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Developing fully compatible taper and volume equations for all stem components of *Eucalyptus globoidea* Blakely trees in New Zealand

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Abstract

Background: Individual-tree taper and volume equations are essential for forest management. They provide estimates of volume that are incorporated into plot-level volume equations and also into growth and yield models to estimate volumes per hectare in forest crops. Moreover, taper equations allow forest managers to estimate dimensions of logs that can be cut from stems in their forests when they have measured diameters at breast height and heights of trees in inventories. Compatible taper and volume equations have the property that the same individual tree volume can be estimated either from the tree volume equation or by integrating the taper equation. Durable eucalypt species such as *Eucalyptus globoidea* Blakely, however, have especially valuable heartwood and so managers require estimates of the volumes and shapes of heartwood zones within trees. Simple overall wood taper and volume equations would therefore be inadequate.

Methods: 74 *Eucalyptus globoidea* trees were destructively sampled in 8 different trial plots throughout New Zealand. Tree ages were 7 to 29 years old, the DBHs were 11 to 67.6 cm and the heights were 7.2 to 35.4 m. All trees were felled and lengths and taper diameters outside bark were measured. To measure taper diameters of heartwood and sapwood, discs were cut at irregular intervals along the stems. Heartwood and sapwood components were identified by applying methyl orange dye and quantified using image analysis. In this study we extended compatibility so that sums of estimated volumes of separate components of stems, bark, sapwood and heartwood, would equal overall tree volume estimates. In addition, taper equations were made for outside bark, inside bark and heartwood that were compatible with their respective volume equations. Parameters of five volume equations for whole stem, whole wood, bark, sapwood, and heartwood were simultaneously estimated. Compatible taper equations for whole stem, stem wood and heartwood were estimated so that they were compatible with the volume equations, thereby creating a fully compatible system.

Results: Root mean squared error (RMSE) of volume models were 0.1248298 m^3 for heartwood, 0.05496807 m^3 for sapwood, 0.1539625 m^3 for wood inside bark, 0.1108201 m^3 for bark and 0.1439786 m^3 for wood including bark. RMSE of taper models was 2.572765 cm for heartwood, 2.348552 cm for wood inside bark and 2.474088 cm for wood including bark.

Conclusions: A compatible system of multiple taper and volume equations can be fitted through the simultaneous fitting of parameters with minimal bias and precision levels of $\pm$ 0.055-0.154 m^3 for volume equations and $\pm$ 2.35 to 2.57 cm for taper equations. Leave-one-out cross-validation of the fitted models yielded very similar levels of precision and bias to those encountered when fitting models with the entire dataset.

Keywords: Volume, taper, mensuration, *Eucalyptus*, heartwood

Introduction

A plan to diversify New Zealand's planted forests by expanding the establishment of *Eucalyptus* species in dryland areas requires information for decision-making based on data from forest inventory and techniques to evaluate these data. Information obtained from forest inventories is required so that forest managers can understand current growing stock and growth potential. These data can be used to provide accurate and precise estimates of volume production per tree, log types, and, in association with stand inventories, stand value (Casnati 2016). Taper and volume equations can provide diameter estimates at any point on a stem, height estimates at which a given diameter occurs along the stem, total stem volume, merchantable volume to any merchantable height and individual log volumes (Methol 2001). This information is crucial as it can be used to determine the wood products and timber that can be produced from forest stands. Taper and volume equations can provide estimates of tree stem volumes and shapes with input data of only tree height and diameter at breast height.

Tree stem volume

Measurement of stem volume describes the amount of wood in a tree's stem or, collectively, in a stand of trees. The unit of measurement is cubic meters (m^3) or cubic meters per hectare (m^3/ha). Volume can be calculated in two variants: over-bark or under-bark (Avery & Burkhart 1994). The stem volume of the individual standing trees is characterised as shown in Equation 1.

$$v = \frac{1}{4}\pi\, d^2\, h\, f \tag{1}$$

where, v is calculated stem volume, d *is* diameter at breast height (1.4 m), h is height of the tree and f is a form factor.

Volume is usually estimated from equations with diameter at breast height (DBH), DBH and height, or DBH, height and an upper-stem diameter as predictor variables (Van Laar & Akça 2007). The shape of a tree bole or stem can be estimated approximately with three mathematical solids: a conoid, a paraboloid or a neiloid. However, to provide better volume results Equations 2, 3 and 4, are specific to different parts of a tree stem shape (Vanclay 1994):

Conoid $$v = \frac{\pi}{12}\, l\, (d_1^2 + d_2^2 + d_1 d_2) \tag{2}$$

Paraboloid (Smalian) $$v = \frac{\pi}{8}\, l\, (d_1^2 + d_2^2) \tag{3}$$

Neiloid $$v = \frac{\pi}{16}\, l\, (d_1^2 + d_1^{\frac{3}{2}} d_1^{\frac{1}{2}} + d_1^{\frac{1}{2}} d_1^{\frac{3}{2}} + d_2^2) \tag{4}$$

where, v is calculated stem volume, l is length of the tree section, d_1 is the large-end diameter of the section, d_2 is small-end diameter of the section.

Tree stem taper

Measurement of tree stem taper describes the shape of the tree bole, which influences volumes of specific products from a forest (Gomat et al. 2011). The most challenging part of developing a taper equation is to describe changing shapes along the stem. The first attempt in this area was conducted at the beginning of the 20th century and since then significant progress has been made (Loetsch et al. 1973). To find the best taper equation fit, different factors should be taken into consideration, including tree species, available data, tree size classes, climatic conditions and region (Li et al. 2012). The long history of compatibility between taper and volume equations began in the early 1970s, when Demaerschalk (1971, 1972) published papers on converting individual tree volume equations to compatible taper equations. The advantage of this system is that volume can be calculated by both the tree volume equation or by integrating the taper function, with both methods providing the same answer (Casnati 2016). Compatible taper and volume models are widely used in the forestry sector (Brooks et al. 2008; Cao et al. 1980; Fang et al. 2000; Jiang et al. 2005; Jordan et al. 2005; Özçelik & Brooks 2012).

