8	0	100	5
8	5	0	100	4
8	9	0	100	3
9	8	0	100	3
9	10	0	100	2
10	9	0	100	4
10	11	0	100	1
11	10	0	100	2
7	12	0	100	0
12	7	0	100	0
5	12	0	100	0
12	5	0	100	0
11	12	0	100	0
12	11	0	100	0
1	12	1	1	0

OUTPUT FROM THE PROBLEM

ORIG	DEST	LOWER BOUND	UPPER BOUND	VAR COST	ACCUM COST
2	1	0	100	6	6
3	2	0	100	5	11
4	3	0	100	2	13
6	4	0	100	1	14
7	6	0	100	5	19
12	7	0	100	0	19
1	12	1	1	0	19

This solution is consistent with Carson's original one. The algorithm used is not the same as Carson used and this network diagram required a closing link and the addition of a "super skid" to ensure a balanced flow was obtained in the network.

In this example the super skid 12 represents the total flow of wood through landings 5, 7 and 11, and hence to the mill. It allows the feedback link 12-1 to be included and balances the total wood supplied with that arising from the various landings.

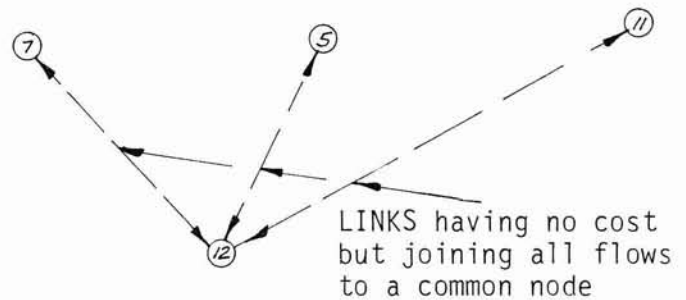


Figure 5 - A "Super Skid"

REFERENCES

Carson, Ward W; Dykstra, Dennis P - (1978) : "Programs for Road Network Planning US Forest Service", General Technical Report PNW-67.

Sessions, J (1985) : "Network Analysis Using Microcomputers for Logging Planning, in Improving Mountain Logging Planning Techniques and Hardware" Vancouver Proceedings p 87 - 91.

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