Heartwood and sapwood components of tree stems

Heartwood and sapwood are two components within a tree's stem. Heartwood is the central core of the tree stem while sapwood describes the newer growth rings being found between the heartwood and the bark. The sapwood transports water and minerals upward while phloem transports photosynthates downward between leaves and other components of the tree. Sapwood is lighter, often less durable, softer, and contains more moisture compared to heartwood. As trees age and new sapwood rings are formed, older sapwood changes into heartwood (Bamber 1961). Heartwood is an inner layer of the wood, which does not contain living cells (Taylor et al. 2002). Heartwood is naturally strong, often durable and more resistant to deterioration by insects and microorganisms than sapwood (Taylor et al. 2002). Heartwood has many important functions in living trees. It provides structural support for a tree, although it does not differ structurally from the original sapwood. Significant radial strength differences between sapwood and hardwood are the result of radial changes in wood density and cell wall ultrastructure (Panshin 1980). As the heartwood is newly formed it recycles nutrients back to sapwood (Bamber 1961) while building up anti decaying substances (Stewart 1966) which provides its natural durability (Shain 1995). In some species heartwood can be distinguished from sapwood by its darker colour, lower permeability, different moisture content and increased decay resistance (Taylor et al. 2002). Heartwood, due to its relatively high density and durability, is used to produce furniture, flooring, roofing and other outdoor applications. Sapwood, due to its lower density and non-durability, is often used as pulp or for indoor applications.

Durable heartwood is a valuable commodity and an important timber component in the wood industry. Due to its natural durability, it can be a significant supplement and potential replacement for non-durable woods treated with toxic preservatives. Estimating the amount and shape of the heartwood in a tree stem is challenging due to fact that it is an internal part of the tree and cannot be measured from the outside without special tools or destructive intervention within tree (such as collecting discs or cores). For growers of *Eucalyptus globoidea* Blakely, which grows heartwood rated at class 2 durability (Bootle 1983; Nguyen et al. 2020), estimation of dimensions of heartwood zones within stems is crucial for estimating product types and financial value.

Objective

The objective of this study was to develop compatible taper and volume equations for *Eucalyptus globoidea* to estimate the following:

- stem taper from DBH and height,
- stem volume from DBH, height, and taper,
- heartwood taper and volume from DBH and height.

There are five volume components; heartwood, sapwood, wood inside bark, bark, and wood including bark and three taper components; heartwood, wood inside bark, and wood including bark. In Brown's (2019) study the pattern of heartwood development was examined in relation to several tree characteristics. Cross-sectional discs of northern red oak *(Quercus rubra* L.) were analysed to detect changes in heartwood radius. The study showed that age of the tree, DBH size class, tree height, and inside bark radius were significantly related to heartwood diameter. In the study described here a similar cross-sectional analysis approach was applied, however the context of research was extended to five tree components, including heartwood. The goal of this study was to create compatibility within taper and volume models for all components of stems of *Eucalyptus globoidea*. Moreover estimates of heartwood and sapwood volume combined produce the same estimate as wood inside bark, and estimates of wood inside the bark and bark combined produce the same estimate as wood including bark. Integrating these taper equations produces identical volume estimates to those obtained with tree volume equations. Compatibility at this level, where all stem volume components depend on each other, is a new approach to taper and volume modelling.

Methods

Study sites

New Zealand is situated between 34°S and 47°S, between the South Pacific Ocean and Tasman Sea, and comprises three main islands: the South Island (151,215 km^2), the North Island (113,729 km^2), and Stewart Island (1,746 km^2) (Moot et al. 2009). The total land area is almost 270,000 km^2 (McKinnon et al. 1997). Mean annual temperatures in New Zealand range from 10°C in the south to 16°C in the north. The warmest month is usually January or February and the coldest month is usually July. Most areas of New Zealand have between 600 and 1600 mm of rainfall with increased rainfall during the winter months, although some extreme areas have rainfall as high as 4000 mm/annum (Mackintosh 2001). The desired area for future large-scale *Eucalyptus* plantations is located in the dryland areas of New Zealand, which are spread along the east coasts of the North and the South Islands. Drylands are defined and characterised by deficiency of water; over the long term, natural moisture inputs such as precipitation are outweighed by moisture losses through evaporation from surfaces and transpiration by plants. This potential water deficit affects both natural and managed ecosystems, and constrains the production of crops, forage, and other plants and trees (Safriel et al. 2005). To study growth and species acclimatisation in these areas, multiple trial plantations of *Eucalyptus globoidea* were established in different years throughout the dryland areas of New Zealand. For this study, data were collected from eight different study sites, as shown in Figure 1.

The climate data presented in Table 1 came from Land Environments of New Zealand (LENZ). The data represent the year 2009 and were mainly obtained from summaries of climate observations published by the New Zealand Meteorological Service.

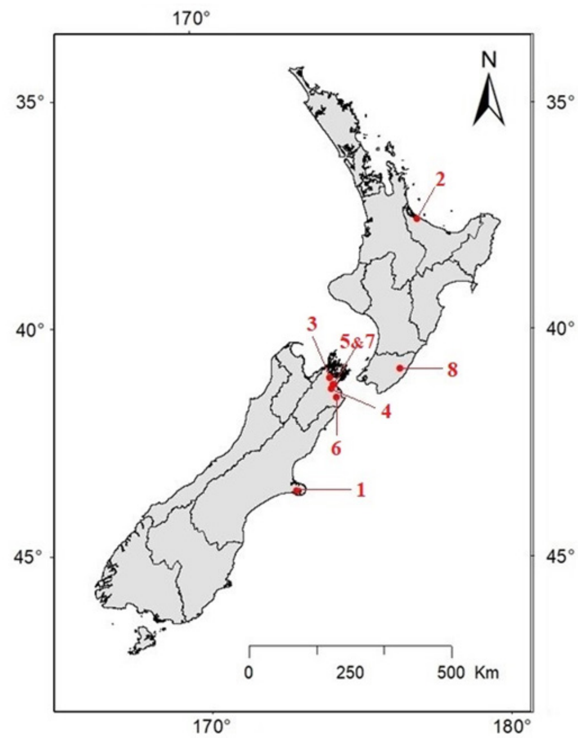


FIGURE 1: Locations of study sites in New Zealand (red dots). Scales are in latitude and longitude.

TABLE 1: Description of the study sites includes location and climate data, which were sourced from the Land Environments of New Zealand (LENZ 2020).

Site No.	Establishment Date	Plantation Age (as at 2019)	Location	Coordinates	No. trees sampled	Mean annual temperature (°C)	Mean minimum temperature of the coldest month (°C)	Mean annual solar radiation (MJ/m²/day)	Winter solar radiation (MJ/m²/day)	Annual water deficit (mm)
1	1990	29	South Island, Canterbury, Okuti Valley	43°47’32.0”S 172°50’20.8”E	28	11.7	3.4	13.8	4.5	176
2	1998	22	North Island, Bay of Plenty, Welcome Bay, Tauranga	37°43’55.8”S 176°13’20.9”E	13	14.1	5	15.1	6.0	18
3	2003	16	South Island, Marlborough, Havelock	41°17’27.9”S 173°48’56.0”E	1	10.6	0.5	15.1	4.7	5
4	2006	13	South Island, Marlborough Fairhall	41°32’29.2”S 173°52’20.5”E	5	12.5	2.2	15	4.9	274
5	2009	10	South Island, Marlborough, Lower Wairau Valley	41°26’32.7”S 173°56’07.6”E	8	12.5	1.7	15.1	4.8	231
6	2009	10	South Island, Marlborough, Seddon	41°43’08.4”S 174°02’02.4”E	5	12.2	2.9	14.9	5.5	222
7	2010	9	South Island, Marlborough, Lower Wairau Valley	41°26’31.7”S 173°56’17.5”E	4	12.5	1.7	15.1	4.8	231
8	2011	7	North Island, Stronvar, Masterton, Wellington region	41°02’42.7”S 175°52’37.2”E	10	12.2	3.7	14.0	4.6	107

Data

Tree-selection standards

To select study trees on each site, preliminary assessments were conducted to find a diverse range of combinations of diameter at breast height (DBH) and height, among the same aged groups. They were based on two measurements: the DBH at 1.4 m height, measured by a DBH tape, and the height of the tree measured by a Vertex IV Hypsometer (Haglöf, Sweden). The most important criteria for tree selection were straight stems, lack of stem damage, absence of forking, and no epicormic branches. The selected study trees had straight stems without any deformation (Figure 2). Before felling, each tree was spray painted in two places: at breast height around the stem at a height of 1.4 m, and in a straight line from the ground up to the DBH, which was needed later for correct orientation of the discs. After this preparation, trees were felled. 74 *Eucalyptus globoidea* trees were destructively sampled across the 8 trial sites throughout New Zealand. Tree age varied from 7 to 29 years old with mean value of 19. DBH varied from 11 to 67.6 cm with mean value of 32.3 cm and standard deviation of 15.1 cm. Height varied from 7.2 to 35.4 m with mean value of 19 m and standard deviation of 8.2 m. Total volume varied from 0.036 to 4.859 m^3 with mean value 0.88 m^3 and standard deviation of 0.82 m^3.

Measurement of felled stems

The first step after felling was to measure the length of the stem using a tape measure. The tape was extended along the stem, from the base of the felled stem to the tip of the apically dominant leader in a straight line, and was pulled tightly before taking the final measurement. Then, the height of the stump was measured by ruler. In case of sloping land the stump measurement was taken from the highest ground level. The total height of the tree was obtained from the sum of the felled stem length and the height of the stump.

The next step was to measure diameters along the stem from the bottom to the top including the measurement from the stump, located as close to ground level as possible. Two diameter measurements at 90-degree angles were obtained using a calliper and repeated at defined increments along the stem up to the tip of the apically dominant leader. All diameter measurements had corresponding height measurements obtained from the tape extended along the stem. Diameter measurement points were spray painted along the stem so the painted markings were visible for subsequent disc cutting (Figure 3).

Distances between stem diameter measurements, called the taper step distance, were established depending on the diameter (D) of the stem. Where D ≤ 25 cm, stem diameter was measured at 2.5 cm taper increments; where 25 cm < D < 50 cm, stem diameter was measured at 5 cm taper intervals; and where D ≥ 50 cm, stem diameter was measured at 7.5 cm taper intervals.

The taper step distance was treated as a guideline for measurements. During fieldwork the taper step distance was adjusted, particularly near the base of the felled stem, and towards the tip of the apically dominant leader to find the exact height at which the heartwood ended. In Brown's (2019) study of northern red oak, the author produced satisfactory results for heartwood taper based on 5-6 discs per tree. In the study reported here there were 993 diameter measurements collected from 74 trees, resulting in a mean value of 13 diameter measurements per tree.

Bark measurement

Bark of *Eucalyptus globoidea* is flaky and has a tendency to fall off, especially for older trees and lower trunk sections. For this reason, bark measurements were taken before cutting discs from felled stems, or using a ruler just after cutting stem sections. For every disc, four bark measurements were collected at intervals of 90 degrees around the stem in the same orientation.

FIGURE 2: Examples of selected study trees from the trial plot located in the Okuti Valley, Canterbury.

FIGURE 3: Paint marks showing the diameter measurement points along a sample log.

Disc collection and preparation

After collection of diameter measurements along the stem, the next step was to collect tree discs corresponding to the location of each diameter measurement. The process of selecting discs within a tree was based on taper, which is the most commonly used procedure for similar kinds of studies (Ojansuu & Maltamo 1995). The first disc was cut from the stump; after felling the tree chainsaw operator cut a disc from the remaining stump. Subsequent discs were cut from the felled log at the spray paint marks, where the previous taper measurements had been made (Figure 4). The disc thickness was between 3–5 cm. To keep the correct order every disc was labelled with a unique number after being cut. During the initial stage of research the discs were transported to the wood technology laboratory at the University of Canterbury and analysed there in a green state. However, due to transport logistics, some of the discs where dyed and measured in the field.

To help with heartwood and sapwood identification, an indicator dye was applied with a brush to the discs to dye the heartwood pink and the sapwood bright yellow. The indicator dye was 1% methyl yellow diluted in 40% ethanol, which after correct preparation is clear orange. Colour differences in the indicator dye arise from differences in pH between sapwood and heartwood. The methyl yellow indicator turns pink below pH 2.9 and yellow above pH 4.0 (Tawarah & Abu-Shamleh 1991). When both heartwood and sapwood were visible, the next step was to take photographs. All discs were placed on a level surface, in order. Each disc was photographed in good light to minimise shadow and under/overexposed areas. Every image included a ruler to scale image size in subsequent steps (Figure 5).

Data pre-processing - Image analysis

Images of the discs were analysed using ImageJ software version 1.52a (Rasband 2018). The images were loaded into the software and analysed individually. The first step was to set a scale for the image. By using a "set scale" tool, the software calculated the area occupied by a pixel based on the scale visible on each image. With this information, the software calculated any pixel unit and converted it to the unit of interest, such as cm or cm^2. The next step was to manually select the area of heartwood, sapwood bark circuit using a "polygon selection" tool. After completing each circuit, the area inside it was calculated in cm^2 using the "analysis measure" tool. From the area measured in cm^2, the diameter and radius of the different tree components were calculated and used for the subsequent analysis.

Volume modelling

In this study, volumes of five different tree components were modelled: heartwood, sapwood, wood inside bark, wood including bark, and bark. To calculate sectional volumes of heartwood, wood inside bark, and wood including bark components the Smalian formula (Equation 3) was used. To calculate sapwood and bark volume, simple subtractions of the above calculated components were applied:

sapwood volume = wood inside bark volume – heartwood volume;

bark volume = wood including bark volume – wood inside bark volume.

At the beginning of the modelling process it was important to ensure that all variables were normally distributed. Modelling was attempted with a variety of dependent variables, including tree diameter squared times height, but after preliminary evaluation of bias, each volume component was modelled as a function of tree diameter times height (dh) with both volume components and dh was transformed using scaled power transformations (Sakia 1992) (Equation 5), with λ values chosen to make distributions of values as normal as possible. Normality of frequency distributions of correlated variables makes relationships between them as linear as possible, and generally helps to avoid heteroscedasity of residuals if dependent variables are transformed.

FIGURE 4: Cutting a disc from the sample stem with visible "wavy" thick bark structure typical of *E. globoidea* at this site.

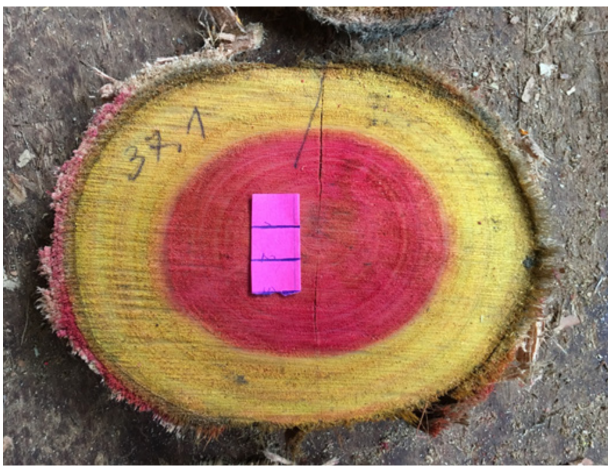


FIGURE 5: An example of a disc with the visible scale after applying dye.

$$x^{(\lambda)} = \begin{cases} (x^{\lambda} - 1)/\lambda & \lambda \neq 0 \\ log(x) & \lambda = 0 \end{cases} \quad (5)$$

where, $x^{(\lambda)}$ is the scaled power transformation of x, λ is a parameter that defines the curvature of the relationship between x and $x^{(\lambda)}$.

After applying transformations, the following assumptions were fulfilled:

- the Y values could be expressed as linear functions of X values;
- variation of observations around the regression line was homoscedastic;
- for given values of X and Y, error values were normally distributed.

The next step was to run a linear regression using command lm in R (R Core Team 2021) for each tree volume component separately, to determine the starting values for a non-linear regression. A non-linear regression was required when all models were fitted simultaneously because all transformation terms had to be on the right hand sides of equations to ensure additivity of stem components. The obtained starting values were used, in the next step, as the starting coefficients for non-linear models (nls) (Hamann et al. 2007), for each tree volume component separately. Both sides of volume equations were weighted by 1/dh (tree breast height diameter times height) which ensured that the residuals of small values were not biased and the impacts of heteroscedasity on bias for small estimates were reduced by the weighting. Right hand sides of equations were back-transformed during fitting so that component volume predictions could be made compatible, and so this weighting was necessary to avoid bias.

The next step was to create compatible volume equations. The following dependencies were created between heartwood, sapwood, wood inside bark, bark, wood including bark:

- volume of wood inside bark = heartwood volume + sapwood volume
- volume of wood including bark = volume of wood inside bark + bark volume

The R library systemfit (Hamann et al. 2007) and command nlsystemfit was used to fit equations to the data simultaneously using the Ordinary Least Squares method (OLS). Different systemfit methods were considered in this study (Weighted Least Squares (WLS), Seemingly Unrelated Regression (SUR), Two-Stage Least Squares (2SLS), Weighted Two-Stage Least Squares (W2SLS) or Three-Stage Least Squares (3SLS). However the OLS method converged more easily and effectively than other methods.

Taper modelling

Taper modelling of heartwood, wood inside bark and wood including bark was based on Demaerschalk's (1971, 1972, 1973) method for estimation of tree taper and volume. The idea is to create compatibility between taper and volume equations. Taper and volume equations are defined as compatible when integration of the taper equation yields the same total volume as that given by the volume equation (Demaerschalk 1973). Taper equations were polynomial (Equations 8-11). The constraint is that the coefficients of the taper equation fit must be restricted so that they sum to one. To solve it in the non-linear fitting procedure one of the coefficients was set to one minus the other coefficients.

Heartwood height prediction

Predicting the height at which heartwood stops is important for applying the heartwood taper model. To predict the heartwood height a linear mixed effects model was executed by the lme function from R package nlme (Pinheiro et al. 2019). The heartwood height prediction was modelled based on the correlation between heartwood height and tree height. No transformation was applied as there was a linear relationship between those variables.

Validation

The leave-one-out cross-validation technique was used to validate taper and volume models. To find estimators of parameters, each observation has to be systematically left out from the dataset to calculate estimates and fit new models, and then residuals for the record left out are computed. This technique is a cross-validation method (Refaeilzadeh et al. 2009).

Due to the large number of observations and the complexity of the compatible taper and volume fitting process, a loop was created in R to obtain new models with each successive tree left out, and to compute residual values for that tree. Each time from the dataset of 74 trees, one tree was excluded and the analyses were conducted using the 73 remaining trees.

Statistical interpretation

All taper and volume models were compared statistically with root mean squared error (RMSE), mean absolute bias (MAB), and model efficiency (EF).

$$RMSE = \sqrt{\frac{\sum(Y-Y')^2}{N}}$$

$$MAB = \frac{\sum(|Y-Y'|)}{N}$$

$$EF = 1 - \frac{\sum(Y-Y')^2}{\sum(Y-\bar{Y})^2}$$

Where: N = number of observations, Y = observed value, Y' = expected value, $\bar{Y}$ = overall mean.

Results

Volume modelling

The volume equations were all of the form shown in Equation 6:

$$\frac{V_i}{dh} = \frac{\left(\lambda_i \ a_0 + \ a_1 \left(\frac{dh^{\lambda dh}_{-1}}{\lambda_{dh}} \right) + 1 \right)^{\left(\frac{1}{\lambda_i}\right)}}{dh} \quad (6)$$

where, a_0 and a_1 are fitted coefficients, λ_i is value unique to each variable, *i* is the heartwood, wood inside bark or wood including bark component.

Parameters, λ values for each volume component and standard errors are shown in Table 2. Wood inside the bark and wood including bark values were obtained using initial non-linear regression; the parameters were used as the starting values in systemfit modelling. Wood inside the bark and wood including bark were created as simple sums of tree components in the systemfit model: heartwood + sapwood = wood inside bark, and wood inside bark + bark = wood including bark. Plots of residuals are presented in Figure 6.

Graphs in Figure 6 present residual vs fitted values. Residuals were also plotted against explanatory variables. Overall the residuals were well distributed in the volume range and all models converged without difficulty. As can be seen, back transformed residuals are heteroscedastic, but transformations and weighting used during fitting ensured that relationships were relatively unbiased and that residuals for small predictions had similar impacts on fitting to those of large predictions. Back-transformed equations were required so that volume estimates could be summed to ensure compatibility. The residual range around the x axis varied from -0.15/0.15 m^3 (for sapwood) to -0.6/0.6 m^3 (for wood including bark). The rest of the components were between -0.4/0.5 m^3 (heartwood), -0.4/0.6 m^3 (wood inside bark), and -0.3/0.5 m^3 (bark).

Model validations showed there were only small differences between leave-one-out cross-validation techniques and full model fits. Both methods yielded normal residuals.

A summary of fitting statistics for five compatible volume equations is presented in Table 2. Differences between the components were small. The RMSE value varied from 0.05496807 m^3 (for sapwood) to 0.1539625 m^3 (for wood inside bark). The MAB value varied from 0.0382446 m^3 (for sapwood) to 0.08887412 m^3 (for wood inside bark). The EF value varied from 0.805 (for sapwood) to 0.971 (for wood including bark).

Heartwood height

Height of the heartwood was found to be a linear function of tree height. The residuals versus fitted values of heartwood height prediction are visible in Figure 7. The model is presented in Equation 7.

$$hht = -6.2994 + 1.1208\ h \quad (7)$$

where, *hht* is heartwood height, *h* is tree height, -6.2994 and 1.1208 are model coefficients.

The root mean squared error (RMSE) was 1.6461 m. In the heartwood height prediction model there is one visible outlier – it is the tree with an unusual characteristic: heartwood height is substantially smaller than the tree height. It was unusual, but we decided to keep the record.

Taper modelling

Compatible taper equations were created and the final version is presented in Equation 8. This equation predicts diameter at any height up the stem.

$$d = \sqrt{\frac{(b_1 z_1 + b_3 z_3 + b_4 z_4 + b_5 z_5) V_c}{0.00007854H}} \quad (8)$$

whe t predicted by the app quation 6.

The essential parts needed to create Equation 8 are z values (z_n) and height ratio (HR) which is the relative height at which the diameter was predicted (Equation 9). The value of HR is in the range from 0 (the small-end diameter) to 1 (the large-end diameter).

$$HR = (H - h)/h \quad (9)$$

TABLE 2: λ values, individual model parameters and statistics from fitting the five compatible volume equations for *Eucalyptus globoidea.*

Component	***λ***	a_0	a_1	a_2	**RMSE (m^3)**	**MAB (m^3)**	**EF**
Heartwood	0.22	-5.1118424	0.2089528	-	0.1248298	0.06782255	0.905
Sapwood	0.38	-2.5616701	0.0673768	-	0.05496807	0.0382446	0.805
Wood inside bark*	0.25	-1.114294*	0.050221*	0.064701*	0.1539625	0.08887412	0.91
Bark	0.12	-4.9560452	0.1991522	-	0.1108201	0.06808297	0.905
Wood incl. bark*	0.19	-0.743484*	0.054786*	-	0.1439786	0.08347402	0.971
dh	0.28	-	-	-	-		
Tree height	0.32	-	-	-	-		

*Wood inside bark and wood including bark values with asterisk (*) were calculated individually rather than by systemfit, and within systemfit they were assigned as the sums of their components in order to ensure compatibility. For the wood inside bark model, transformed tree height was added as an independent variable (coefficient a_2).

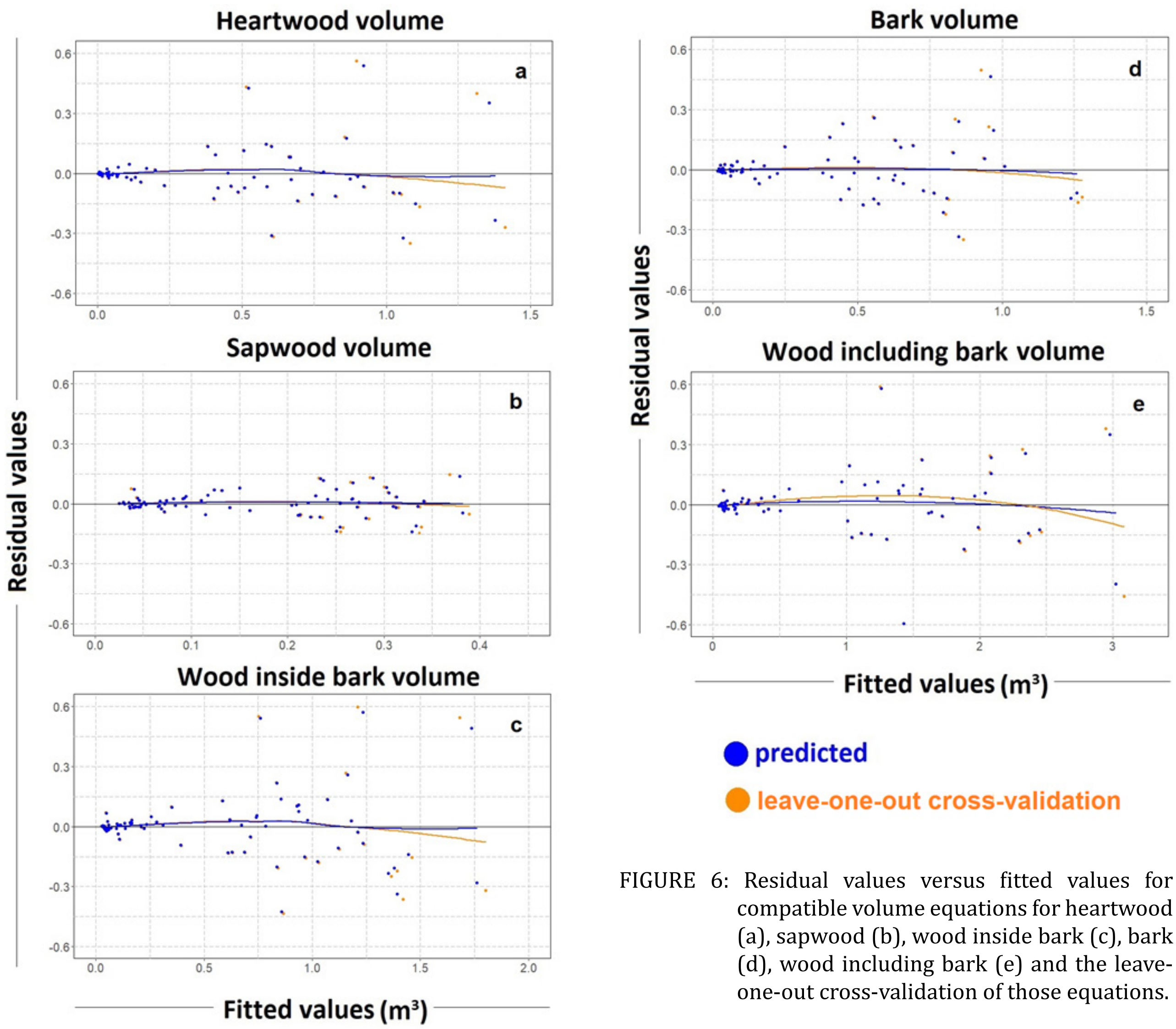


FIGURE 6: Residual values versus fitted values for compatible volume equations for heartwood (a), sapwood (b), wood inside bark (c), bark (d), wood including bark (e) and the leave-one-out cross-validation of those equations.

where, H is height of the tree and h is height of the diameter measurement.

Z values (z_n) are power functions of height ratio (HR) and they represent the different shape along the stem. To calculate z values(z_n) Equation 10 was used. The z values of larger power are responsible for shape starting from the large-end stem, when the z values of smaller power are responsible for shape ending to small-end stem. In the fitting process to ensure the taper and volume compatibility the best results were obtained with z values of power 1, 3, 4 and/or 5 kept and the z value of power 2 excluded.

$$z_n = (n + 1)HR^n \tag{10}$$

where, z values are powers of the height ratio used in multilinear equation, n =1, 3, 4, and/or 5.

Equation 11 was created using Equation 8, it predicts any log length volume and demonstrates compatibility between taper and volume models. This volume equation created from taper equation (integration of the taper equation) yields the same total volume as given by the volume equation (obtained by summation of sections).

$$V_l = V_{c_{h_S}}^{h_L}[b_1HR^2 + b_3HR^4 + b_4HR^5 + b_5HR^6] \tag{11}$$

where, h_s is height to the small-end diameter, h_L is height to the large-end diameter.

Taper function parameters are shown in Table 3 and are all statistically significant. The best taper functions for heartwood included coefficients for 1st, 3rd, 4th and

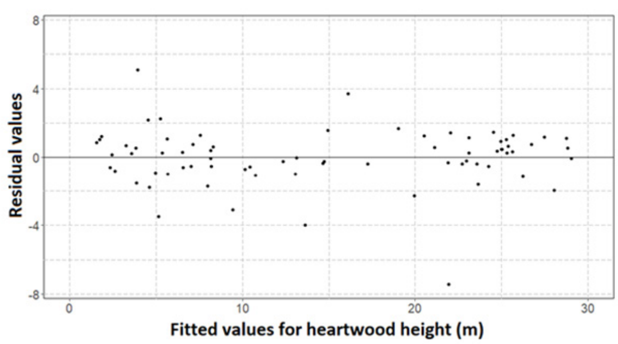


FIGURE 7: Heartwood height prediction residual values versus fitted values.

TABLE 3: Parameter estimates of three compatible taper equations fitted for *Eucalyptus globoidea*.

Component	b_1	b_3	b_4	b_5
Heartwood	0.2419945	4.9174316	-7.9903269	3.830901
Wood inside bark	-	6.416621	-9.975695	4.559074
Wood including bark	-	6.001335	-9.781425	4.780089

5th powers, while for wood inside the bark and wood including bark included coefficients for 3rd, 4th and 5th powers. Residuals for predictions of diameter by the taper function are shown in Figure 8.

Figure 8 presents graphs of residual versus fitted values of the three compatible taper models. In all the graphs we can observe heteroscedastic patterns, which means that variance is increasing; larger predicted values are associated with larger errors of residuals. Smaller diameters had lower residuals compared to larger diameters, as the values of residuals and diameter increased proportionately. The residual range around the x axis was similar for all three components: -11/16 cm for heartwood, -12/15 cm for wood inside bark and -14/16 cm for wood including bark.

The model validation showed that there were differences between leave-one-out cross-validation technique and full model fits, but the ranges and distributions of fitted and validation residuals were similar. Both methods showed normal residuals.

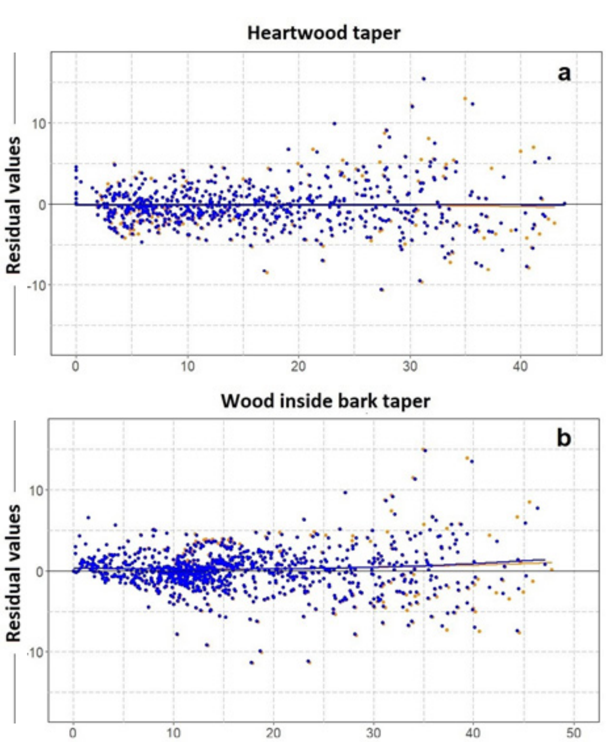
A summary of the fitting statistics for three compatible taper equations is presented in Table 4. The RMSE values varied from 2.348552 cm for wood inside bark to 2.572765 cm for heartwood. The MAB valued varied from 1.585148 cm for wood inside bark to 1.816375 cm for heartwood. The EF valued varied from 0.959 for heartwood to 0.977 for wood inside bark.

Taper and volume interactive tool

The above results from taper and volume modelling were combined and used to create an interactive tool which can graph tree shapes including three components of heartwood, wood inside bark and wood including bark. The tool is in an Micrsoft Excel® spreadsheet available from the authors and in Additional File 1. The user of the tool inputs tree DBH and height. If the inputs are within the range model will show "OK" in the adjacent cell. If inputs are beyond the range of the model then "Extrapolated" will appear. The tool predicts the height of the heartwood and plots the graphs. Examples for three different trees that vary in size are shown in Figure 9. Information about the trees is provided in Table 5 below.

Discussion

In this study a compatible taper and volume equation approach, developed by Demaerschalk (1971, 1972, 1973) was used. Its simple approach allowed integration of many simultaneously harmonised components in one whole system of compatible models. In this study the total volume of heartwood, wood inside bark and wood including bark, obtained by summation of sections whose volumes are defined using taper equation, is almost identical to the volume defined by the volume



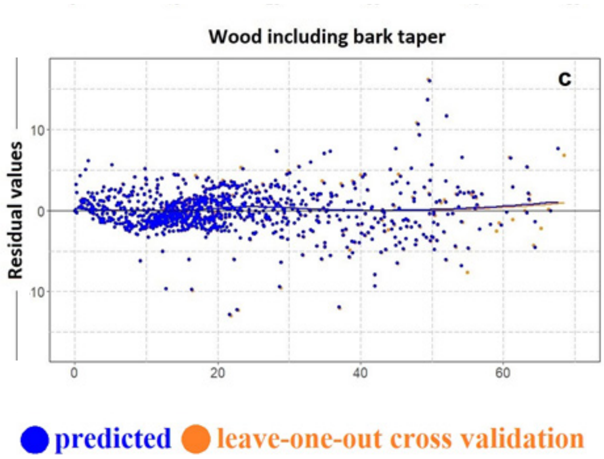


FIGURE 8: Residual values versus fitted values for compatible taper equations for heartwood (a), wood inside bark (b), wood including bark (c) and the leave-one-out cross-validation of those equations.

TABLE 4: Statistics of fit of the three compatible taper equations for *Eucalyptus globoidea.*

Component	RMSE (cm)	MAB (cm)	EF
Heartwood	2.572765	1.816375	0.959
Wood inside bark	2.348552	1.585148	0.977
Wood including bark	2.474088	1.647877	0.972

TABLE 5: Information about trees projected in Figure 9.

Parameter	Tree number		
	1	2	3
DBH (cm)	11	32	62
Tree height (m)	8	19	34
Heartwood height (m)	2.7	15	31.8
Heartwood proportion in wood inside bark (%)	13	57	79
Heartwood volume (m^3)	0.004	0.2	1.62
Wood inside bark volume (m^3)	0.03	0.35	2.05
Wood including bark volume (m^3)	0.05	0.57	3.57

equation. Integration of the taper equation of those components yields the same total volume as given by the volume equation.

Compatible volume equations of heartwood, sapwood, and bark were built using non-linear regression. The compatible volume equations of wood inside bark and wood including bark were built using simple summary of above components (wood inside bark = heartwood + sapwood; wood including bark = wood inside bark + bark). The compatible taper equations of heartwood, wood inside bark and wood including bark were built using non-linear regression.

From a silvicultural and economic point of view, the most valuable part of the log is the lower end, with larger diameters, but residuals are larger for predictions of that part of the tree. This is unavoidable, and the best we can say is that predictions across the entire range of diameters are relatively unbiased.

There are a few publications about *Eucalyptus* species taper and volume equations. Rachid's study compared several taper and volume equations with emphasis on testing the suitability of compatible taper equations (Casnati et al. 2014). Among two species one of them is *Eucalyptus grandis* W.Hill*.* Muhairwe (1999) developed taper equations for *Eucalyptus pilularis* Sm. and *Eucalyptus grandis.* A comparison of statistics of fit of wood including bark volume equations from three studies (the study reported here, Casnati and Muhairwe) provides the following results. The *Eucalyptus grandis* volume equation (Casnati et al. 2014) with its RMSE value of 0.0422 m^3 was comparably more precise than the *Eucalyptus globoidea* model developed herein *(*RMSE values 0.1439786 m^3). The *Eucalyptus globoidea* MAB value was higher than the lowest (0.0245 m^3) reported for *Eucalyptus grandis.* Model efficiency was the best for *Eucalyptus globoidea* based on EF value (0.971). The *Eucalyptus globoidea* taper equation was a less precise model as it had a larger RMSE value (2.474088 cm) compared to Rachid's model (0.8937 cm) (Casnati et al. 2014). The MAB value of 1.647877 cm was also higher than the lowest (0.6747 cm) reported for *Eucalyptus grandis.* Model efficiency was quite similar for *Eucalyptus globoidea* based on EF value (0.972) compared with *Eucalyptus grandis* (0.989).

It is important to underline that the study described here had only 74 *Eucalyptus globoidea* trees (age from

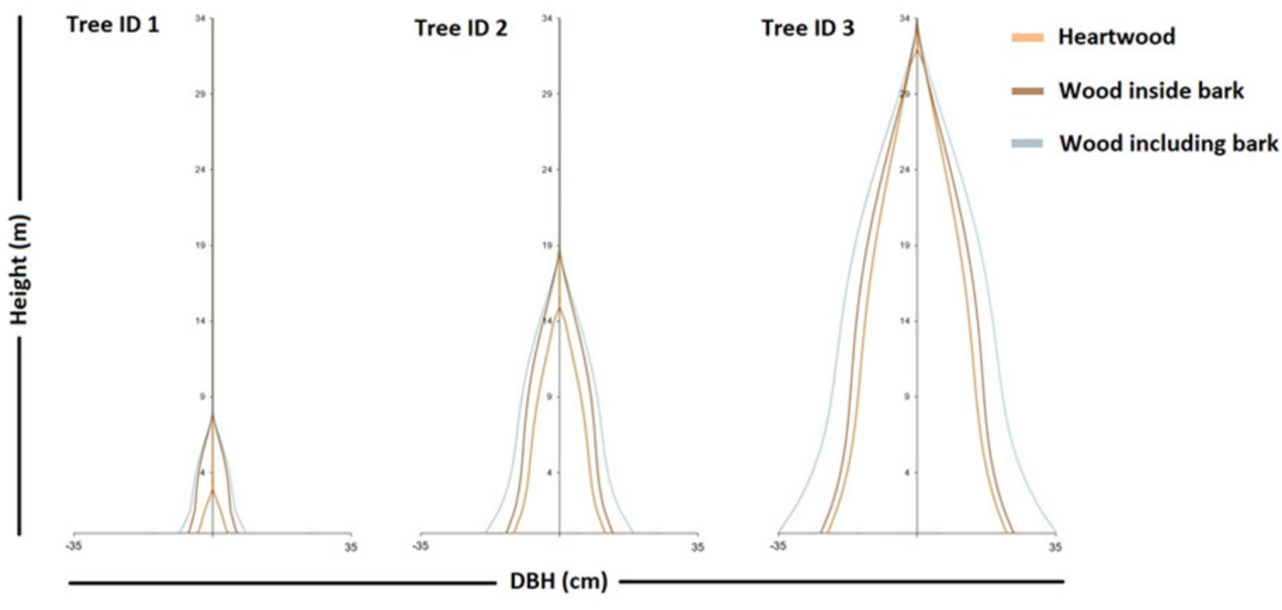


FIGURE 9: Projection of three different trees using tool based on taper and volume models.

7 to 29 years old) compared to 932 *Eucalyptus grandis* trees (age from 2 to 23 years old) (Casnati 2016) or 526 *Eucalyptus piluraris* trees and 645 *Eucalyptus grandis* trees (varied in age; *Eucalyptus piluraris* DBH range 11.2 – 192.4 cm; *Eucalyptus grandis* DBH range 5.9 to 94.3 cm) (Muhairwe 1999).

Another interesting finding was obtained by analysing the predictions of the interactive tool. By increasing DBH and keeping tree height constant at harvest we observed that the proportion of heartwood in wood inside bark increased with DBH. We deduced the silvicultural implication that lower stockings may result in a higher proportions of heartwood. This implication was borne out in a study by Gominho (2005). He studied 27 *Eucalyptus globulus* Labill. trees and found that the proportion of heartwood volume increased with spacing from 20% to 40% of tree volume, respectively for 2 × 1 and 3 × 3 spacings. Another study found that heartwood percentage of *Eucalyptus grandis* was also positively correlated with DBH; however it didn't find this correlation in case of *Eucalyptus grandis* × *Eucalyptus urophylla* S.T.Blake (Brito et al. 2019).

The results of this study are specific to *Eucalyptus globoidea* within ranges of DBHs of 11-67.6 cm, tree heights of 7.2-35.4 m, and volumes of 0.036-4.86m^3. Using the models outside the ranges of fitting data is not recommended as they can predict unrealistic values. The standard error tends to be small for smaller trees and bigger for big trees. The models shouldn't be generalised to other species or beyond the range of tree sizes included in the training data. Future research could be focused to make the models presented herein more specific by incorporating spatial effects and different genotypes of the species. Sites with different water-relations, for instance, may influence retention of sapwood, and sites or genotypes with greater leaf area index may also result in greater retention of sapwood based on the sapwood-pipe theory.

Conclusions

The taper and volume equations presented in this paper were created using simultaneous fitting with the following constraints: the wood inside the bark is the sum of the heartwood volume and sapwood volume; both of these components never exceed the volume of the wood inside the bark; and the wood including the bark is the sum of the heartwood, the sapwood and the bark volumes. These three components never exceed the volume of the wood including bark. These two constraints ensure that the system is compatible and correct. Moreover, taper equations compatible with the volume equations were created for the entire stem, the wood within the stem and the heartwood. These models performed well during leave-one-out cross-validation presenting similar results to those obtained during fitting with the full dataset.

Additional File

NZJFS 52_6 2022 Boczniewicz et al. Interactive tool

Author contributions

JM and EM secured funding for the study. DB and EM initiated the study, and developed the methods and R code required to fit and validate the models. DB collected all the data. DB wrote the draft version of the manuscript, with EM and JM revising the draft resulting in the final manuscript.

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